

**UNSTEADY SHEAR-STRESS-DRIVEN RIVULETS FLOW OF  
NEWTONIAN AND NON-NEWTONIAN POWER-LAW FLUIDS  
WITH STRONG SURFACE-TENSION EFFECT**

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## ABSTRACT

Studying thin-film flow provides several significant advantages in both scientific and practical contexts. Understanding the behaviour of thin-films under various conditions is crucial for optimising a wide range of industrial processes. This research investigates the thin-film flow of rivulets for both Newtonian and non-Newtonian power-law fluids on an inclined plane, emphasising the role of shear stress at the free surface in driving the flow under strong surface tension effects. The study investigates the dynamics of rivulets under different parameters of fluid viscosity, power-law indices, and shear stress through a combination of theoretical modelling and numerical simulations. The lubrication approximation is used to solve the continuity equation and the Navier-Stokes equations. A fourth-order governing partial differential equation is obtained by applying these equations to the kinematic condition, the no-slip and no-penetration boundary conditions, and the balances of normal and tangential stress. The similarity transformation method is then used to reduce the governing equation to an ordinary differential equation. The governing fourth-order ODE was solved numerically using the Runge-Kutta-Fehlberg Fourth Fifth (RK45) in Maple. The results show that the single-humped cross-sectional profiles is achieved to be the physically realisable solution. The results show that the rivulet widens and thickens at any given time, while narrowing and thinning at any given position. These results are consistent with previous studies and are independent of the power-law index  $N$ . This research enhances understanding of rivulet behaviour, which is important for applications like coating and microfluidic systems. It provides a foundation for future experimental studies and broader parameter investigations.

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