Mathematical Sciences and Informatics Journal
Vol. 5, No. 2, Nov. 2024, pp. 95-104
http://www.mijuitmjournal.com DOI : 10.24191/mij.v5i2.1199

Comparative Analysis of Euler and Runge-Kutta Fehlberg Methods in Solving the Lotka-Volterra Competitive Model

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Article Info

Article history:

Received Apr 20, 2024
Revised Aug 21, 2024
Accepted Sept 18, 2024

Keywords:

Lotka-Volterra
Numerical Solutions
Dynamical Behavior
Carrying Capacity
Runge-Kutta Fehlberg

ABSTRACT

This study addresses the problem of determining equilibrium and stability in ecosystems where species compete for the same resources, as outlined by the principle of competitive exclusion. It is often challenging to ascertain the rate at which exclusion happens and whether species can coexist in competitive environments. To discover these issues, the study investigates the competitive interactions between lions (Panthera Leo) and leopards (Panthera pardus) in the Sabi Sand Game Reserve, South Africa, focusing on whether the dominant competitor, lions, limit the population and distribution of leopards. Using the Lotka-Volterra Competitive model, the research compares numerical solutions obtained through Euler and Runge-Kutta Fehlberg (RKF) methods. It also examines how carrying capacity and initial conditions influence equilibrium and stability in this competition. Data on dietary overlaps between lions and leopards were used to test their competitive dynamics, with lions targeting larger prey and leopards focusing on smaller prey. The findings indicate that the RKF method provides more accurate approximations than Euler's method, with leopards showing a higher carrying capacity and greater resilience in the face of competition from lions. This indicates that leopard populations are less affected by lion presence. The study emphasizes the role of carrying capacity in species survival during competition and highlights the utility of numerical methods for predicting competition outcomes without extended experimentation. These findings contribute to wildlife management strategies, particularly in efforts to restore large carnivores in ecosystems, and improve understanding of competitive exclusion in ecological systems.

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1. Introduction

The Lotka-Volterra equation is used to interpret population dynamic in which two organisms interact in one of two ways, either competing for shared resources or associated with a prey-predator system. This study focuses on competitive interaction between two different species solved by Competitive Lotka-Volterra model. Species interactions can be classified into two distinct groups whether interspecific interactions or intraspecific interactions. Interspecific interactions are interactions between two species while intraspecific refers to the interactions between two individuals of the same species. The Competitive Lotka-Volterra equation is a simple model used to solve population dynamics of species competing for shared resources. Competition can be defined as a direct or indirect interaction of organisms that leads to changes in fitness when organisms share the same resources [1].

If natural resources are insufficient to sustain both populations, population growth and survival may be impacted [2]. Consequently, an ecosystem's population may be wiped out. There are equations that are often very difficult to solve analytically. Numerical methods can be used to derive approximate solutions to process models for which analytical solutions are unavailable [3]. Therefore, this study systematically analyzes and compares the exact solution of the numerical solutions obtained by the Euler and Runge-Kutta Fehlberg (RKF) methods on the Lotka-Volterra Competitive model. These two techniques will be compared to determine which method solves this model most effectively.

2. Literature Review

The Lotka-Volterra competition model is a set of mathematical equations used to describe the dynamics of two species that are competing for the same limited resources in an ecological community. These equations describe how the populations of the two species change over time in response to their interactions. According to Seytov *et al.* [4], Lemos-Silva *et al.* [5], Akjouj *et al.* [6], the Lotka-Volterra dynamical system is a set of two autonomous and nonlinear differential equations that describe the interaction between two species - the prey and the predator - and how it influences the growth of both populations. The system is important for studying population dynamics because it provides a mathematical model for understanding the dynamics of prey-predator relationships in an ecosystem.

In this study, two numerical methods, the Euler Method and the Runge-Kutta-Fehlberg (RKF) Method, are applied to solve the model. A numerical solution approximates the solution of a mathematical equation. It is often used where analytical solutions are hard or impossible to get. Euler's method is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is a basic explicit method for numerical integration of ODEs [7]. It is the first numerical method for solving initial value problem (IVP) and serves to illustrate the concepts involved in the advanced methods. The RKF method is an algorithm in numerical analysis for solving ODEs with adaptive step sizes to balance accuracy. The RKF method is derived from the calculation of two Runge-Kutta (RK) methods of a different order, where subtracting the results from each other can obtain an estimate of the error [8], [9].

Recently, many researchers applied numerical experiments using different schemes to explore competition of dynamics and coexistence among biological populations [10-12]. Razali *et al.* [13] employed various numerical methods, such as Euler, Taylor Series, and Runge-Kutta (RK) methods, to understand the effect of interspecific competition, and found that the RK method provided the most accurate approximation of the orbit's behaviour. Paul *et al.* [14] conducted a comparison between the RKF method and the Laplace Adomian Decomposition Method (LADM) applied to the Lotka-Volterra model. The results indicated that the RKF method exhibited higher accuracy and reliability in solving differential equation models related to population dynamics.

Previous research by Chandra *et al.* [15], Han [16] has used the methodology known as Heun's and Euler's method, which incrementally approximates the solution to two differential equations using first-order derivatives starting from initial conditions. The increments used are small to ensure high accuracy. Euler's method is a first-order numerical procedure for approximating differential equations given an initial value.

Paul *et al.* [8] stated that RKF method could be used as a numerical approximation in a wide range of deterministic and stochastic, linear and nonlinear problems in physics, biology, and chemistry [17] conducted a numerical experiment to illustrate the competitive dynamics between two species using an invasive difference scheme and found that strong competition between two populations with low growth rates and large diffusion coefficients would lead to the extinction of the weaker population. studied the asymptotic behavior, and the number of coexistence equilibria is shown by a saddle-node bifurcation of the level of resource under conditions on competitive effects relative to the associated growth rate per unit of resource. Analysis in this research proved that the key factor of the competition outcome is the relation between intraspecies and interspecies interference effects and sometimes the resource level importance. Extinction in the Lotka-Volterra model has been studied to show the extinction time scale has a power law which depends on the population size. Previous research considers two herbivorous species, rabbits and sheep, competing over a limited amount of food supply [18].

This research's main objective is to assess and compare the effectiveness of two numerical methods, the Euler and RKF methods, in solving the Lotka-Volterra competitive model. The research focuses on the application of these numerical techniques to analyze mathematical models like the Lotka-Volterra competitive model. To conduct this analysis, the researchers will draw upon data from a study that explored competitive interactions between two prominent top predators, namely lions *(Panthera leo)* and leopards *(Panthera pardus)* observed in the Sabi Sand Game Reserve, South Africa, spanning the years 2010 to 2015 [19]. The emphasis will be on determining the species' stability and equilibrium, and it will also determine whether the species can achieve a stable equilibrium or undergo competitive exclusion.

3. Methodology

This study used data from Manaf *et al.* [9] to examine competitive interactions between two top predators, lions *(Panthera Leo)* and leopards *(Panthera Pardus).* Nutritional data obtained concurrently on the two species were used to determine if lions, as the dominant competitor, would limit the distribution and abundance of leopards. The initial populations of both predator species were 254 lions and 355 leopards, and the population growth was observed based on monthly counts of individual lions and leopards known to be alive in the study area. The Euler and Runge-Kutta-Fehlberg (RKF) methods will be applied for solving the Lotka-Volterra Competitive model and subsequently results will be compared with the exact solutions.

3.1 Exact Solution

Growth of population with logistic equation from Lotka-Volterra model are given by:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) \tag{1}$$

where $x(t)$ is the mean density (in individuals) at time t (in months), r is the instantaneous rate of increase (birth/deaths), and K is the carrying capacity. Assume constant K and r, linear density dependence, no time lags, no migration, no age structure, and limited resources. This equation is to determine K and r values with simple iterative process for lions and leopards.

The solution for the initial condition can be determined by solving equation (1), which gives:

$$x = \frac{K}{1 + \frac{1}{69} e^{(-rt)} (K - 69)} \tag{2}$$

to utilize equation (2), values for K and r can be determined through a process of curve fitting by referring Figure 1 and Figure 2. The fit is reasonable for:

Species 1 is lions *(Panthera Leo)* where $K = 85.4648$ and $r = 0.2168$:

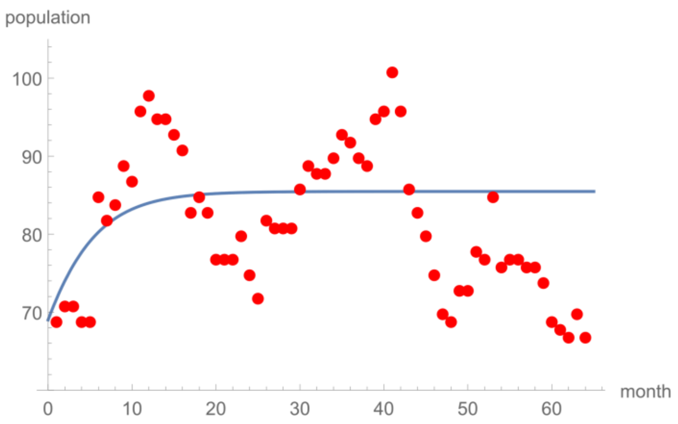

Figure 1. Curve Fitting for growth of lions

Species 2 is leopards *(Panthera Pardus)* where $K = 88.4648$ and $r = 0.0177$:

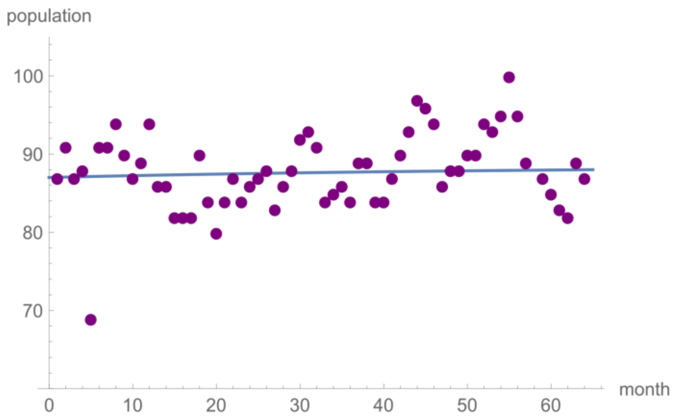

Figure 2. Curve fitting for growth of leopards

By analyzing Figures 1 and 2, values of r and K for both species can be obtained through the application of the best fit of the fitting curve. This process is facilitated using Mathematica 13.2 software. The values of r and K will then be utilized to compute the exact solution using Equation (2). Subsequently, these determined values will be inserted into Equation (1) to implement the numerical methods, namely the Euler and RKF methods.

3.2 Euler Method

Euler method is the simplest one-step method which uses the concept of local linearity to join multiple small line segments so that they make up an approximation of the actual curve of y versus t. Euler formula can be written as:

$$y_1 = y_0 + hf(x_0, y_0) \quad (3)$$

with the initial values are $x_0 = 0$ and $y_0 = 0$. The step size is $h = 1$ as it represents observation time. Data are taken in an average per month, every month through almost 6 years from 0 until 64 months.

3.3 Runge-Kutta-Fehlberg (RKF) Method

RKF method is a one-step algorithm method with an adaptive step size which automatically organizes the step size as a recompositing to the calculation truncation errors [5].

Consider the following system of i^{th} equation with the initial value problem:

$$y'(t) = f_i(t, y_1...., y_n); \\ y_i(t_0) = y_0 \tag{4}$$

where i = 1, 2…n.

The equation solves the initial value problem using RK methods of order 4 and order 5. By the first definition:

$$\begin{aligned} k_1 &= hf_i(t, y_i), \\ k_2 &= hf_i(t + \frac{1}{4}h, y_i + \frac{1}{4}k_1), \\ k_3 &= hf_i(t_i + \frac{3}{8}h, y_i + \frac{3}{32}k_1 + \frac{9}{32}k_2), \\ k_4 &= hf_i(t_i + \frac{12}{13}h, y_i + \frac{1932}{2197}k_1 + \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3), \\ k_5 &= hf_i(t_i + h, y_i + \frac{439}{216}k_1 + 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4), \\ k_6 &= hf_i(t_i + h, y_i - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5), \end{aligned} \tag{5}$$

where h is step size, t_i is time, and $k_1, k_2, ..., k_6$ is every step of accuracy test.

Then, an approximation to the solution of initial value problem is made using RK method of order 4:

$$y_{i+1} = y_i + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 - \frac{1}{5}k_5 \tag{6}$$

A better value for the solution is determined using RK method of order 5:

$$z_{i+1} = y_i + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6 \tag{7}$$

Calculation is repeated by using those values.

3.4 Lotka-Volterra Competitive Model

The Lotka-Volterra competitive model is a set of equations that defines how two species interact in a competitive environment. Research from Manaf [9] stated that the Lotka-Volterra model of interspecific competition is based on two other models of population growth, namely the exponential growth and logistic sigmoid growth models.

$$\begin{aligned} \frac{dN_1}{dt} &= r_1 N_1(t)\left[\frac{k_1 - N_1(t) - \beta_{12}N_2(t)}{k_1}\right] \\ \frac{dN_2}{dt} &= r_2 N_2(t)\left[\frac{k_2 - N_2(t) - \beta_{21}N_1(t)}{k_2}\right] \end{aligned} \tag{8}$$

where, N represents the population density of species for lions and leopards. The term r represents the instantaneous rate of increase of both species and K represents the carrying capacity of the species. The parameter β_{21} represents the per capita effect of lions on the population growth of leopards' species and β_{12} represents the per capita effect of leopards on the population growth of lions. Equation (8) must be solved numerically using the Mathematica 13.2 software because an exact analytical solution would take a long time to obtain.

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4. Results and Discussion

Logistic equations for the population of lions and leopards tested using numerical approximation methods (Euler and RKF methods) are shown as below. Based on Figures 3 and 4, the graph shows that RKF method values are more accurate than Euler method values because the RKF curve is closer to the exact solution curve. For solving the Lotka-Volterra competitive model, it was found that the RKF method is more reliable than the Euler method and this is applicable to both species.

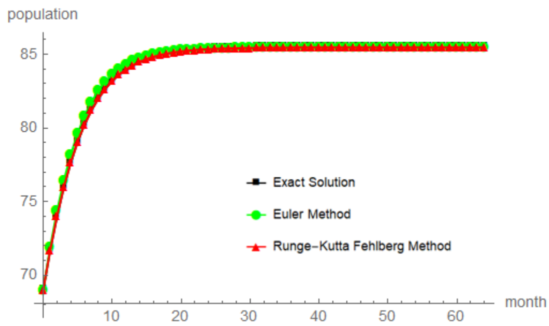

Figure 3. Numerical approximation graphs for lions' logistic curve

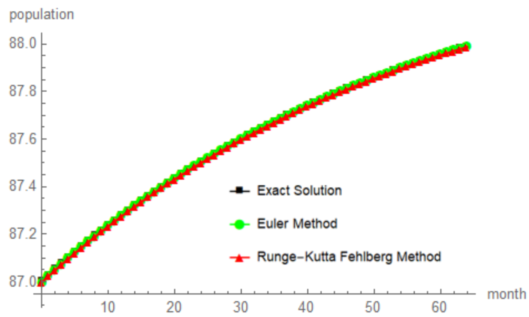

Figure 4. Numerical approximation graphs for leopards' logistic curve

Table 1. The Mean Absolute Error (MAE) for lion and leopard

Method	**Euler**	**Runge-Kutta Fehlberg**
Lion	0.1148731077	0.0001373077
Leopard	0.0022830769	0.0019056154

Table 1 presents a comparison of the Mean Absolute Error (MAE) values for the Euler method and the Runge-Kutta Fehlberg method for these two species. These findings suggest that the Runge-Kutta Fehlberg method is generally more precise than the Euler method.

Figures 5 and 6 show a close-up of the comparison between the exact solution and Euler and RKF methods to compare the accuracy among these two numerical methods. Via the equilibrium and stability tests, this study unveiled four possible cases in the competition dynamics. Case I, species one winning; case II, species two winning; case III, the existence of an unstable equilibrium; and case IV, the coexistence of both species.

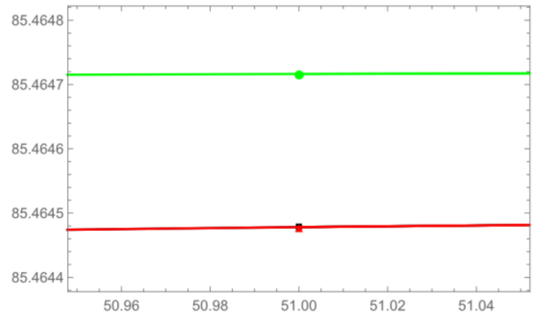

Figure 5. Comparison of numerical approximations for lions

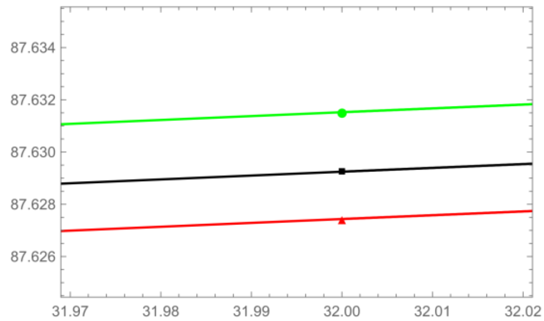

Figure 6. Comparison of numerical approximations for leopards

Case I: $\frac{K_2}{\beta_{21}} < K_1$ and $\frac{K_1}{\beta_{12}} > K_2$, and for Case II is $\frac{K_1}{\beta_{12}} < K_2$ and $\frac{K_1}{\beta_{12}} < K_2$.

Figure 7 below shows that the lions' isocline is above and to the right of the leopards. As a result, the leopards are driven to extinction, and the lions' population increase until they reach it carrying capacity (K_1). This stable equilibrium signifies that the lions consistently outcompete the leopards, resulting in the competitive exclusion of the leopards by the lions. While Figure 8 shows that the isocline of leopards is above and to the right of the isocline of lions. Leopards always outcompete lions in this situation, and lions are competitively excluded by leopards.

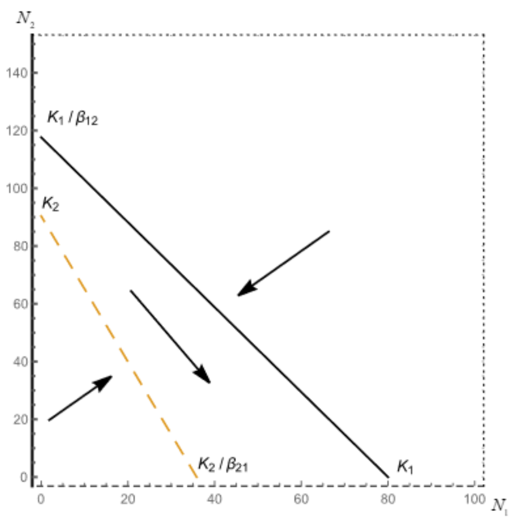

Figure 7. Case I (Species 1 wins)

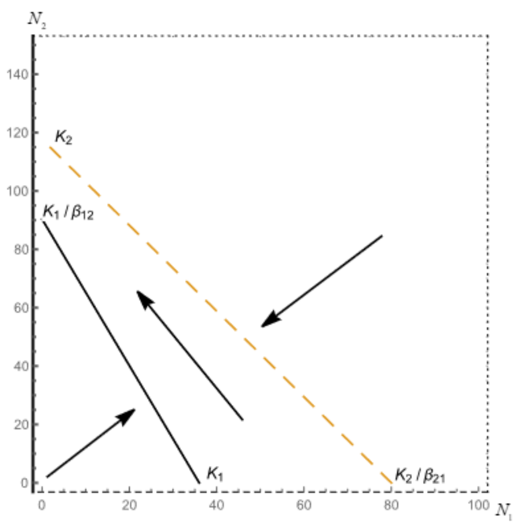

Figure 8. Case II (Species 2 wins)

Case III: $\frac{K_2}{\beta_{21}} < K_1$ and $\frac{K_1}{\beta_{12}} < K_2$, while Case IV: $(\frac{K_2}{\beta_{21}}) > K_1$ and $(\frac{K_1}{\beta_{12}}) > K_2$.

Figure 9 shows the isoclines of both species cross one another. The carrying capacity of lions (K_1) is higher than the carrying capacity of leopards divided by the competition coefficient $(\frac{K_2}{\beta_{21}})$. Besides, the carrying capacity of leopards (K_2) is higher than the carrying capacity of lions divided by the competition coefficient $\left(\frac{K_1}{\beta_{12}}\right)$. For Region 2, the point above the dashed line and below the solid line represents the occurrence of competitive exclusion of leopards by lions. While the point above the solid line and below the dashed line in region 1 shows the occurrence of competitive exclusion of lions by leopards.

Figure 10 shows that the isoclines representing the two species, leopards and lions, cross each other. In this case, both species' carrying capacities are lower than the other's carrying capacity divided by the competition coefficient. The species can coexist at a stable equilibrium point, which is the intersection point between both isoclines. At this point, the balance between the two species is achieved because intraspecific competition (competition within the same species) which is stronger than interspecific competition (competition between different species).

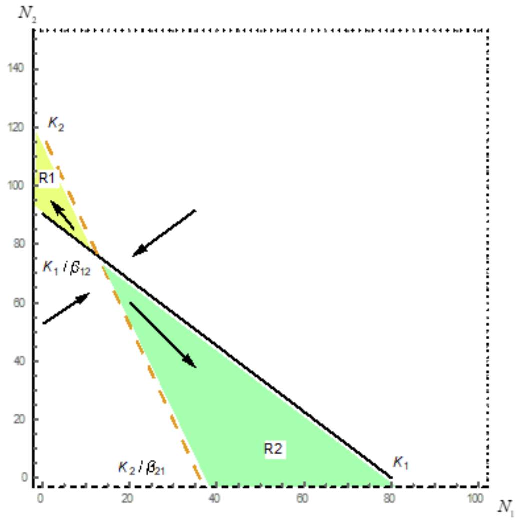

Figure 9. Case III (Unstable equilibrium)

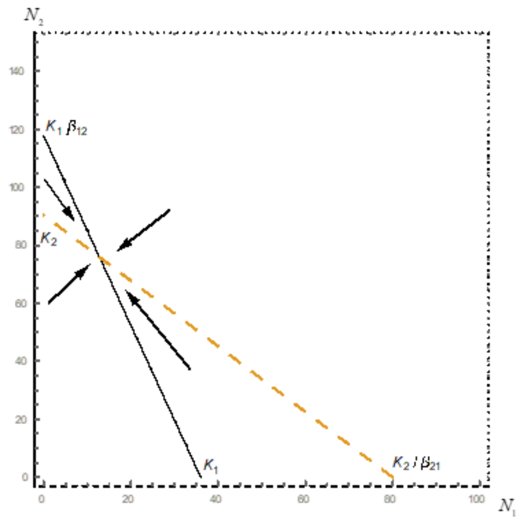

Figure 10. Case IV (Coexistence of both species)

5. Conclusion

In conclusion, this study highlights the efficacy of the Euler and RKF methods for solving the Lotka-Volterra Competitive model. It provides valuable insights into the dynamics of species competition. The results showed that the RKF method provided a more accurate approximation of the numerical methods. The findings also indicated that the study of numerical approaches could serve as reliable prediction tools to eliminate the need for extended observation durations. The estimation of carrying capacity revealed the importance of resource availability in species competition. This suggests that a larger carrying capacity corresponds to a greater ability for a species to thrive and survive in competition. The findings underscored the importance of carrying capacity and initial conditions in understanding equilibrium and stability in competitive interactions. Equilibrium and stability of competition interactions were influenced by the initial conditions and population sizes of the competing species. This study contributes to population dynamics and offers practical implications for wildlife management and conservation efforts. For future research, it is recommended to include more species and consider various other obstacles in species competition. It is suggested to investigate environmental issues, such as the consequences of climate change and how it will affect predator-prey or competitive relationships. Future studies can also examine the performance of numerical approaches while dealing with more complicated interactions and higher-dimensional systems. Aside from that, additional constraints can be included in the model such as spatial barriers or migration patterns. This can provide a more accurate representation of ecological dynamics. Other more sophisticated numerical methods can also be used, such as adaptive step-size methods, implicit methods, and higher-order Runge-Kutta methods. Moreover, it is fascinating to test solving the model using multiple step methods like Adam-Bashforth, Adam-Moulton and others, and investigate how these methods compare in terms of accuracy and computational efficiency when solving complex ecological models. Other than that, the Lotka-Volterra Competitive model can also be explored for various fields, including business, economic politics, medicine, social sciences and others.

Acknowledgements
The authors would like to thank Universiti Teknologi MARA (UiTM), Perak branch as the organizer for providing the opportunity and support of this research.

Conflict of Interest
The authors declare there is no conflict of interest in the subject matter or materials discussed in this manuscript.

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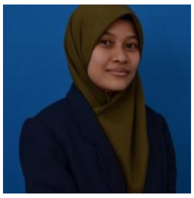
Picture	Biography	Authorship contribution
	Nur Hidayah Abd Rahaman with Bachelor of Science (Hons.) Management Mathematics from College of Computing, Informatics and Mathematics, Universiti Teknologi MARA Perlis.	Design the research work and data collection.
	Nurizatul Syarfinas Ahmad Bakhtiar obtained her first degree in Pure Mathematics from the University Sains Malaysia, Malaysia in 2011. She received a master’s and PhD degree in Mathematical Modeling from the same university in 2012 and 2020 respectively. Currently, she is a senior lecturer at the Universiti Teknologi MARA Perlis. Her research interests include dissipative solitons and dynamical systems and analysis.	Design the research work and drafting article.
	Hafizah Hajimia obtained herfirst degree in TESL at Universiti Pendidikan Sultan Idris. She received a master’s and PhD from Universiti Utara Malaysia. Currently, she is a senior lecturer at the Universiti Utara Malaysia.	Design the research work.
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