Performance Analysis of BPSK and QPSK Using Error Correcting Code through AWGN

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***Abstract*–** This paper presents the performance analysis of BPSK and QPSK using error correcting code. To calculate the bit error rate, different types of error correcting code were used through an Additive White Gaussian Noise (AWGN) channel. Three error controls types; Bose- Chaudhuri-Hocquenghem (BCH), Cyclic code and hamming code were used as the encoder/decoder technique. Basically, the performance was determined in term of bit rate error (BER) and signal energy to noise power density ratio (Eb/No). Both BPSK and QPSK were also being compared in the symbol error capability known as t in which expected that the performance is graded in response to the increasing of value of t. The maximum codeword length (n) used in the hamming code is 63 and the message length (k) is 57. For BCH code, the maximum N is 63, K is 36 and error-correction capability, t is 5. For cyclic code, the maximum N is 31, K is 21 and error-correction capability, t is 5. All simulations were done using MATLAB R2007b software. The results show that the best performance occurs when the communication system uses a BCH code with N=31, K=11 and t=5 with BPSK modulator/demodulator. The higher the values of N, K and t, the better the performance of the system using BPSK and QPSK. In general BCH codes are better than Hamming code and Cyclic code for both BPSK and QPSK.

Keywords: Additive White Gaussian Noise (AWGN) channel, Bose-Chaudhuri-Hocquenghem code (BCH), Hamming code, Cyclic code, energy per bit to noise power spectral density ratio (Eb/No), Bit Error Rate (BER), codeword length (N), message length (K), error-correction capability (t).

1. Introduction

1.1 BPSK

BPSK is the simplest form of phase shift keying (PSK). It uses two phases which are separated by 180°, hence is also be termed as 2-PSK. It does not particularly matter exactly where the constellation points are positioned, and in this figure 1 they are shown on the real axis, at 0° and 180°. This modulation is the most robust of all the PSKs since it takes the highest level of noise or distortion to make the demodulator reach an incorrect decision. It is, however, only able to modulate at 1 bit/symbol and so is unsuitable for high data-rate applications when bandwidth is limited [1].

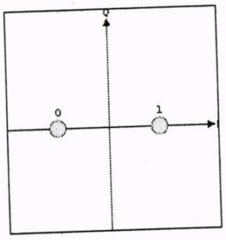


Figure 1: Constellation diagram for BPSK

Implementation

Binary data is often conveyed with the following signals:

S_0 (t) = ($\sqrt{2E_b/T_b}$) Cos ($2\pi fc + \pi$) = - ($\sqrt{2E_b/T_b}$) Cos ($2\pi fc + \pi$) .... (1) (For binary "0")

S_1 (t) = ($\sqrt{2E_b/T_b}$) Cos ($2\pi fc$) .... (2) (For binary "1")

Where f_c is the frequency of the carrier-wave

Bit error rate

The bit error rate (BER) of BPSK in AWGN can be calculated as [3]:

BER = 0.5ERFC [sqrt (Eb/No)] .... (3)

1.2 QPSK

QPSK uses four points on the constellation diagram, equispaced around a circle. With four phases, QPSK can encode two bits per symbol [1]. Analysis shows that this may be used either to double the data rate compared to a BPSK system while maintaining the bandwidth of the signal or to maintain the data-rate of BPSK but halve the bandwidth needed.

As with BPSK, there are phase ambiguity problems at the receiver and differentially encoded QPSK is used more often in practice [1].

Implementation

The implementation of QPSK is more general than that of BPSK and also indicates the implementation of higher-order PSK. Writing the symbols in the constellation diagram in terms of the sine and cosine waves used to transmit them [1]:

S_i (t) = ($\sqrt{2E_b/T_b}$) Cos [$2\pi fc + (2i+\pi)\ \pi/4$] .... (4)

i = 1, 2, 3, 4.
This yields the four phases $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ as needed.
Comparing with for BPSK shows clearly how QPSK can be viewed as two independent BPSK signals. Note that the signal-space points for BPSK do not need to split the symbol (bit) energy over the two carriers in the scheme in the BPSK constellation diagram

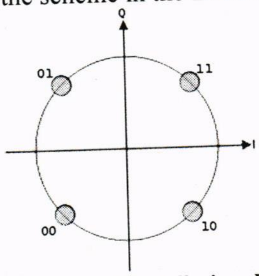


Figure 2: Constellation diagram for QPSK

Bit error rate

Although QPSK can be viewed as a quaternary modulation, it is easier to see it as two independently modulated quadrature carriers. With this interpretation, the even (or odd) bits are used to modulate the in-phase component of the carrier, while the odd (or even) bits are used to modulate the quadrature-phase component of the carrier. BPSK is used on both carriers and they can be independently demodulated [1].
The bit error rate (BER) of QPSK in AWGN can be calculated as [3]:
BER = ERFC [sqrt (Eb/2No)] .... (5)

1.3 AWGN channel

In communications, the additive white Gaussian noise (AWGN) channel model is one in which the only impairment is a linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for the phenomena of fading, frequency selectivity, interference, nonlinearity or dispersion [2]. However, it produces simple and tractable mathematical models which are useful for gaining insight into the underlying behavior of a system before these other phenomena are considered [1].

1.4 Error coding techniques

1.4.1 Hamming Code

The Hamming Encoder block creates a Hamming code with message length K and codeword length N. The number N must have the form 2^M-1, where M is an integer greater than or equal to 3. Hamming codes can detect up to two simultaneous bit errors, and correct single-bit errors; thus, reliable communication is possible when the Hamming distance between the transmitted and received bit patterns is less than or equal to one. By contrast, the simple parity code cannot correct errors, and can only detect an odd number of errors [1].

1.4.2 Cyclic Code

Cyclic code are the subset of the class linear codes of linear codes that satisfy the every codeword $c = (c_1,\ldots,c_n)$, then the word $(c_n, c_1,\ldots,c_{n-1})$, obtained by a cycle shifts of the elements of C, is also code word.

1.4.3 BCH Code

The BCH Encoder block creates a BCH code with message length K and codeword length N. The input must contain exactly K elements. The output is a vector of length N For a given codeword length N, only specific message lengths K are valid for a BCH code. No known analytic formula describes the relationship among the codeword length, message length, and error-correction capability [1].

1.5 Objective

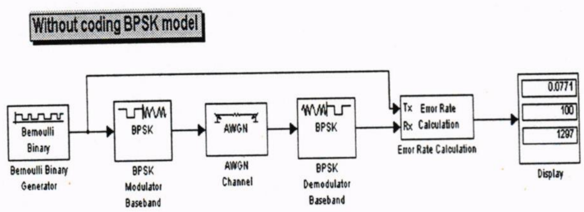
This project has several objectives such as to analyze and simulate the performance of BPSK and QPSK using three types of error coding namely Hamming code, BCH code and Cyclic code. Besides, this project purposely compares both BPSK and QPSK in order to yield the best performances in the term of BER as well as Eb/No. Furthermore, the performance of three types of error coding in term of symbol error correcting capability (t) is also highlighted for both BPSK and QPSK.

2. Methodology

This project begins with the construction of the block diagram of the communication system. This block diagram play an important role in order to make sure this project is satisfied with the communication system

The simulation was divided into four parts: simulation without block codes, simulation with hamming code, simulation with cyclic code and simulation with BCH code using the BPSK/QPSK modulator/demodulator.
During this project, all simulations process representing the performance both BPSK and QPSK is done using software known as Matlab simulink.

2.1 Block Diagram for Simulation of a Communication System without Error Correcting Code.



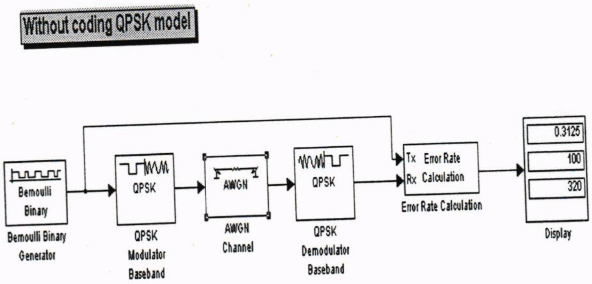


Figure 3: simulation without error correcting codes for BPSK and QPSK

In transmission of information or signal applying in communication system, the process involved can be shown clearly by the communication block diagram system constructed as in Figure 3. According to this block diagram, communication system consists of three stages which are transmission part, communication channel and receiving part. In detail transmission consist several steps which are encode and demodulating. Besides receiving part consist of demodulation process in which is reversed process for modulation and encode process respectively. On the other hand, there are no processes involved in communication channel except it is act as medium where the signal propagates. Noise is injected to the signal in which AWGN channel has been used in this project.

In this simulation, Bernoulli generator is used for sending the data to the BPSK/QPSK modulator. The Bernoulli Binary Generator block generates random binary numbers using a Bernoulli distribution. Then for modulator, it uses the BPSK/QPSK modulator baseband and BPSK/QPSK demodulator baseband. The channel used in this simulation is AWGN channel.

During simulation, the value of (Eb/No) in AWGN channel is varied from 0(dB) to 10(dB). This is to observe the performance of BER as (Eb/No) varies. The Error Rate Calculation block compares the input data from a transmitter with the input data from the receiver. It calculates the error rate as a running statistic, by dividing the total number of unequal pairs of data elements by the total number of input data elements from one source. The display is to show the Bit Error Rate at the end of the simulation. BER of BPSK and QPSK can be calculated theoretically using Equation 3 and 5 respectively [4].

2.2 Block Diagram for Simulation of a Communication System with Error Correcting Code

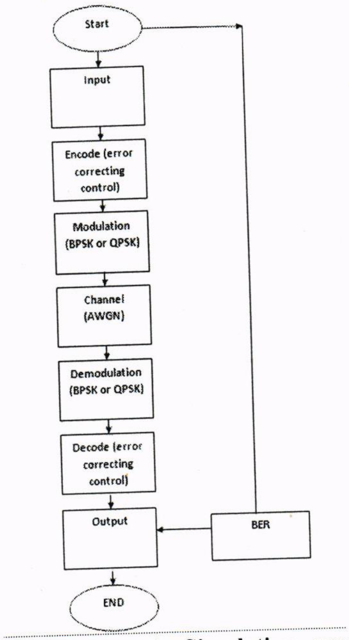


Figure 4: Simulation process of 3 stages Flow Chart.

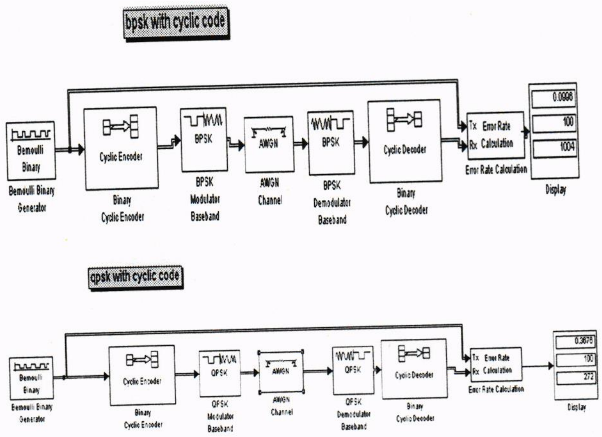
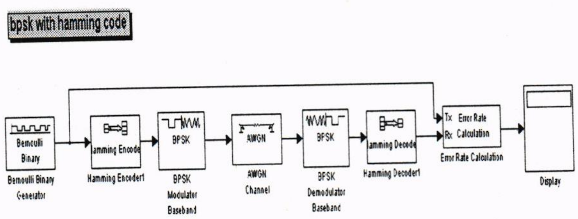


Figure 5: Simulation for BPSK and QPSK Modulator/demodulator with Cyclic code.

The Binary Cyclic Encoder block creates a systematic cyclic code with message length K and codeword length N. The number N must have the form 2M-1, where M is an integer greater than or equal to 3. The input must contain exactly K elements. If it is frame-based, then it must be a column vector. The output is a vector of length N.



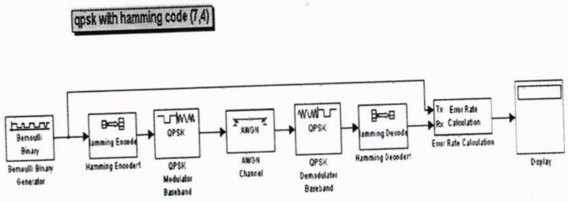


Figure 6: Simulation for BPSK and QPSK Modulator/demodulator with Hamming code.

For this simulation, hamming encoder and hamming decoder were used. The output from the Bernoulli Binary Generator is frame based. The Hamming Encoder block creates a Hamming code with message length K and codeword length N. The number N must have the form 2^M-1, where M is an integer greater than or equal to 3. Then K equals N-M. The value for the sample per frame in Bernoulli Binary Generator must be the same as the value of the massage length (K) in Hamming encoder. The outputs of the hamming encoder are sent to QPSK/BPSK modulator baseband. In QPSK modulator, the phase offset (rad) is set to $\pi/4$ and for BPSK set to 0. For mode in AWGN channel, it changes to ratio of signal energy to noise power spectral density (Es/No).The Equation for (Es/No) is:

$$Es/No = (Eb/No)\ dB + 10\log (K/N) \quad \ldots\ldots (6)$$

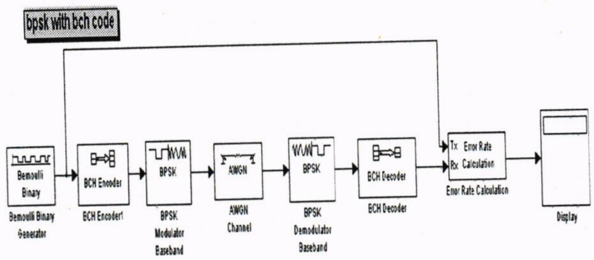
The (Eb/No) varies from 0dB to 10dB. The value of the codeword length (N) increases from 7, 15, and 31 to 64. The Hamming Decoder block recovers a binary message vector from a binary Hamming codeword vector. For proper decoding, the parameter values in this block should match those in the corresponding Hamming Encoder block. Equation is the BER theory calculation for block codes [4].

$$BER = \frac{dmin * Pe}{n} \quad \ldots\ldots\ldots (7)$$

$$Pe = 1 - \sum P(i,n)$$

$$P(i,n) = \frac{n!}{i!\,(n-i)!}\, p^i (1-p)^{n-i}$$

Where dmin = 2t+1, error-correction capability, t.
n = codeword length
Pe = probability of error in block code
P(i,n) = probability of i erroneous symbols in a block of n symbol.



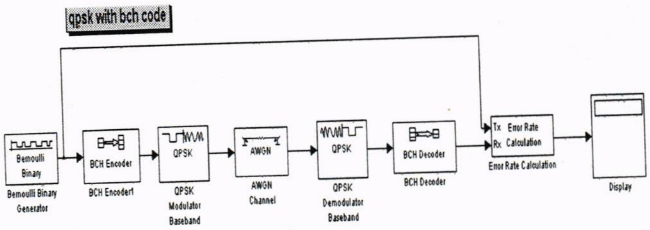


Figure 7: Simulation for BPSK and QPSK Modulator/demodulator with BCH code.

For this simulation, BCH encoder code and BCH decoder code were used. The BCH Encoder block creates a BCH code with message length K and codeword length N. The input must contain exactly K elements. The output is a vector of length N For a given codeword length N, only specific message lengths K are valid for a BCH code. No known analytic formula describes the relationship among the codeword length, message length, and error-correction capability.

The BCH Decoder block recovers a binary message vector from a binary BCH codeword vector. The value of codeword length (N) and message length (K) in the BCH encoder must be the same with the BCH decoder.

3. Results and Discussion

3.1 Performance of a Communication System Of BPSK without Error Correcting Codes.

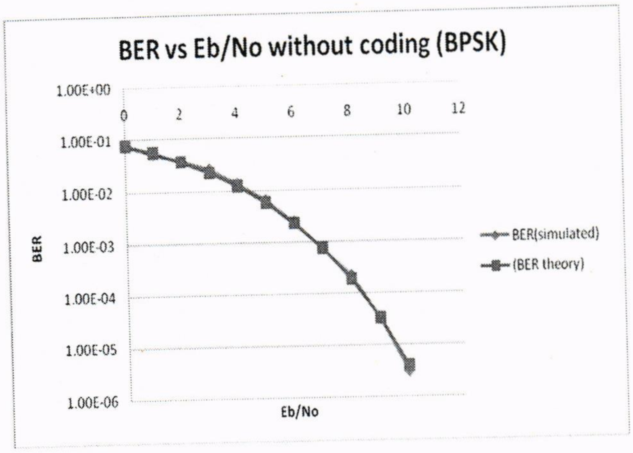


Figure 8 BER performance of BPSK without error correcting codes.

Referring to Figure 8, a comparison between simulated BER and theoretical BER has been made. It shows the BER value as Eb/No varies from 0dB to 10dB. At 0dB, simulated BER is 0.0771 and theoretical BER is 0.07856. There are slight difference and the values is 0.00155. At 10dB, the simulated BER is 3.00E-06 and the theoretical BER is 3.87E-06. The difference is 8.72E-07. Since the difference is very small, it shows that, the value of the theoretical BER is similar to the simulated BER.

3.2 Performance of a Communication System (using QPSK) without Error Correcting Codes.

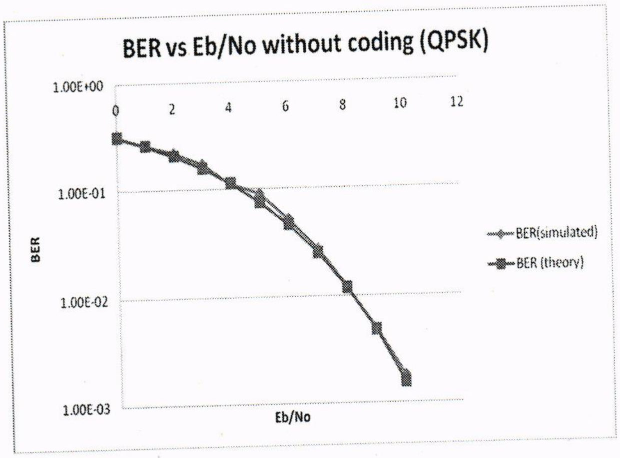


Figure 9 : The BER performance of QPSK without error correcting codes

At 0dB, the simulated BER is 0.3125 and theoretical BER 0.317310813 . The difference between simulated BER and the theoretical BER is 0.004811. At 10dB, the simulated BER is 0.001805 and the theoretical BER is 0.001565402. The difference between the simulated BER and the theoretical BER is very small which is 0.00024. As a conclusion, theoretical and simulations values are almost the same.

3.3 Performance of communication system of BPSK without and with error correcting codes using Hamming code

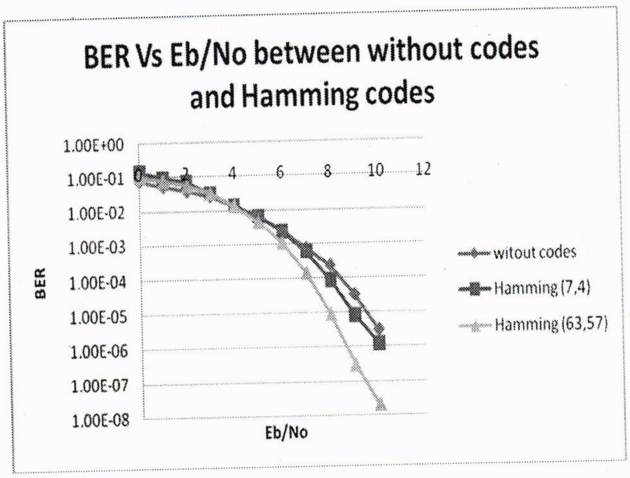


Figure 10 BER vs (Eb/No) of a communication System (using BPSK) without error correcting codes, Hamming code (7,4) and hamming code(63,57)

From the graph of Figure 10 above, the results show that BPSK using Hamming code (63, 57) has the lowest BER compared to others. BER for BPSK using Hamming (63, 57) is 0.1042 at 0dB and 2.00E-08 at 10dB. Thus it can be concluded that BPSK using hamming (63, 57) performs better than others.

3.4 Performance of communication system of QPSK without and with error correcting codes using Hamming code.

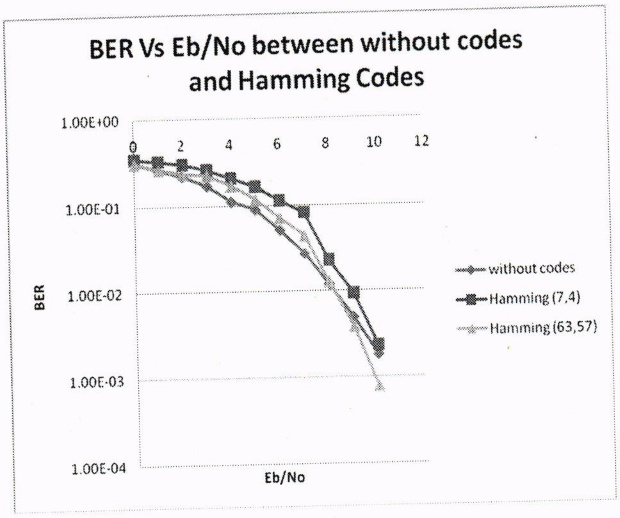


Figure 11 BER vs (Eb/No) of a communication System (using QPSK) without error correcting codes, Hamming code (7,4) and hamming code(63,57)

From the graph of Figure 11, the performance of Hamming (7,4), Hamming (63, 57) and without codes are observed in term of BER. At 0dB, Hamming (7, 4) is the highest at this point other than the rest. At some point when Eb/No reach 8dB, the value of Hamming (63, 57) and without code is almost the same. Those values are 0.003848 and 0.004934 respectively. At 10dB, the value of BER of Hammings (63, 57) state the smallest than Hamming (7, 4) and without code. It can be concluded that Hamming (63, 57) gives better BER.

3.5 Performance of communication system of BPSK without and with error correcting codes using BCH code.

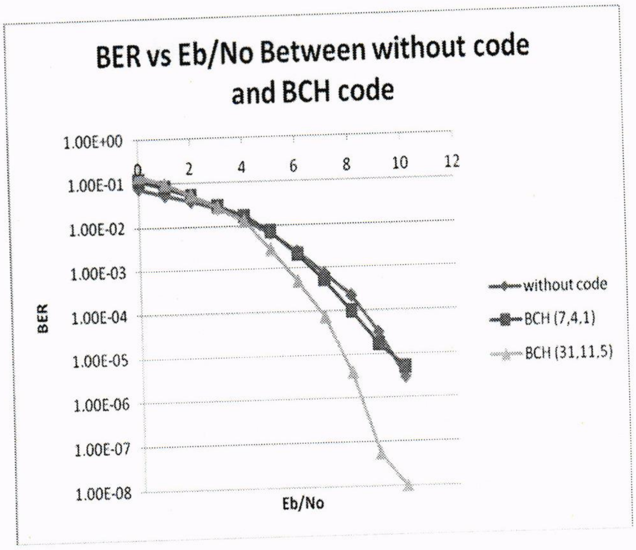


Figure 12 BER vs (Eb/No) of a communication System (using BPSK) without error correcting codes, BCH code (7,4,1) and BCH code(31,11,5)

From the graph of Figure 12, at 0 dB, the highest value is 0.1481using BCH (31,11,5) coding, followed by BCH (7,4,1) and without coding. Then, at 10dB the lowest BER is 0 using BCH (31,11,5) while the BER values of BCH (7,4,1) and without coding at 10dB are 0.000005 and 3.00E-06 respectively. Thus, it can be concluded that BCH (31,11,5) has better performance than BCH (7,4,1) and without code. It can be said that the higher error correction capability, t will produce the better performance in communication system.

3.6 Performance of communication system of QPSK without and with error correcting codes using BCH code

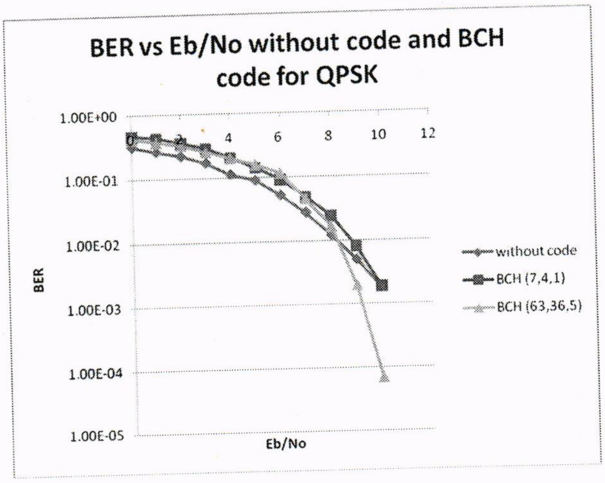


Figure 13 BER vs (Eb/No) of a communication System (using QPSK) without error correcting codes, BCH code (7, 4, 1) and BCH code (63, 36, and 5)

At 0dB, the highest BER value is BCH (7,4,1) with 0.463 and the lowest BER is without code which is 0.3125. At 10dB, the simulated BER of BCH (63,36,5) is lowest compared to the BER of BCH(7,4,1) and without code. The value BER of BCH (63,36,5) at 10dB is 7.00E-05 and value of BCH(7,4,1) and without code are similar which is 0.00189 and 0.001805. Once again it shows when the higher value of t, the better performance of BER.

3.7 Performance of communication system of BPSK without and with error correcting codes using cyclic code.

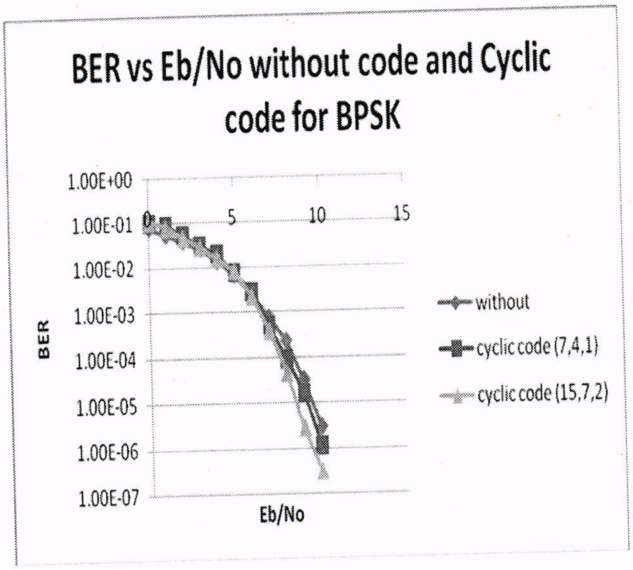


Figure 14 BER vs (Eb/No) of a communication System (using BPSK) without error correcting codes, cyclic code (7, 4, 1) and cyclic code (15, 7, 2)

Referring to Figure 14, BPSK also has been compared for various value of symbol error correcting capability, t in term of BER. Here, it can be seen that the BER of Eb/No from 0dB to 6dB is almost the same. Similarly with value of t is being compared for t equal to 1 and t equal to 2. The fact that the higher value of t will cause the better performance in term of BER that has been achieved for Cyclic code (15,7,2) with 3.00E-07 at 10dB while 0.1091 at 0dB.

3.8 Performance of communication system of QPSK without and with error correcting codes using cyclic code

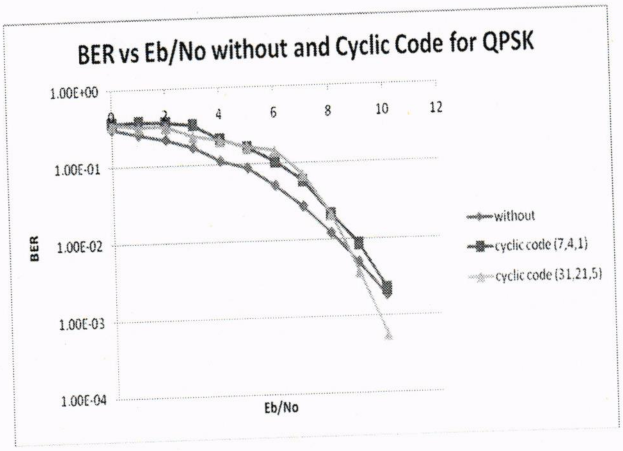


Figure 15 BER vs (Eb/No) of a communication System (using QPSK) without error correcting codes, cyclic code (7, 4, 1) and cyclic code (31, 21, 5)

In response to this graph, it can be summerized that the smallest value of BER is when using Cyclic code (31, 21, 5).

3.9 Performance Comparison between Communication System with BCH Code, cyclic code and with Hamming Code Using BPSK modulator/demodulator and QPSK modulator/demodulator.

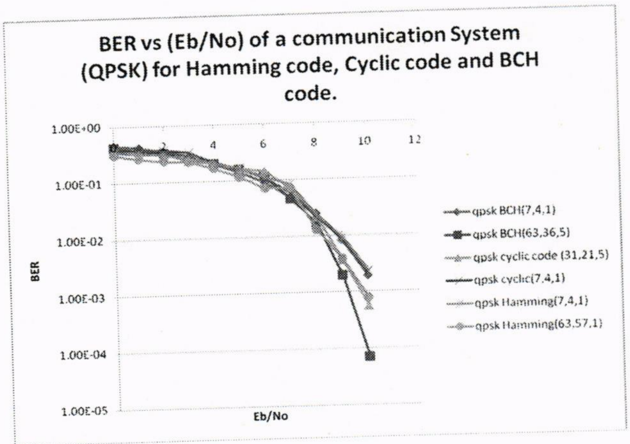


Figure 16 BER vs (Eb/No) of a communication System (QPSK) for Hamming code, Cyclic code and BCH code.

The results show that the best performance occurs when the communication system uses a BCH code with N=63, K=36 and t=5 with QPSK modulator/demodulator.

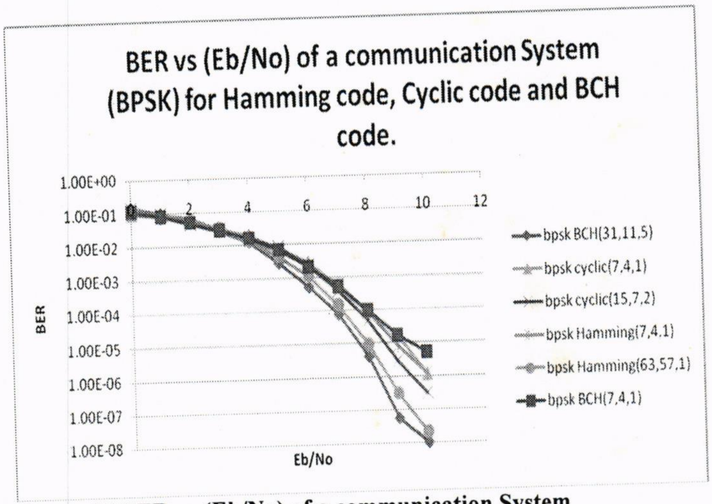


Figure 17 BER vs (Eb/No) of a communication System (BPSK) for Hamming code, Cyclic code and BCH code

The results show that the best performance occurs when the communication system uses a BCH code with N=31, K=11 and t=5 with BPSK modulator/demodulator.

In general, the BCH codes are better than Hamming code and Cyclic code. This is because Hamming codes and Cyclic code are capable of detecting and correcting single errors only whereas BCH codes are capable of detecting and correcting multiple errors.

4. Conclusion

The simulation shows that the performance of QPSK and BPSK is dependent on several factors. The symbol error correcting, t, codeword length, N and message length, K. Based on the result obtained, it can be concluded that the best performance is graded when the value of t is increased. The best performance occur when communication system use a BCH code with N=31, K=11 and t=5 with BPSK modulator/demodulator. The higher of value of N, K and t is better the performance and in general BCH codes are better than Hamming code and cyclic code Hence, the objective of this project is successfully achieved in which this project success to analyze and simulates the performance of BPSK and QPSK using different types of error correcting codes through AWGN channel.

5. Recommendation

In future, this project can be improved in term of the performance of higher order of PSK when using error correcting codes. It also can use Reed Solomon as error coding. This is because the Reed Solomon particularly use for burst error correction and if there is error has not been correct, Reed Solomon unable to repeat the transmission.

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