UNIVERSITI TEKNOLOGI MARA

A GLOBALLY CONVERGENT HYBRID CONJUGATE GRADIENT METHOD FOR SOLVING UNCONSTRAINED OPTIMIZATION PROBLEMS

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ABSTRACT

The Conjugate Gradient (CG) method is utilized among researchers in solving optimization problems. It possesses characteristics that distinguish the steepest descent and Newton’s method. For example, it has a faster convergence rate than steepest descent method and a lesser computational cost than Newton’s method. In 2016, the Aini-Rivaie-Mustafa (ARM) method, a modified CG method, was introduced and presented a good performance compared to the previous CG method that it was tested with. However, sometimes the ARM method generates a negative CG coefficient value. Therefore, this paper intends to propose a new hybrid CG method for solving an unconstrained optimization problem, where the main focus of this study is to improve the ARM method. It is combined with another CG method to solve this problem that always generates a positive CG coefficient value. The proposed method demonstrates that it could globally converge towards the minimizer and possessed sufficient descent conditions under a strong Wolfe line search. Besides that, the numerical observation was made by testing the method with 20 standard test functions that vary in shape. The testing was also made on four different variables on three different initial points, ranging from close to far from the solution point. Moreover, the purpose is to test the method's efficiency and reliability in solving different types of functions with various ranges and numbers of variables. The testing was made using MATLAB R2013A, and the results were recorded and compared employing the performance profile introduced by Dolan and More (2002). The result indicates that it could outperform both original methods regarding the Central Processing Unit (CPU) time and the number of problems solved where the proposed method (A-ARM method) could solve 100% of the test functions surpassing both of its original methods.

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CHAPTER ONE
INTRODUCTION OF THE RESEARCH

1.1 Introduction

In this world, many problems involve decision-making, and part of this process involves solving optimization problems. Optimization is associated with decision-making as it helps to find the best solution to a problem. It could be modeled into a mathematical model to provide a solution that includes finding either the maximum or minimum of the function. Note that the calculation process starts with the input values selection of the function that will initiate the calculation for the corresponding values. These problems could then be applied in the business field, where they might minimize the loss or maximize the profit. Several methods could aid in providing a solution to optimization problems. However, improvement regarding the efficiency of these methods could still be made. Henceforth, this research aims to improve one of the methods for solving the optimization problem.

1.2 Research Background

Optimization commonly occurs during decision-making in which the problem is presented mathematically. It is considered the preference to solve a problem in the best way possible, which led to optimization being popularized in nearly every application area. This process aims to attain the best solution for a function by tackling its minimum and maximum value. Optimization can be applied in real situations from many different domains, such as engineering, chemistry, biology, and aeronautics. For example, in the aeronautics field, optimization will ensure the aircraft is manufactured properly to meet the standards specified, averting serious disasters during flight. Meanwhile, in the chemistry and biology fields, optimization is employed for the formulation and processing of creating, for instance, any medicine or pathogen inactivation by optimizing its format. Hence, optimization is an essential technique that can be applied in many disciplines.

There are two types of optimization: constrained and unconstrained optimization. These two types of optimization differ by the condition of the variables