Identification of Nonlinearity in Bolted Beam using Response-Controlled Testing and Force-Controlled Testing

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ABSTRACT

Nonlinear phenomena such as super-and sub-harmonics and unstable branches in frequency response functions (FRFs) often occur in slender structures, which are common in the aerospace, civil and wind power domains. The widely used methods of linear modal analysis are invalid for the identification of these phenomena, which play an important role in nonlinear structural analysis as they can significantly affect the behaviour and stability of structures under various excitations. This paper proposes the methodology of Response-Controlled Stepped Sine Testing (RCT) and Force-Controlled Stepped Sine Testing (FCT) to identify nonlinear phenomena in a bolted beam. The bolted beam was excited with two different sets of predefined displacement and force amplitudes to measure the Nonlinear Frequency Response Functions (NLFRFs) of the bolted beam. The measurements were performed with RCT and FCT, using Stepped Sine signals to control the displacement and force amplitudes. The Harmonic Force Surface (HFS) technique was used as means of obtaining the NFRFs. It was found that RCT with HFS successfully identified

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the nonlinearity that is very similar to the identification results obtained with FCT. In addition, RCT was computationally faster than FCT in identifying nonlinear bolted beams and avoided dealing with jump phenomena. These advantages of RCT and HFS can provide engineers and researchers with a great technique to accurately and efficiently identify nonlinearities in structures.

Keywords: *Nonlinear Frequency Response Function; Harmonic Force Surface; Stepped Sine; Response-Controlled Testing; Force-Controlled Testing*

Introduction

Modern engineering designs consist of advanced materials, a large number of connections and long, slender structures that are often used in aerospace, bridges, buildings and wind turbines. These structures exhibit high cubic stiffness due to geometric nonlinearity [1]-[4], nonlinear material behaviour and complex boundary conditions [5]-[11], leading to responses containing sub- and super-harmonics and frequency response functions that contain unstable branches. In such cases, linear modal analysis, which is widely used to determine the modal parameters of the structures, is invalid. To overcome this barrier, an accurate and efficient technique to identify and characterise the dynamic response of nonlinear structures is required.

Among the various methodologies for identifying nonlinear structures in both time and frequency domains are summarised in [7], the frequency domain is widely regarded as one of the methodologies with the greatest potential [6]. The frequency domain offers the advantages of simplicity, efficiency, and accuracy over the time domain-based techniques, such as Time-Series-Based Linearity plots (INTL), the Nonlinear Resonant Decay method (NLRDM), the Control-Based Continuation (CBC) and the Phase-Locked-Loop (PLL), which require considerable computational and experimental effort and also special measurement equipment that may not be commercially available [2], [12].

One of the efficient frequency domain methodologies is Nonlinear Modal Analysis (NLMA), which is based on the Describing Function Method (DFM) [1], [13]-[14]. This method was introduced in the early 1990s [15] and was primarily developed to analyse the harmonic oscillations of nonlinear systems. Tanrikulu et al. [16] later extended this method by including a wider range of describing functions. An interesting series of efforts were made by Budak et al. [17] to apply this method in their various studies, the main aim of which was to incorporate describing functions in modal analysis. Özer et al. [18] proposed the use of the nonlinearity matrix approach to determine the type and parametric values of nonlinear elements in systems with multiple degrees of freedom, both with and without grounding.

Another promising method for the experimental and computational analysis of nonlinear structures, which is not based on the methods mentioned above, is the use of Force-Controlled Stepped Sine Testing (FCT). This technique offers specific control of the input amplitude, improved signal quality and a very good signal-to-noise ratio compared to other methods [19]-[21] for the identification and characterisation of nonlinear behaviour, even for complex structures with multiple modes [22]-[23]. Due to these significant advantages of FCT, this technique is widely used in industry and academic research to identify nonlinear structures. For example, FCT is used in aerospace [6]-[7], [24], model updating [25]-[27], control [26], [28] and wind turbines [29]-[31]. However, one of the biggest drawbacks encountered in the application of FCT is the inaccurate and inefficient identification of nonlinear behaviour. This is due to the fact that FCT does not account for nonlinear phenomena such as unstable branches that theoretically exist between two jump phenomena in the NLFRF [5]. These limitations of FCT have led researchers to investigate alternative methods such as CBC and the PLL. The main advantage of these methods over FCT is their ability to track backbone curves and unstable branches in the response of nonlinear systems to harmonic excitations. However, none of these methods have been used in industry, so there is no standard equipment for these methods that is suitable for the identification of nonlinear behaviour.

Efforts to accurately and efficiently account for nonlinear behaviour have led to the development of a new technique called Response-Controlled Stepped-Sine Testing (RCT) [32]. The main idea of this technique is to keep a response amplitude constant at the excitation point so that the system can be considered linear. By keeping the amplitude constant, nonlinear phenomena can be identified based on established methods of linear modal analysis. The advantage and power of RCT were tested by [2] to identify unstable branches, both computationally and experimentally, incorporated with the theory of arc length continuation. Recently, the significant benefits of using RCT have been demonstrated by [33]. They incorporated the RCT methodology into Ansys and used Harmonic Surface Force (HSF) to identify nonlinear phenomena, resulting in highly satisfactory outcomes. There is very little research on the identification of nonlinear phenomena using the RCT methodology, warranting a more in-depth computational and experimental investigation of its accuracy and performance. Therefore, it is imperative to conduct comparative research on the accuracy and efficiency of the RCT methodology to offer a new perspective for engineers and researchers who work in nonlinear identification.

In this study, the methodologies of RCT and FCT for identifying the nonlinear behaviour of a bolted beam are thoroughly tested and critically evaluated. Nonlinear Frequency Response Functions (NLFRFs) are measured using both methodologies by keeping the predetermined amplitudes of displacement and force constant at the location of the excitation point. The nonlinear behaviour in the bolted beam is observed from the measured NLFRFs and V-shaped curve-based HFS plots. Finally, a comparison of the performance of RCT and FCT in terms of accuracy and efficiency is carried out.

Theoretical Background

Linear and nonlinear MDOFs system

The FCT employed in conventional methods controls at a predetermined amplitude of harmonic forces applied at different degrees of freedom denoted $\{F\}$, as described by:

$$\{F\} = \{f\} e^{(\omega t)} \tag{1}$$

where $\{f\}$ denotes the vector of amplitudes and ω denotes the frequency excitation

In linear modal testing analysis, the dynamic response of a linear Multi Degrees of Freedom (MDOF) system can be expressed by a second-order differential equation, as shown in Equation (2).

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\}e^{(\omega t)} \tag{2}$$

where $[M]$ is the mass, $[C]$ is the damping and $[K]$ is the stiffness matrices. $\{\dot{x}\}$ and $\{x\}$ are velocity and displacement vectors respectively.

For a nonlinear MDOFs system subjected to harmonic excitation, the matrix formulation for the equation of motion can be expressed as follows:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + \{F(x, \dot{x}\}_{NL}\} = \{f\}e^{(\omega t)} \tag{3}$$

where $\{F_{NL}\}$ is a nonlinear force that depends on displacement and velocity. For RCT, this nonlinear equation of motion can be cast into:

$$-\omega^2 [M]\{X\} + i\omega[C]\{X\} + [K]\{X\} + \{F_{NL}\} = \{f\} \tag{4}$$

where the vector $\{X\}$ represents the amplitude of the harmonic response of the external force, while the vector $\{F\}$ denotes the corresponding amplitudes. The internal nonlinear forces can be expressed by:

$$\{F_{NL}\} = \{Y\} e^{i\omega t} \tag{5}$$

where {Y} denotes the amplitude of the internal nonlinear forcing vector represented by:

$$\{Y\}=[\Delta]\{X\} \tag{6}$$

where [*Δ*] denotes the nonlinearity matrix and can be calculated using the describing functions [7], [10], [13] depending on the type of nonlinearities to be analysed.

Various computational and experimental methodologies for the identification of nonlinear systems based on Equation (4) have been developed over many years. Identification involves determining the response $\{X\}$ as a result of a force excitation $\{F\}$ applied to a system with known parameters. Conversely, for a system with unknown parameters, nonlinearities can be determined by fitting Equation (4) to the measured response $\{X\}$ resulting from the application of a constant or controlled force amplitude. However, it is reported that these methodologies are not able to directly determine unstable branches and that their success is highly dependent on other numerical methods, e.g. the path continuation method.

To overcome the above-mentioned obstacles, a method proposed by [7], the so-called RCT, is further investigated experimentally in the present study. In this testing, the displacement amplitude of the excitation point determined in Equation (4) is kept constant. This strategy allows the calculated Frequency Response Functions (FRFs) to be in linear form due to the V-shaped harmonic force spectrum calculated in the frequency domain. This strategy facilitates the effective application of linear modal analysis to identify important system parameters such as resonance frequency, damping ratio, modal constant and modal amplitude. The calculated values of these parameters in Equation (7) are used in conjunction with the response $\{X_j\}$, to determine the NLFRFs for all unstable branches by the path continuation method.

$$X_j(q_r)=\frac{\bar{A}_{jkr}(q_r)F_k}{\bar{\omega}_r^2(q_r)-\omega^2+i2\varsigma_r(q_r)\omega\bar{\omega}(q_r)} \tag{7}$$

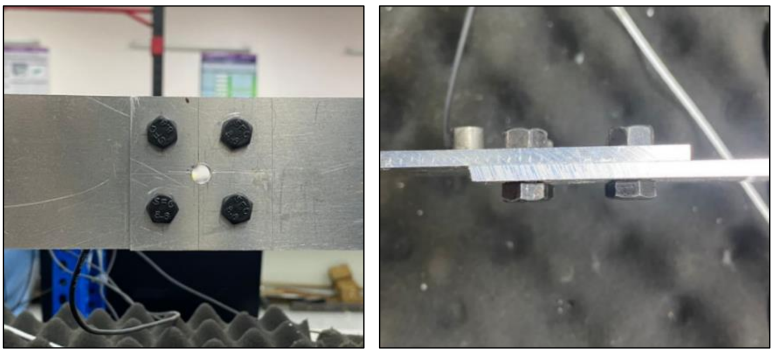
where X_j is the response at point j, $\bar{A}_{jkr}$ is the modal constant, (q_r) is the modal amplitude, F_k is the given load at point k, $\bar{\omega}_r$ is the resonance frequency, ς_r is the damping ratio, and ω is the excitation frequency.

It is worth mentioning that the V-shaped curves of the harmonic force spectrum obtained by RCT provide a simple yet accurate alternative to the calculation of NLFRFs, which is also suitable for the calculation of unstable branches. These calculations can be performed efficiently by constructing the

HFS using linear interpolation and slicing the surface with the predefined force magnitude.

Experimental Modal Analysis (EMA) setup

Two thin aluminium beams with a length of 482 mm and a rectangular cross-section of 52 mm width and 3 mm thickness are joined together by four sets of M5 bolts and nuts. Figure 1 shows the bolt assembly for the bolted beam as (a) top view and (b) side view of the assembly. The fixed preload value for these bolted joints is 4 N.



(a) (b)

Figure 1: M5 bolt assembly for the bolted beam (a) top view and (b) side view

The total length of the test structure is 920 mm. This bolted test beam is used for the nonlinear test. Figure 2 shows the EMA setup for linear and nonlinear modal analysis. The bolted beam is held horizontally with one end of the bolted beam fixed in the *z* and *x* directions by a rigid and stiff clamp, while the other end of the bolted beam is free for bending modes in the y-direction.

Figure 3 shows a schematic representation of the EMA setup. The accelerometers are placed at positions A, B, C and D as shown in both Figures 2 and 3. In this setup, two black ferrite permanent magnets with self-adhesive metal epoxy were attached to the free ends of each side of the bolted beam. The total weight of the bolted beam is 432 g including the attached magnets on the end. To create a magnetic attraction, especially at the free end of the bolted beam, two electromagnets were also positioned next to both sides of the free end of the bolted beam. The two-cylinder electromagnets of 30 mm height and 65 mm width have the ability of 60 kg holding force and are controlled by a power supply for each of the electromagnets. The experimental setup was carefully arranged to maintain a constant distance of 75 mm between the magnets in equilibrium, with equal distances of 35 mm on each side of the plate.

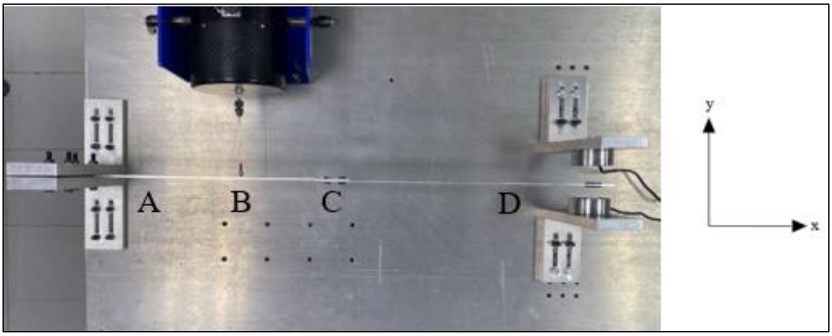


Figure 2: EMA setup for linear and nonlinear modal analysis

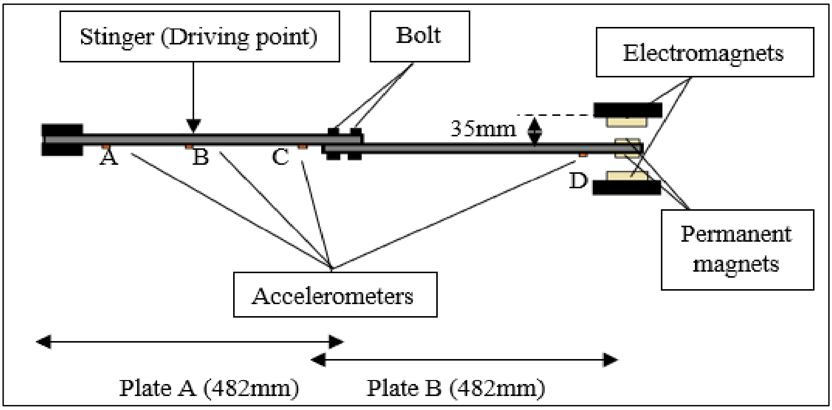


Figure 3: Schematic representation for the EMA setup

EMA was performed with a dynamic excitation setup consisting of an electromagnetic shaker mounted on a robust steel module and driven by an amplifier. This fixed shaker was positioned close to the clamp and a force transducer was securely attached to the bolted beam with a strong adhesive to measure the excitation force exerted by the shaker via a stinger. The interaction between the shaker and the bolted beam was carefully adjusted to minimise external influences and misalignment. Four accelerometers were used to measure the fundamental frequency, as shown in Figure 3. The actual sensitivity and weight of the accelerometers are listed in Table 1.

Accelerometer A was placed near the clamped end to monitor the fixed boundary condition. Accelerometer B was located at the driving point and served as a reference channel for control purposes. Accelerometer B was placed 15 mm from the driving point to measure the vibration response where the fundamental mode shape has a large deflection. Accelerometer C was placed near the joints of plate A and plate B. Accelerometer D was located near the magnet at the free end and not at the tip of the bolted beam to ensure accurate measurements while avoiding interference with magnetic attraction.

Table 1: Detailed specifications of the equipment

Equipment	Brand, Model	Actual	Weight (g)
Accelerometer A	Metra Meß- und Frequenztechnik (MMF), KS91C	10.516 mV/g	1.3
Accelerometer B		10.509 mV/g	1.3
Accelerometer C		10.74 mV/g	1.3
Accelerometer D		10.262 mV/g	1.3
Force transducer (Driving point)	PCB Piezotronics, 208C03	2.262 mV/N	22.7

EMA was performed according to the steps in the flowchart in Figure 4. In the first step of EMA, sweep sine testing was performed to obtain the linear frequency response function of the bolted beam. The result was a fundamental frequency for the first mode with a large deflection in the y-axis. EMA was continued with the nonlinear dynamic test. The frequency of the first mode was used as a reference value from the FRFs of the linear dynamic test. Different force and displacement amplitudes were selected as control parameters for FCT and RCT. The comparison of the measured NLFRFs of FCT and RCT was used to detect nonlinearity due to the bolted joints and magnetic attraction.

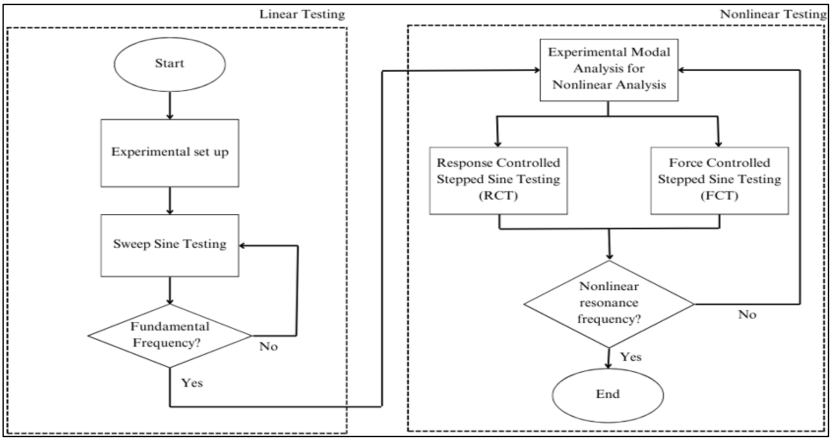


Figure 4: Flowchart of linear and nonlinear testing

Results and Discussion

Linear EMA

Linear EMA with the sweep sine testing of the excitation force of 0.5 N to identify the first mode was performed on the bolted beam with different magnetic settings: 0 V no magnetic effects (0 V), 3 volts, and 6 volts with the distance between the electromagnet and the permanent magnets kept constant at 35 mm. In order to determine the linear dynamic characteristics, the FRFs were measured and analysed at the position of accelerometer B. The linear resonance frequency and modal damping ratio were estimated using the LMS Test.Lab Polymax identification algorithm.

Figure 5 shows the FRFs measured from the linear EMA with the swept sine testing of the excitation force of 0.5 N and different magnetic settings. The frequency range used for the FRF measurement was between 15.0 Hz and 25.0 Hz with a frequency step increment of 0.05 Hz. From the measured FRFs, it was found that the magnetic attraction of 3 V and 6 V affected the resonance frequency with a reduction of 0.21 Hz compared to 0 V. On the other hand, the amplitudes calculated from the FRFs of 3 V and 6 V did not show the same trend, with the former decreasing at about 0.05 mm/N and the latter increasing at about 0.7 mm/N, as shown in Table 2. Based on this result, the 6 V magnetic field was chosen for the FCT and RCT.

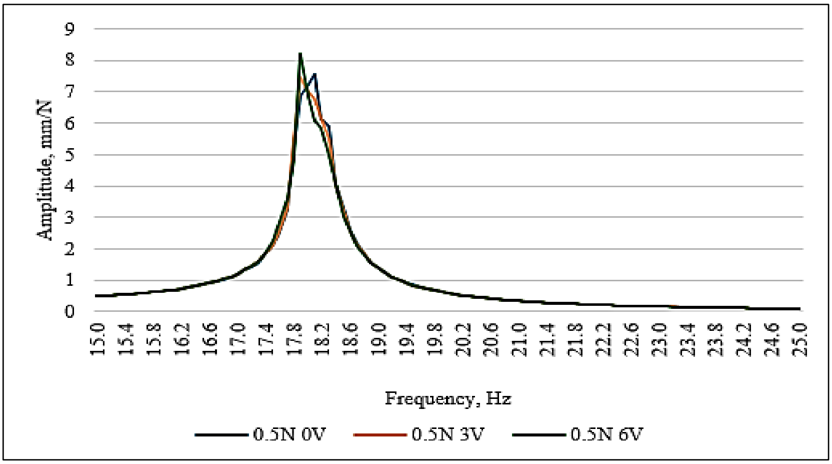


Figure 5: Measured FRFs using sweep sine testing

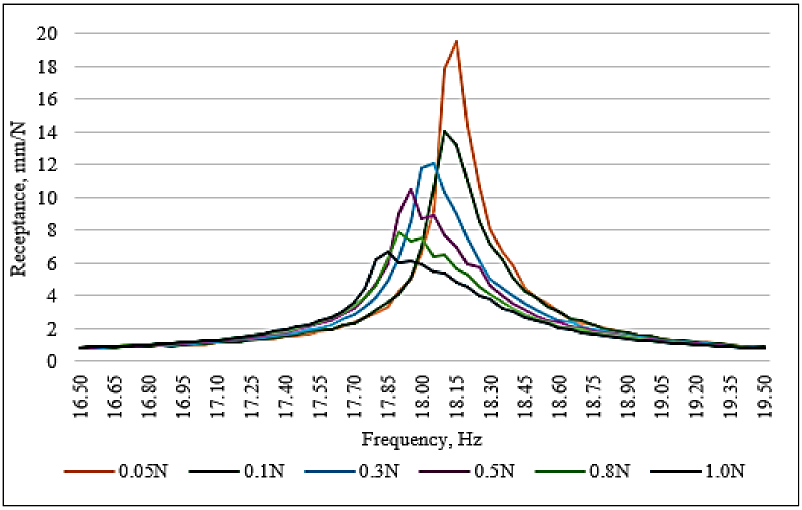
Table 2: Natural frequencies and amplitudes measured from linear EMA

Distance between magnets (mm)	Magnetic effect (V)	Natural frequency (Hz)	Receptance (mm/N)
35	0	18.1	7.52
35	3	17.9	7.47
35	6	17.9	8.17

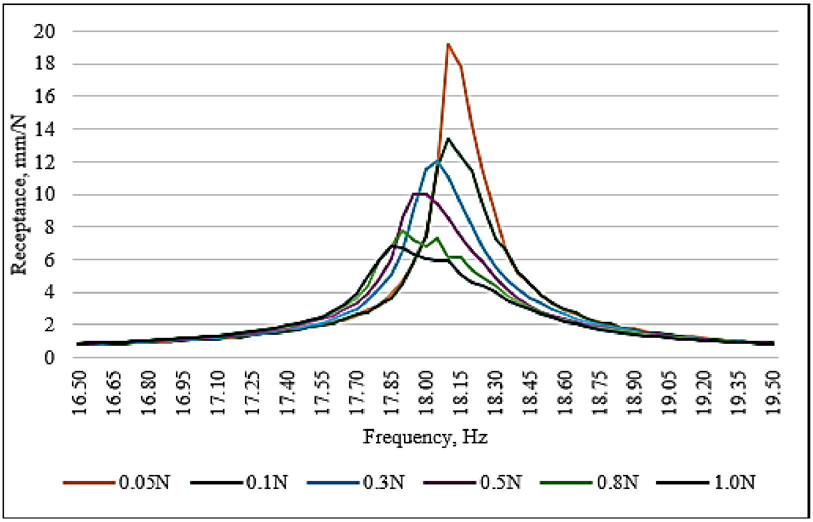
Force-controlled stepped-sine testing (FCT)

The FCT was performed on the bolted beam to measure its NLFRFs. The FCT covered only the first mode with a frequency range between 16.5 Hz and 19.5 Hz, which was smaller than that of the linear test. In addition, the FCT used seven different values of controlled force amplitudes, which were controlled at the driving point (accelerometer B). Moreover, 12 different runs with six different constant forces (0.05 N, 0.1 N, 0.3 N, 0.5 N, 0.8 N, and 1.0 N) were performed for the FCT, with six runs for the swept-up and the other six for the swept-down. The nonlinear FRFs measured with the FCT are shown in Figure 6(a) and 6(b).

It was observed that the application of different constant forces at the excitation point resulted in different FRFs for both swept up and swept down. The use of a larger constant force resulted in the measurement of highly distorted FRFs. Another important observation was that the larger the constant force, the lower the amplitude of the FRF. This scenario shows the presence of nonlinearity in the bolted beam and the capability of the FCT to accurately identify the nonlinearity.



(a)



(b)

Figure 6: Measured FRFs of (a) swept up and (b) swept down

The different resonance frequencies obtained from the FRFs are tabulated in Table 3. The decrease of resonance frequency indicated that the nonlinearity in the bolted beam was of a softening type. Meanwhile, Table 4 shows the times required to execute FCT with swept up and swept down for different forces. The total time recorded from the measurements was 4010.12 seconds.

Table 3: Resonance frequencies obtained using FCT

Runs	Controlled forces (N)	Swept up frequencies (Hz)	Swept down frequencies (Hz)
1 & 2	0.05	18.15	18.10
3 & 4	0.1	18.10	18.10
5 & 6	0.3	18.05	18.05
7 & 8	0.5	17.95	18.00
9 & 10	0.8	17.90	17.90
11 & 12	1.0	17.85	17.85

Table 4: Execution time of the 12 runs of FCT

Force (N)	Time (s)		Sum time (s)
	Up	Down	
0.05	368.31	347.83	716.14
0.1	347.83	306.87	654.7
0.3	429.75	368.31	798.06
0.5	306.87	306.87	613.74
0.8	327.35	286.39	613.74
1.0	327.35	286.39	613.74
		Total time taken:	4010.12

Response-controlled stepped-sine testing (RCT)

In the field of dynamic analysis, modal analysis of linear structures has been widely used to identify essential modal parameters. However, it must be pointed out that the assumption of linearity may not hold true in practical situations where test structures are being examined. Even with the existing nonlinear identification methods, there are often challenges in determining accurate nonlinear parameters. The main objective of adopting the RCT in this study was to identify the nonlinearity in the bolted beam, which theoretically may present difficulties with the FCT [1]-[2], [12], [33].

RCT was performed on the bolted beam using the same experimental setup as FCT. The main difference was that various controlled displacement amplitudes were required for RCT. During this testing, 10 different displacements, ranging from 0.76 mm to 6.7 mm with an increment of 0.66 mm as shown in Figure 7, were used to measure the harmonic force spectrums of the bolted beam.

It is imperative to note that the accuracy of RCT is highly dependent on the correct selection of displacement amplitudes. This selection can be determined based on the displacement versus frequency plot of FCT, as shown in Figure 8. The highest and lowest peaks of the amplitude were used as benchmarks for calculating the controlled displacement amplitudes required, which comprised a total of 10 different displacement amplitudes.

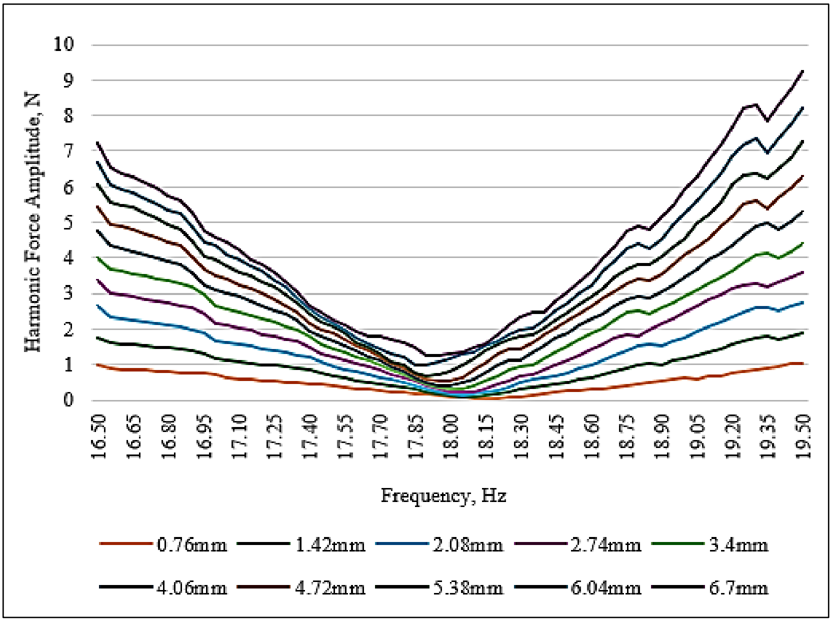


Figure 7: Measured harmonic force spectrums of accelerometer B

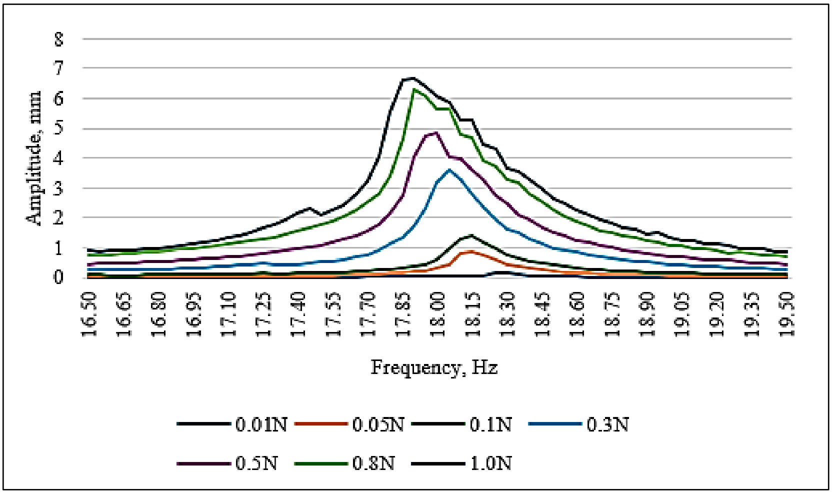


Figure 8: Measured displacement vs. frequency of FCT

From the constant-response FRFs of RCT shown in Figure 9, it is evident that the resonance frequencies, represented as peaks in the FRFs, decreased as the controlled responses increased. This indicates a significant softening nonlinear behaviour. Additionally, an increase in controlled-responses led to a decrease in the amplitudes of the FRFs, indicating an increase in the damping values. Another important observation of the measured constant-response FRFs is that they showed a symmetrical shape for all applied displacements, in contrast to the constant-response FRFs, which showed a distorted shape for all applied forces.

The symmetrical shape of the FRFs shows the significant advantage of the RCT in identifying nonlinearity in the bolted beam based on linear modal analysis. Another significant advantage of the RCT is that the measurements of the constant-response FRFs were faster than those of the constant-force FRFs. The total time for the measurements was only 2003.74 seconds, which is half that of the FCT. The outstanding feature of the RCT is more evident when the time recorded for a specific equivalent constant response to a constant force is directly compared. This can be observed from the direct comparison between the constant response of 0.67 mm (Table 4) and the constant force of 1 N (Table 3), from which RCT was 3.5 times faster than FCT. In addition, the capability of the RCT in measuring the resonance frequencies is as accurate as the FCT. Table 5 shows the detailed results of the RCT performed on the bolted beam.

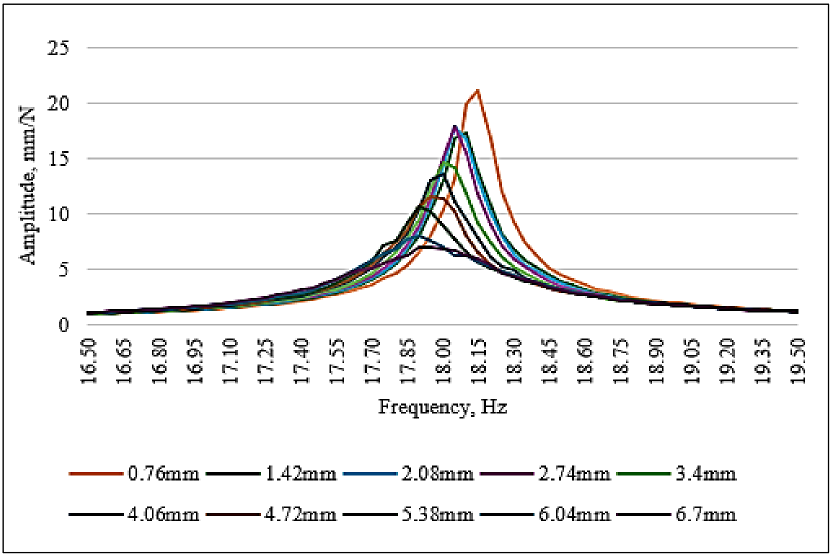


Figure 9: Measured constant-response FRFs of RCT

Table 5: Detailed results of RCT

Run	Controlled displacement, mm	Frequency, Hz	Time taken, s
1	0.76	18.15	183.99
2	1.42	18.10	183.99
3	2.08	18.05	204.47
4	2.74	18.05	204.47
5	3.40	18.00	204.47
6	4.06	18.00	183.99
7	4.72	17.95	183.99
8	5.38	17.90	224.95
9	6.04	17.90	245.43
10	6.70	17.90	183.99
		Total time taken:	2003.74

Harmonic Force Surface (HFS)

HFS is a powerful alternative technique for calculating NLFRFs. HFS allows NLFRFs of structures to be calculated accurately and efficiently at any given force level. This striking advantage is evident, especially when identifying nonlinearities in structures with varying excitation amplitudes. Detailed explanations and derivations of HFS can be found in [12] and [33].

In this study, HFS was used to calculate the NFRFs of the bolted structure as a result of controlling displacement amplitudes. The calculation was carried out by slicing the HFS plot at 1 N and 0.5 N, which were specifically selected for this case study. In other words, the HFS plot can be sliced at any force level as long as the selected force level is within the force levels used in the study. Figure 10 shows the HFS plot that can be sliced at any given force level required.

The accuracy of the HFS technique is evident in Figures 11 and 12. The NLFRFs calculated with HFS showed almost similar patterns to those of FCT swept up for 1 N and 0.5 N. This achievement, which is consistent with the results in [12], and [33], can be attributed to the accuracy and efficiency of RCT in measuring the V-shaped curves used in the HFS technique to plot HFS plots. However, in this study, the powerful HFS technique, especially the identification of unstable branches could not be fully demonstrated as shown in [1]-[2], [12], [33]. This scenario is due to the observation that the intensity of nonlinearity in the bolted beam was not as strong as in the test structure they studied.

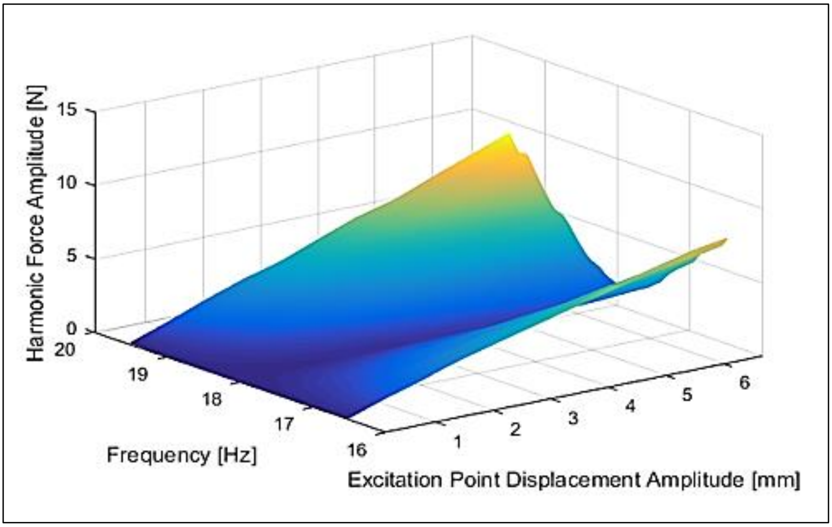


Figure 10: HFS plot of the force-constant spectrums

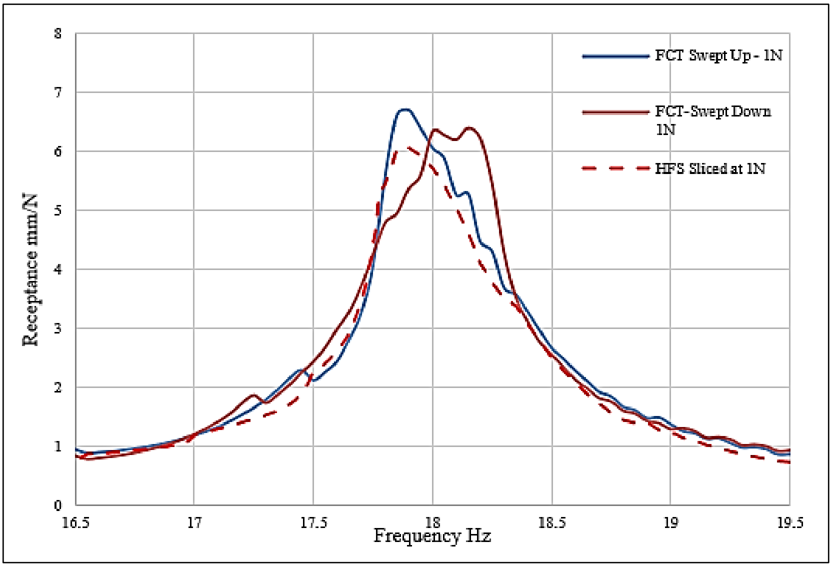


Figure 11: Comparison between RCT-HFS sliced at 1 N and FCT swept up and down

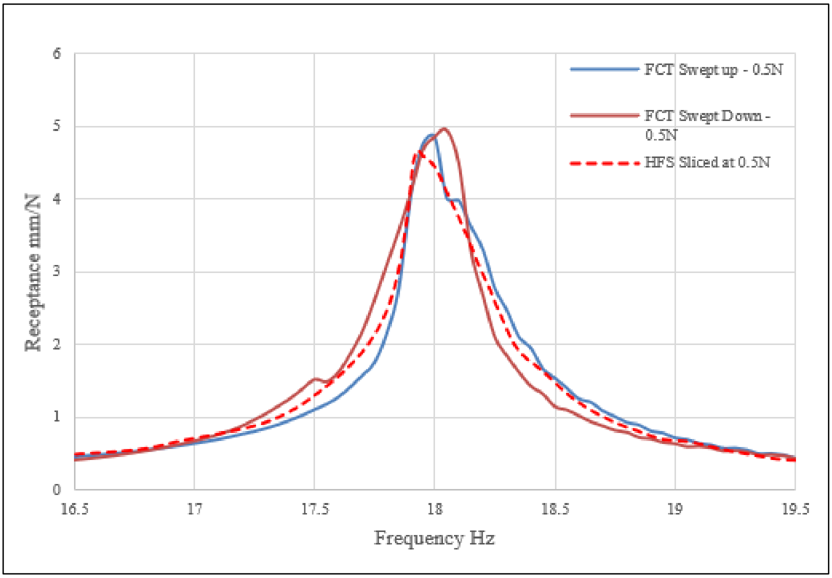


Figure 12: Comparison between RCT-HFS sliced at 0.5 N and FCT swept up and down

Conclusions

This study focuses on the identification of nonlinear behaviour using the methodologies of RCT and FCT techniques and examines the effects of excitation amplitudes on the NLFRFs of the bolted beam. The study has clearly shown that RCT and HFS are powerful and efficient techniques for identifying nonlinearity in the bolted beam. By using RCT, the process of identifying the nonlinearity is faster than FCT. Moreover, HFS identifies the NLFRFs of the bolted beam with the same accuracy as FCT.

Overall, the most striking and completely new perspective observed in this study is that RCT and HFS are better than FCT in terms of nonlinearity identification and measurement time, which is almost twice as fast as FCT. This advantage may provide engineers and researchers with a powerful technique for accurate and efficient identification of nonlinear structures.

Contributions of Authors

The authors confirm the equal contribution in each part of this work. All authors reviewed and approved the final version of this work.

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Conflict of Interest

All authors declare that they have no conflicts of interest.

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