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Application Of Cubic Trigonometric Hermite Interpolation Curve with Shape Parameters In 2 Dimensional Objects

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ABSTRACT

Achieving accurate mathematical representations of an object's geometry for computer-aided design and manufacturing is a complex challenge. Designing such objects requires a method that offers the necessary flexibility and smoothness in curves, which traditional approaches often lack.. The primary aim of this study is to explore and implement cubic trigonometric Hermite interpolation as a method for constructing two-dimensional objects. Additionally, this study seeks to examine the properties of this interpolation method and determine the best value of the shape parameter that produces the smoothest curves. Cubic trigonometric Hermite interpolation is used for constructing two-dimensional objects, with the process carried out using Mathematica. Different values of the shape parameter are employed to achieve the desired curve characteristics in the geometric models. The correct images of the two-dimensional objects are also located, and the suitable shape parameters can be obtained after studying the behavior of the curve using free parameters. Two dimensional objects can be formed using cubic trigonometric Hermite interpolation without making any easy blunders or errors. This project has significant potential to enhance geometric modeling by determining the optimal shape parameters that improve the precision and efficiency of creating two-dimensional geometric models. The results can greatly benefit technical drawing, computer-aided design, and manufacturing by providing a more flexible and accurate method for representing the geometry of objects.

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1. Introduction

Curved objects are designed, computed, and digitally represented in Computer-Aided Geometric Design (CAGD) [1]. Thus, it's unsurprising that CAGD has historically been closely linked to traditional mathematical fields. These include approximation theory (emphasizing polynomial and

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piecewise polynomial functions), differential geometry (focusing on parametric surfaces), algebraic geometry (concerning algebraic surfaces), functional analysis and differential equations (for surface design through minimization of functionals), and numerical analysis. Vlachkova in 2020, identifies the interpolation of scattered data as a critical issue in both approximation theory and CAGD, noting its application across various industries such as architecture, computer graphics, and the design of ships and airplanes [2].

Numerous multivariate interpolation algorithms have been introduced. A vital element in modeling interpolation curves is the cubic Hermite interpolation curve, which, however, suffers from three major drawbacks: poor continuity, difficult shape adjustment, and an inadequate capability to accurately represent several common engineering curves [3]. To overcome these limitations, the cubic trigonometric Hermite interpolation curve has been developed. This curve accurately represents the elliptical arc, circular arc, quadratic parabolic arc, cubic parabolic arc, and astroid arc that frequently occur in engineering designs. It also achieves C2 continuity and offers both local and global adjustability [4].

Therefore, the primary goal of this project is to explore the construction of two-dimensional objects using cubic trigonometric Hermite interpolation. Curves are generated using the same control point across different methods. The resulting curves are then examined, and their behaviors are analyzed. Furthermore, this project introduces the concept of variable parameters that can control the characteristics of the cubic trigonometric Hermite interpolation, including adjustments in the coordinates and the tangents.

2. Literature Review

Cubic Hermite parametric curves and surfaces have been widely applied in Computer Aided Graphic Design (CAGD). Several authors have explored the Hermite trigonometric interpolation problem. Han in 2015, presented explicit piecewise trigonometric Hermite interpolation methods based on the symmetric, nonnegative, and normalized basis of trigonometric polynomials. These trigonometric Hermite interpolants are local and easy to compute [5],[6].

The bivariate rational Hermite interpolation method was developed in 2008 to create a space surface using the function values and the function's first-order partial derivatives as the interpolation data. Additionally, an explicit and simple mathematical expression for the interpolation function with a parameter was provided [7].

A new class of cubic trigonometric Hermite parametric curves and surfaces with shape parameters was introduced in 2012, similar to regular cubic Hermite parametric curves and surfaces. These new surfaces and curves inherit the properties of standard cubic Hermite surfaces and curves in polynomial space while also offering additional beneficial modeling qualities [8].

Furthermore, Li and Liu [4] proposed optimization solutions based on internal energy minimization for the cubic trigonometric Hermite interpolation curve, aiming to optimize the shapes of planar and spatial curves. Several modeling examples demonstrated the success of these methods, showing that cubic trigonometric Hermite interpolation curves are more practical than cubic Hermite interpolation curves [4].

Veldin et al. [9] presented a discrete nonlinear Kirchhoff-Love four-node shell finite element based on the bi-linear Coons surface patch. This patch crosses the region between the edge curves of cubic Hermites and their edge curves, producing cubic Hermite edge curves by decreasing the bending curvature of a spatial curve that connects two neighboring nodes of the element [9]. In 2023, Žagar examined the problem of interpolating two locations, two tangent directions, curvatures, and the arc length sampled from a circular arc (circular arc data). Planar Pythagorean-hodograph (PH) curves with a pitch of seven degrees were preferred due to their ample free parameters and ease of interpolating the arc length [10].

Rabbath and Corriveau [11] compared the potential curve fitting techniques of Piecewise Cubic Hermite Interpolating Polynomial (PCHIP), cubic splines, and piecewise linear functions. They introduced the procedures required to generate such piecewise polynomial functions using accessible tools to approximate the aerodynamics of a generic small arms projectile. Their contribution included evaluating the projectile aerodynamics approximation using PCHIP and offering an initial assessment of how the polynomial functions affect flight trajectory forecasts derived from 6-degree-of-freedom simulations of a standard projectile [11].

Han and Yang [12] presented a two-step modeling method for developing a smoothing curve or surface representation model for interpolating interval data. The first step involves developing a cubic Hermite spline model to create a smoothing piecewise parametric curve of continuity that reveals the implicit relationship between variables. In the second stage, sample points are selected from the best parametric spline curve from the first step as interpolation points to build explicit expressions and demonstrate the direct relationship between variables [12].

Albrecht et al. [13] addressed the G2 and C1 Hermite interpolation problem using planar quintic Pythagorean Hodograph B-spline curves with a single free internal knot serving as a shape parameter. This involves interpolating prescribed boundary locations, first derivatives, and curvatures at these points. They provided prerequisites for the data that guarantee solutions and demonstrated how the internal knot affects the values of the absolute rotation index or bending energy, as well as the shape of the resulting interpolant [13],[14].

The Hermite interpolation problem is solved by Arnal et al. [15] by passing the unique rationally parameterized curve of constant width through certain user-controlled points and tangents, provided an allowable denominator and a width value. The outcome provides an easy explanation for such a parameterization and is constructive. Thus, by selecting a set of points with their related tangents, these curves can be designed in a dynamic and participatory manner.

3. Methodology

3.1 Cubic Trigonometric Hermite Interpolation curve

The Cubic Trigonometric Hermite is used to form the two dimensional objects. All the calculations are calculated using Mathematica software. Relying on the research made by Li and Liu in 2022, the cubic trigonometric Hermite interpolation curve is afterwards obtained naturally. The equation (1) shows the basis function of cubic trigonometric Hermite interpolation curve [4]. Figure 1 shows the basis function for cubic trigonometric interpolation.

$$
\begin{aligned}
F_{i,0}(t) &= (1-\alpha_i) + 3(\alpha_i - 1)S^2 + 2(1-\alpha_i)S^3 + \alpha_i C^3 \\
F_{i,1}(t) &= 2(\alpha_{i+1} - 1) + 3(1-\alpha_{i+1})S^2 + \alpha_{i+1} S^3 + 2(1-\alpha_{i+1})C^3 \\
G_{i,0}(t) &= -\beta_i + S + 3(\beta_i - 2)S^2 + (1-2\beta_i)S^3 + \beta_i C^3 \\
G_{i,1}(t) &= \frac{1}{3}\left(2(3\beta_{i+1} - 5) - 3C + 9(2-\beta_{i+1})S^2 + 3(\beta_{i+1} - 8)S^3 + (13 - 6\beta_{i+1})C^3\right)
\end{aligned}
\qquad (1)
$$

where $S = \sin(t)$, $C = \cos(t)$, $t \in \left[0, \frac{\pi}{2}\right]$, α_i, α_{i+1}, β_i, and β_{i+1} are shape parameters.

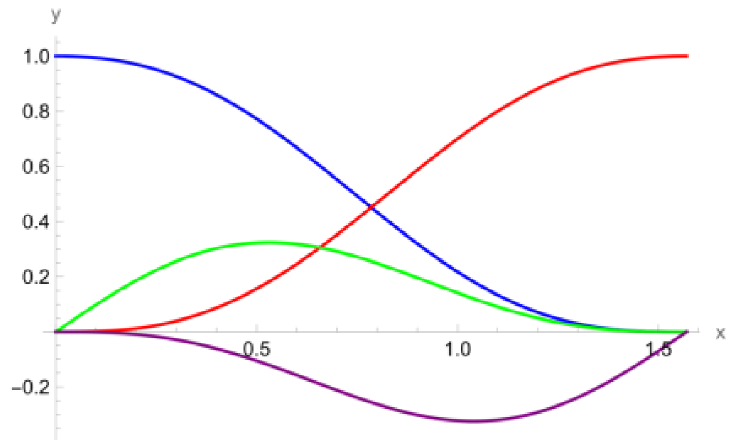


Figure 1. The basis function graph for cubic trigonometric Hermite interpolation

As shown in Figure 1, red curve is formed using $F_{i,0}(t)$, blue curve is formed using $F_{i,1}(t)$. $G_{i,0}(t)$ is used to form green curve while $G_{i,1}(t)$ is used to formed purple curve. Then, all the basis functions are used to form the cubic trigonometric Hermite interpolation curve, also known as the CTHI curve.

3.2 Curve behavior of Cubic Trigonometric Hermite Interpolation

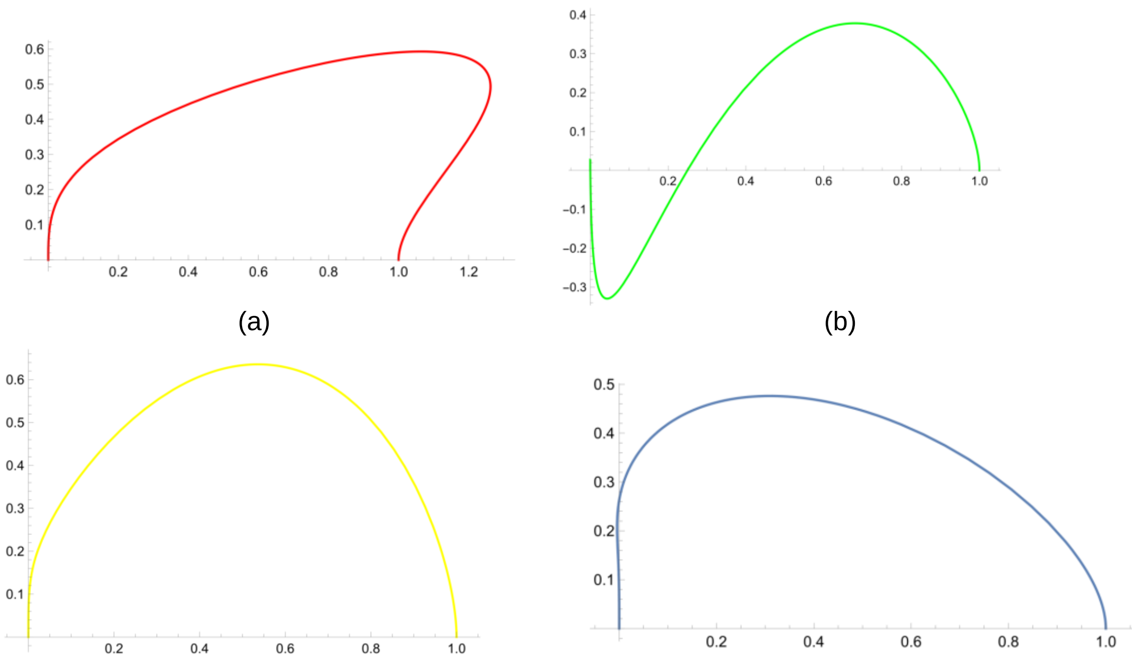
The cubic trigonometric Hermite interpolation curve is defined by equation (2) given a set of points p_j and the corresponding tangent vectors m_j;

$$TH_i = F_{i,0}(t)\, p_i + F_{i,1}(t) p_{i+1} + G_{i,0}(t)\, m_i + G_{i,1}(t)\, m_{i+1} \tag{2}$$

where $i = 0,1,\ldots\ldots, n-1$, $0 \le t \le 1$,

In this paper, point, $p_0 = (0,0)$ and $p_1 = (1,0)$, while the tangent vector $m_0 = (0,1)$ and $m_1 = (0,1)$ are used to form the curve. Position of the point p_j and the tangent vector m_j are maintained while the four shape parameters α_i, α_{i+1}, β_i, and β_{i+1} are changing in order to change the shape of curve.

Figure 2 shows the difference shape of curves obtained if difference values of shape parameters are used. The cubic trigonometric Hermite interpolation curve can be modified by changing the shape parameter value. Curve (a) and curve (b) used the same value of control point which are $P0 = P2 = (0,0)$ and $P1 = P3 = (-1,0)$ while the tangent points also are same for all the curves at $T0 = T2 = (1,0)$ and $T1 = T3 = (-1,0)$. Shape parameter that used to form curve (a) is $\alpha_i = -0.5,\ \alpha_{i+1} = 5,\ \beta_i = 1,\ \beta_{i+1} = 0.5$ and curve (b) is $\alpha_i = 0.5,\ \alpha_{i+1} = 0,\ \beta_i = -5,\ \beta_{i+1} = -1$, while shape parameter for curve (c) is $\alpha_i = 0.5,\ \alpha_{i+1} = 1,\ \beta_i = 0,\ \beta_{i+1} = -0.5$ and the shape parameter for curve (d) is $\alpha_i = -1,\ \alpha_{i+1} = -0.5,\ \beta_i = 0,\ \beta_{i+1} = 0.5$.



(c) (d)

Figure 2. Variety shape of curve obtained by using difference value of shape parameter

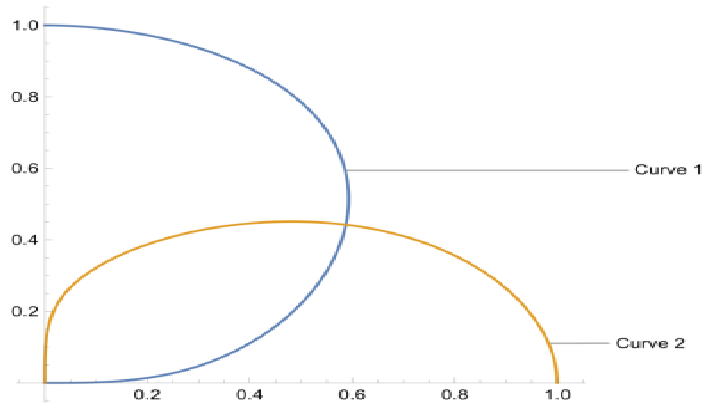
4. Results and Discussion

4.1 Application of Cubic Trigonometric Hermite Interpolation curve in 2-dimensional

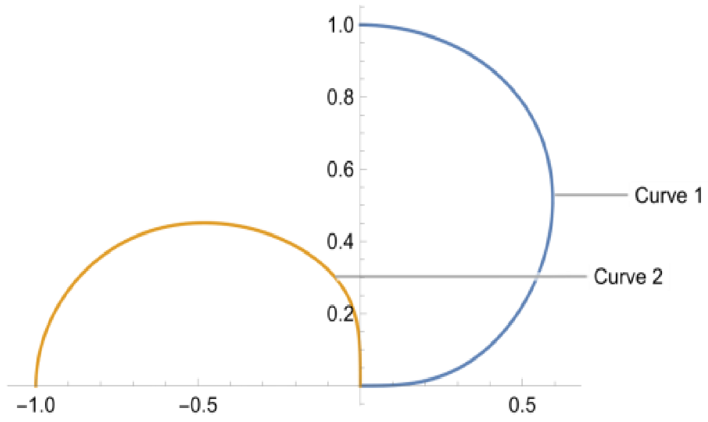
The Cubic Trigonometric Hermite Interpolation method is used to design the 2-dimensional shape which are crescent and circle. Both curve must be on the same quadrant in order to design the crescent. While to design the circle, curve 1 must be on the quadrant that opposite to the curve 2. Then, the control point, the tangent point and shape parameter are need to modify in order to produce desired shape.

Figure 3 (a) shows that the curve 2 is on the first quadrant when the control points $P2 = (0,0)$ and $P3 = (-1,0)$ while the tangent point $T2 = (1,0)$ and $T3 = (-1,0)$ are used. Figure (b) shows that the curve 2 appeared on the second quadrant when the control points $P2 = (0,0)$ and $P3 = (-1,0)$ while the tangent point $T2 = (0,1)$ and $T3 = (0,-1)$ are used.

Table 1 shows the curves produced when each of the shape parameters is changing respectively from the value of curve 1: $\alpha_i = -1, \alpha_{i+1} = -0.5, \beta_i = 0, \beta_{i+1} = 0.5$ and curve 2: $\alpha_1 = 0.5, \alpha_2 = 1, \beta_1 = 0, \beta_2 = -0.5$.



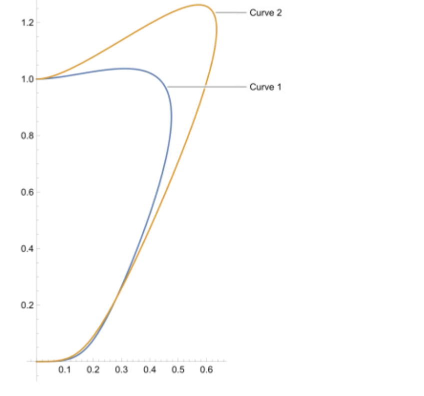
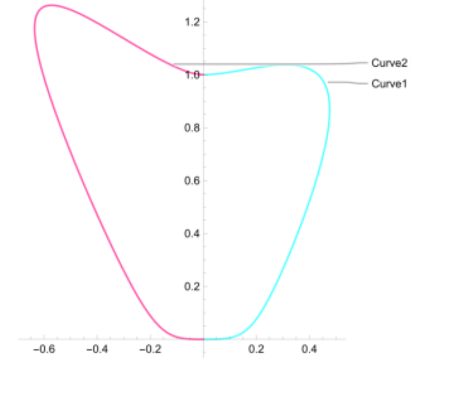
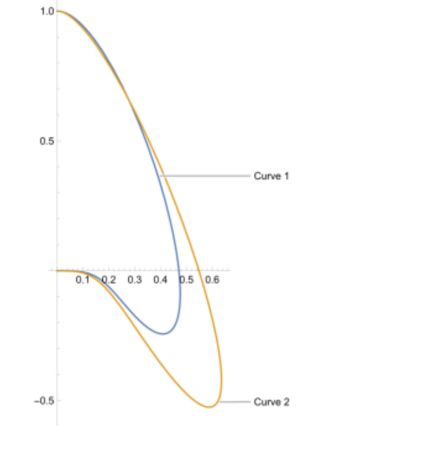
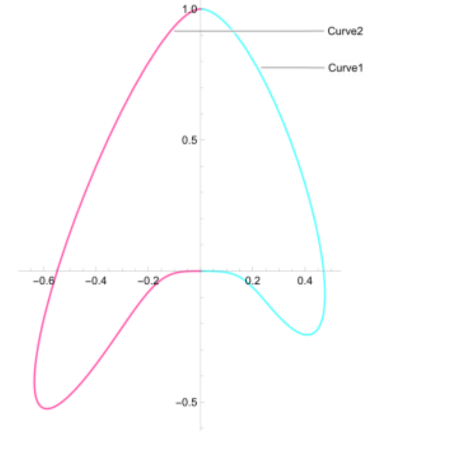
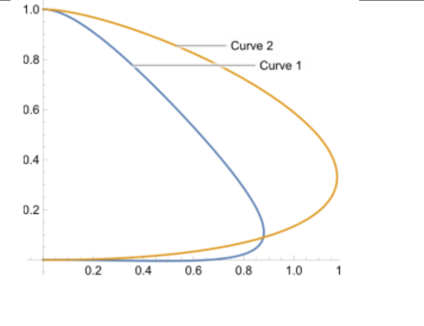
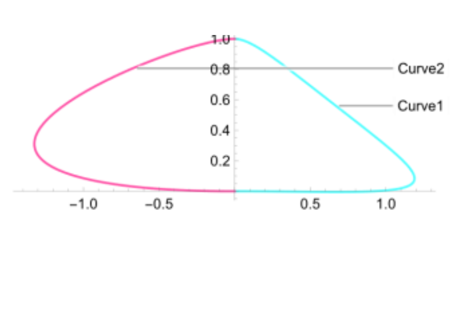
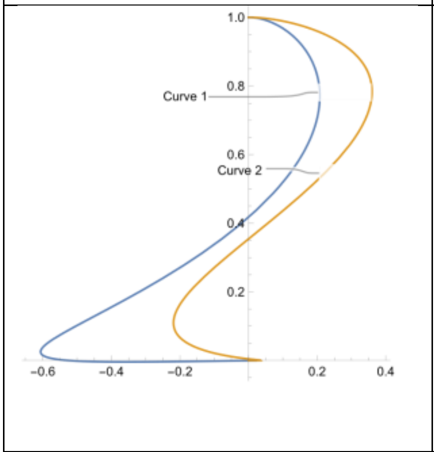
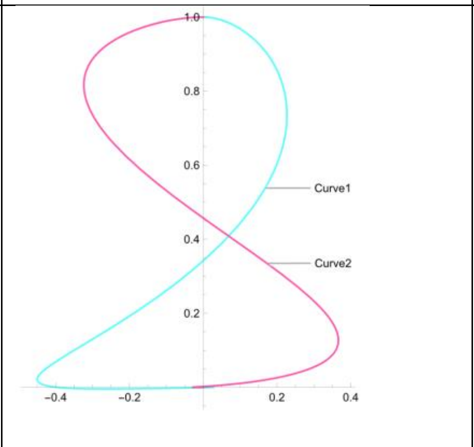
(a)

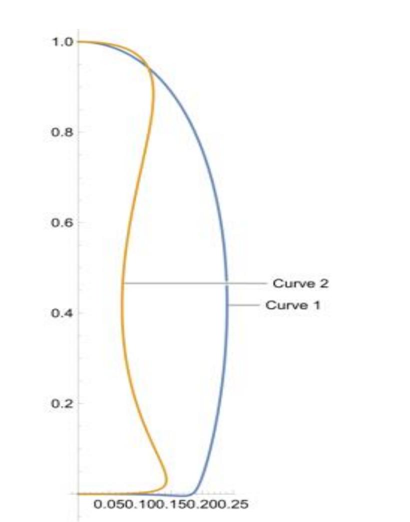
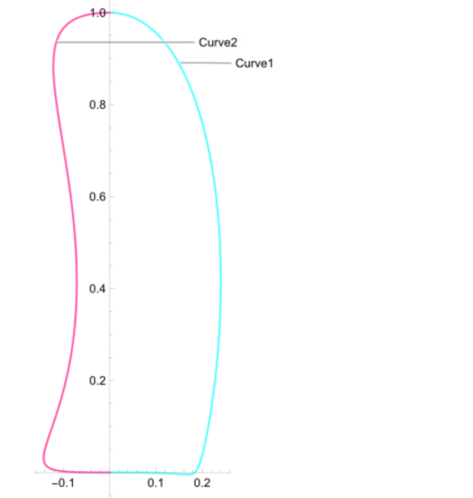
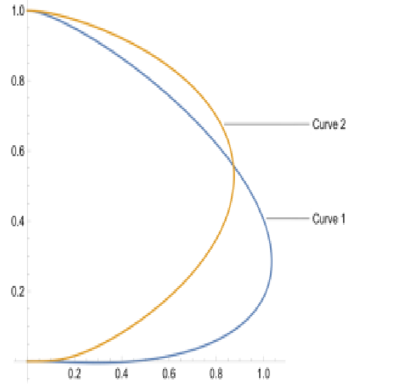
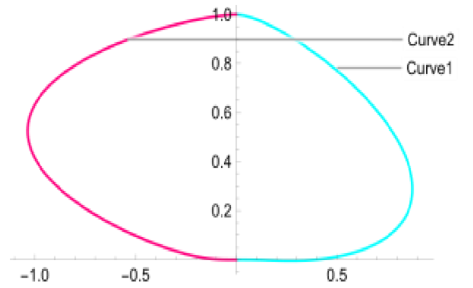


(b)

Figure 3. Position of the curve 2 using different control point and tangent point.

Table 1. The curve obtained when each of the shape parameter is changing respectively

		The value of shape parameter α_{i+1} for curve 1 and α_2 for curve 2 are changed to positive number while other parameters have remained same, the curve 1 and curve 2 will curve upward.
		The value of shape parameter α_{i+1} for curve 1 and α_2 for curve 2 are changed to negative number while other parameters have remained same, the curve 1 and curve 2 will curve downward.
		The curves formed when the value of β_i and β_1 are change to positive number.
		The curves formed when the value of β_i and β_1 are change to negative number.

		The curves formed when the value of β_{i+1} and β_2 are change to positive number.
		The curves formed when the value of β_{i+1} and β_2 are change to negative number.

From table 1, the best shape parameters used has been determined to form the smooth 2-dimensional crescent and circle outline. Figure 4 and Figure 5 shows the crescent and circle outline are design using cubic trigonometric Hermite interpolation. The curve has been constructed based on the control point, and the curve has been drawn according to the shape parameters where all the parameters' values are positive in range 0.5 until 2.

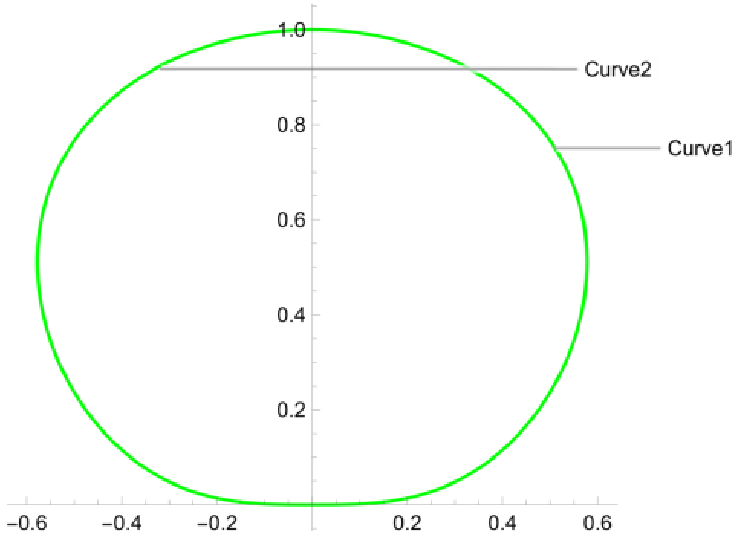


Figure 4. The circle outline formed by using Cubic Trigonometric Hermite Interpolation curves

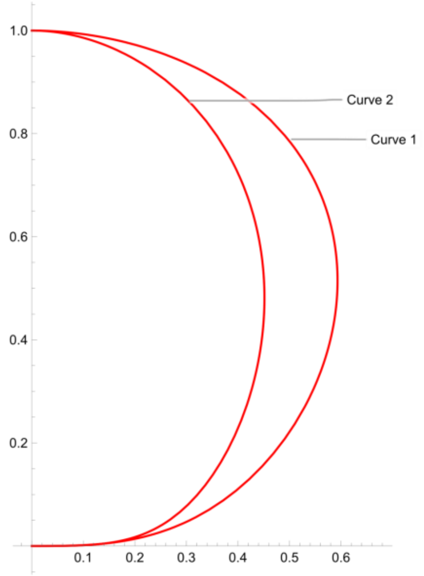


Figure 5. The crescent outline formed by using Cubic Trigonometric Hermite Interpolation curves

For crescent outline, the control points for curve 1 and curve 2 are same at $P0 = P2 = (0,0)$ and $P1 = P3 = (-1,0)$ while the tangent points for curve 1 and curve 2 also are same at $T0 = T2 = (1,0)$ and $T1 = T3 = (-1,0)$. The best shape parameters for each curve have been chosen to generate a two-dimensional crescent shape are shown as in Table 2 below.

Table 2. Shape parameter values for Crescent Outline shape

Curve	α_i	α_{i+1}	β_i	β_{i+1}
Curve 1	2	1.5	1	0.5
Curve 2	2	1	0.5	1

While for the circle outline, the control points for curve 1 and curve 2 are same at $P0 = P2 = (0,0)$ and $P1 = P3 = (-1,0)$ while the tangent points for curve 1 $T0 = (1,0)$ and $T1 = (-1,0)$ and curve 2 is at $T2 = (-1,0)$ and $T3 = (1,0)$. The best shape parameters for each curve have been chosen to generate a two-dimensional crescent shape are shown as in Table 3.

Table 3. Shape parameter values for Circle Outline shape

Curve	α_i	α_{i+1}	β_i	β_{i+1}
Curve 1	2	1.5	1	0.6
Curve 2	2	1.5	1	0.6

5. Conclusion

The findings of this study enable us to reach the conclusion that every one of the research goals has been well achieved. The correct images of the two dimensional objects are also located and the suitable shape parameters can be obtained after studying the behavior of the curve using free parameters.

The behavior of the curve can be changed by adjusting the curve's shape parameters. Although a smooth curve is generated, if some of the shape parameters are modified, a sharp curve will be generated. A relatively small difference between the parameters of two curves may be discernible. The shape that is assigned to the curves constructed using the cubic trigonometric Hermite interpolation function depends on the shape parameters of the shape that is created with the control points and tangent vectors. When it comes to the creation of two-dimensional shapes, the cubic trigonometric Hermite interpolation function results in smooth curves. It is possible to decide the suitable shape parameters of the cubic trigonometric Hermite interpolation function. Additionally, using the appropriate shape parameters is the only way to produce tidy and respectable results. Because each shape has a unique combination of curve features, using the software is quite helpful when undertaking research that involves the development of two-dimensional shapes.

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Conflict of Interest

The authors declare no conflict of interest in the subject matter or materials discussed in this manuscript.

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	Farah Nabilah Azmi is a student of Universiti Teknologi MARA Terengganu Branch, Kuala Terengganu Campus, Terengganu. She is studying for a bachelor's degree of Science (Hons.) in Mathematical Modelling and Analytics. She can be reached at farahnabilahofficial@gmail.com	Introduction, literature review, data collection, methodology, and analysis.
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