Volume 4 Issue 2 (September 2024)
DOI 10.24191/jaeds.v4i2.81
eISSN 2805-5756

Journal of Applied Engineering Design & Simulation

Comparative Computational Modal Analysis of Uniform and Tapered Plates

N.A.Z Abdullah[1], M.S.M. Sani[1,2,*]
[1]Centre for Automotive Engineering, Universiti Malaysia Pahang Al-Sultan Abdullah, 26600 Pekan, Pahang, Malaysia.
[2]Faculty of Mechanical and Automotive Engineering Technology, Universiti Malaysia Pahang Al-Sultan Abdullah, 26600 Pekan, Pahang, Malaysia.
[*]corresponding author: mshahrir@umpsa.edu.my

ABSTRACT

Thickness variation in structures has significant influence in their structural properties which makes it as an important consideration in engineering design. Varying thickness profile has gained many attention in its application due to the potential to optimise performance and reduce material usage. However, the impact of these thickness variations on modal behaviour of structures has not been fully understood. This study aims to investigate the effects of thickness variation on the modal properties of structural plates. Two plates with different configuration were modelled in MSC.Nastrab/Patran Software in which one plate has uniform thickness while the other has tapered thickness. Finite element modal analysis was performed to determine the natural frequencies and mode shapes of both plates. The results revealed that the plate with varying thickness has lower natural frequencies and its mode shapes are more complex and asymmetric compared to the plates with uniform thickness. These findings suggest that thickness variation can significantly alter the vibrational characteristics of structures which is important in design optimization of applications such as automotive and aerospace engineering.

Keywords: Thickness variation, structural plates, vibrational behaviour, finite element analysis.

Abbreviations

FGM	Functionally graded material
PVB	Polyvinyl butyral
UT	Uniform thickness
VT	Varying thickness

1.0 INTRODUCTION

The study of thickness profile in structures is one of the significant topics in engineering application in which the thickness profile is one of important aspects or properties that needs to be considered in functionally graded materials (FGMs). Thickness variation has a significant impact on structural properties and can influence the structure's performance. The changes in modal properties for structures with varying thickness and uniform thickness have also become the topic of interest in material science and structural engineering. There are many studies that have explored the impact of non-uniform thickness variations on the mechanical response of the structure as reported by Sarkar and Rahman and Madan abd Bhowmick [1, 2]. It was found by Sarkar and Madan that FGM disks with non-uniform thickness profile is more efficient in terms of thermo-mechanical behaviour than the classic uniform thickness one [1]. Madan and Bhowmick have done investigations on rotating disks and proved that speed limit increases substantially as the thickness profile transitions from uniform to varyinng thickness [2]. These studies highlighted the importance and need to understand how the thickness variation influences the performance of a structure.

Other previous studies also investigated the effectiveness of varying thickness profile. For example, Meng and Dou conducted an optimization work on new structure of local uniform thickness and the traditional variable thickness designs for radomes [3]. Similarly, Yuan conducted topology optimization work on stamping structures which stated the significance of considering thickness heterogeneity in the design [4]. There are other studies that investigated the optimal design of variable thickness plates to minimize vibration. The study emphasised the importance of considering varying thickness profiles for effective vibration reduction [5]. Lal and Siani also conducted a vibrational analysis on graded circular plates with variable thickness which suggested that the structure with varying thickness is gaining attention in many engineering application [6]. There is also a study that optimized an annular fin profile for heat dissipation and thermal stress reduction which displayed that the concept of varying thickness has been explored to achieve optimal performance [7].

In the field of automotive, the study of profile thickness is one of the important research area as the industry strives to look for weight reduction for fuel efficiency and material savings. Understanding the behaviour with different thickness profile is important when enhancing the efficiency, safety and the functionality of automotive components. For example, Baraotaji et al. conducted a study on a varying thickness profile for crash box structure

Received on 21.08.2024
Accepted on 27.09.2024
Published on 27.09.2024

and it shows promising energy absorption characteristics suitable for vehicle crashworthiness applications [8]. Xu et al. studied the energy absorption capabilities of material with different interlayer thicknesses to evaluate the impact of thickness variation as well as material properties and impact speed with the energy absorption capabilities [9]. Varying thickness of materials such as PVB interlayer as mentioned by Xu et al. in their study shows that it has impact on energy absorption capabilities [9]. Additionally, research on thin-walled titanium alloy sheets has demonstrated that gradient thickness profile affects residual stress and springback, which highlighted the importance of thickness variations in manufacturing process [10].

In summary, the study of thickness profile in engineering structures has emerged as a critical area of research due to its significant impact on mechanical behaviour and performance. Previous study has demonstrated that thickness variation can lead to superiour energy absorption characteristics, better crashworthiness and control on residual stresses in manufacturing process. While the effects of thickness variation on structural properties are acknowledged, our study provides new insights into how varying thickness profiles specifically influence the modal behaviour of structural plates. The novelty of this work lies in its detailed examination of the relationship between non-uniform thickness and modal properties, particularly the natural frequencies and mode shapes of the plates. Previous studies have also explored general impacts of thickness variations, but this study uniquely focuses on quantifying these effects through empirical analysis and comparing the results for uniform and varying thickness profiles. This study intends to explore and quantify the effects of varying thickness profiles on the natural frequencies and mode shapes of structural plates. By comparing the natural frequencies and mode shapes of two plates with uniform and varying thickness profile, this study will discuss on the its effect on the vibrational characteristics of structures. The findings from this study will provide insights on how thickness profile affects the dynamic response of materials which will impact the design and performance of structures in various industries.

2.0 METHODOLOGY

2.1 Structure modelling

Two plates with different configuration were modelled in MSC.Nastrab/Patran Software (refer Fig. 1). The first plate, which is referred to as the uniform thickness plated (UT plate), has constant thickness throughout the entire plate. The first plate was modelled as a rectangular solid with constant thickness of 10mm and the dimensions of the plate was set to 200mm × 500mm. This model was set as a standard baseline model for later comparison. The second plate which is also referred to as the varying thickness (VT) plate has similar dimension to the the uniform thickness plate but features a linear thickness transition from 15mm at one end, and 5mm at the opposite end. This design is used with the aim to investigate the effects of non-uniform thickness on modal behaviour. This design is also chosen to simulate an engineering scenario where material savings and structural optimization can be achived by using structure with varying thickness.

In both plates, the geometric models were constructed as solid so that the thickness variation can be accurately modelled. In fact, the shelle elements is seen as unsuitable for this study because it can only better suited for uniform thickness profile and might not be able to capture the effects of varying thickness very well. The plates model was meshed into isometric hexahedral elements. The mesh size was refined after mesh convergence study was conducted in order to get the accurate results with optimum computational efforts. The mesh convergence study was conducted by refining the mesh size iteratively and observing the changes in natural frequencies value until the stabilised value was achieved. This is to ensure the model reliability where it can be verified that the results obtained will not significantly be affected by the mesh size. The material properties of steel were assigned for both plates with the value of Young’s modulus of 200GPa, density of 7800kg/m^3, and Poisson’s ratio of 0.3. The standard material properties for steel were used and selected based on published studies data and also manufacturer specifications which are derived from standardised experimental tests. This is to ensure that the models behave according to the assigned materials performance. All these properties are the critical input for the finite element model which can directly influence the stiffness and mass distribution of the plates and hence can affect the modal properties of the plates.

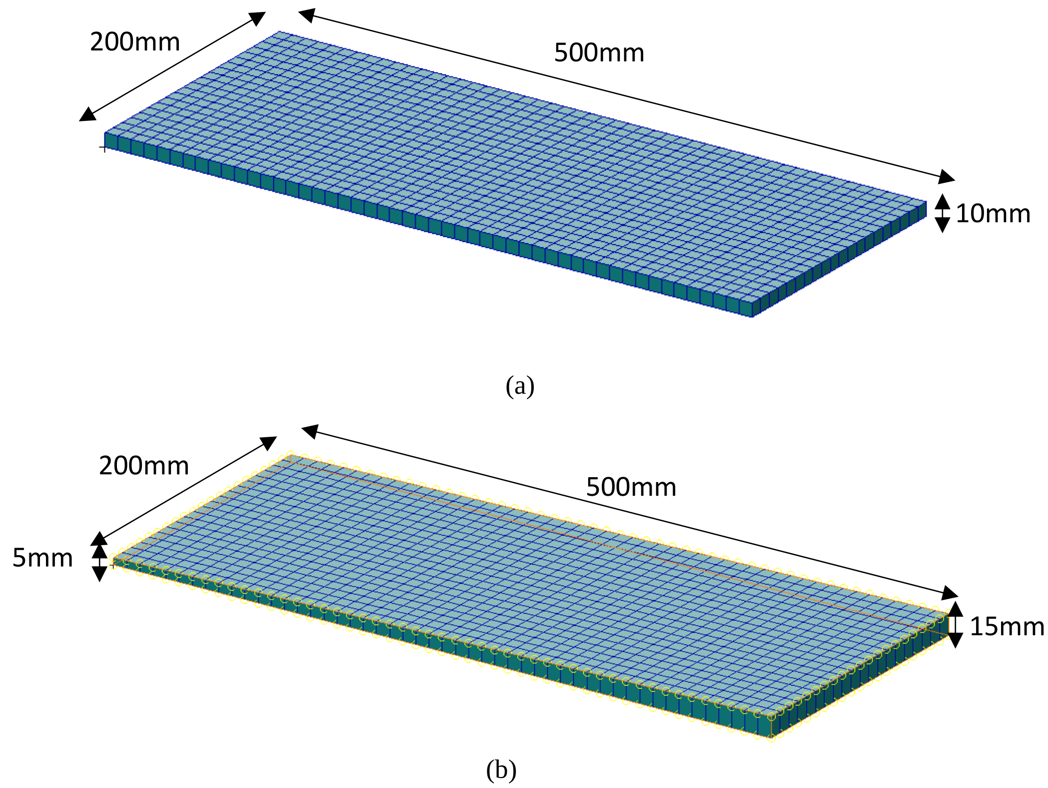


Figure 1. Finite element model of (a) UT plate and (b) VT plate

2.2 Finite element analysis

Both finite element of the plates were set to remain in free-free boundary condition where no loads or constraints were applied. This is because of the intention of this study which is to focus on evaluating the inherent vibrational properties of the plates and therefore the influence of external forces or restraints can be eliminated. The free-free condition is considered by many previous studies and proved to be useful in modal analysis as it allows the natural frequencies and mode shapes to be determined puurely based on the geometry and material properties [11,12].

The analysis was conducted in linear static modal analysis using SOL103 in order to determine the natural frequencies and the mode shapes of both plates. SOL 103 performs a linear perturbation analysis that assumes material behaviour of the structure remains linear with small deformations [13,14]. During the analysis, first ten natural frequencies and their corresponding mode shapes were obtained for both plates. The value of natural frequencies and mode shapes obtained from both plates were compared and analysed to determine the impact of thickness variation on the modal behaviour. The shifts in natural frequencies and difference in mode shapes as well as the percentage differences in natural frequencies value were calculated and compared to quantify the impact of the thickness variation and therefore can provide insights into potential benefits or disadvantages of using tapered design structure. The mode shapes were further examined to classify the nature of vibrations and to see if the thickness variation can show any new any unexpected vibrational patterns. The results are discussed in detail in the subsequent section where the implications of the findings are explored with respect to their suitability for engineering applications.

3.0 RESULTS AND DISCUSSION

The values of the natural frequencies for the UT and VT plates as well as the percentage difference between the natural frequencies value for each mode are illustrated in Table 1. Based on the table, the percentage differences in natural frequencies vary across the modes. The analysis shows a reduction in natural frequencies for the VT plates compared to the UT plates. The percentage differences in natural frequencies between the two plate types vary by mode with initial modes show smaller differences that ranged from 0.84% to 4.17%. Intermediate modes show more significant variations which reflect the impact of thickness changes on these modes. In the intermediate modes (modes 4-7), larger percentage differences are noted with mode 5 and mode 7 showing the highest discrepancies at 13.85% and 18.71%, respectively. These higher values suggest more sensitivity towards the local variations in thickness as the modes involve more complex bending and twisting deformation. In addition, the varying thickness can also has stiffness reduction in some regions which lead to a noticeable decrease in natural frequencies values. Meanwhile, in higher modes which are mode 8 to mode 10, the percentage differences show varying trends. There is a negative difference in mode 8 which indicates that the varying thickness configuration slightly increases the stiffness in mode shapes's region of interest due to the thicker sections that coincide with critical deformation areas. The other high modes also still show sensitivity to the thickness variation with reduced value of natural frequencies. This suggests that stiffness reduction generally outweights any potential of mass reduction benefits as the mass of both plates was kept to be constant [15].

Meanwhile,

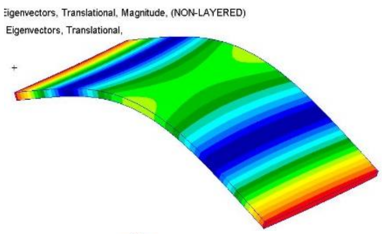
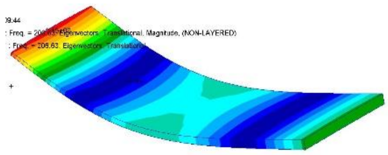
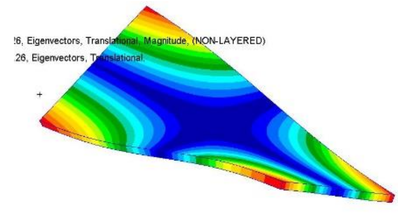
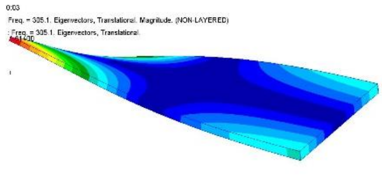
Table 2 shows the comparison of mode shapes in UT and VT plates. It can be seen from the mode shapes that as the modes increase, there are more transitions from pure bending to more complex interaction. There are also variations in mode shapes between the UT and VT plates which indicate how thickness affects vibrational characteristics. The varying thickness causes more complex mode shapes due to the changes in stiffness distribution. It is expected that in UT plate which has uniform distribution of thickness will lead to smoother deformation patterns. The shown mode shapes also exhibit a straighforward progression from lower-order bending modes to more complex higher-order modes which reflect the uniform stiffness and mass distribution throughout the plate. However, VT plate which introduces non-uniform stiffness has the mode shapes to be significantly affected.

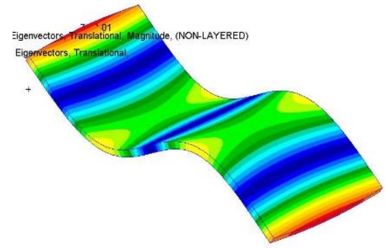
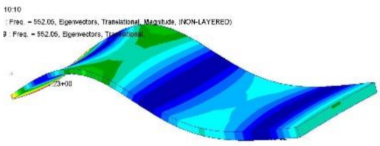
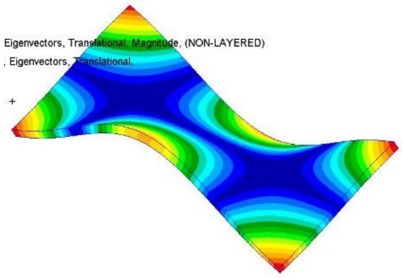
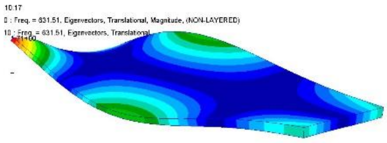
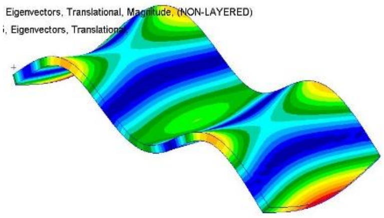
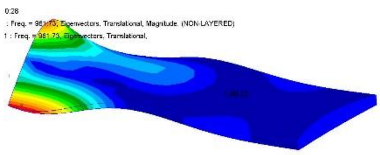
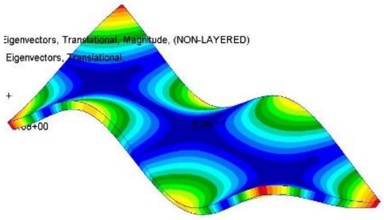
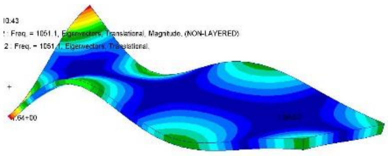
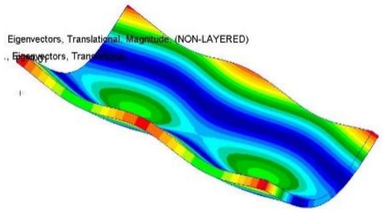
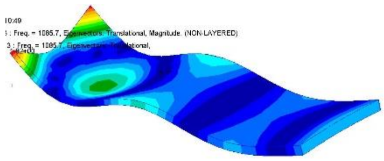
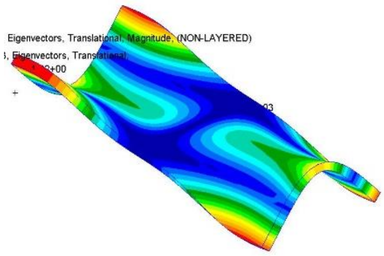
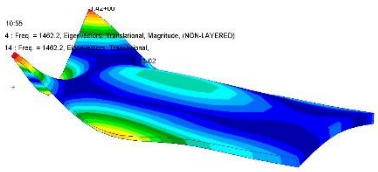
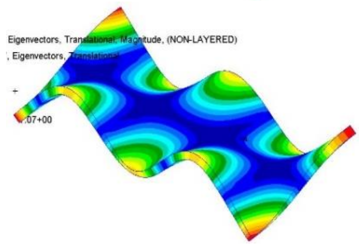
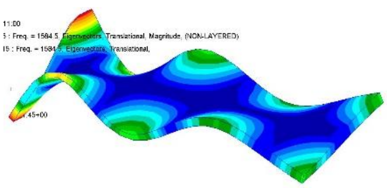
The non-uniformity resulted to more complex deformation behaviour at lower modes and accelerated the transition to higher-order modes [16,17]. The increase complexity in the deformation pattern is shown in the regions with reduced thickness. Also, the varying thickness causes the mode shapes to become asymmetry even in the lower modess. The asymmetry condition can have significant impact in application where uniform stress distribution or predictible vibrational behaviour is critical. Thus, when designing structures with non-uniform thickness, one should pay attention to these differences as the altered mode shapes in varying thickness structure could lead to unexpected resonances or stress concentrations [18].

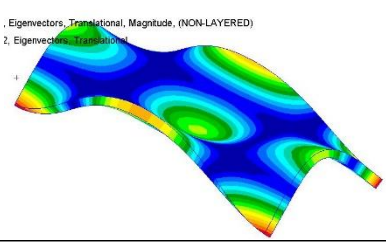
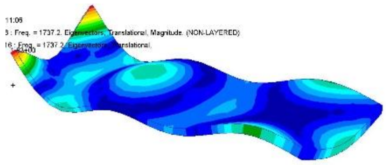
Table 1: Natural frequencies of the uniform thickness plate and the varying thickness plate

Modes	Natural frequencies (Hz)		
	Uniform thickness (UT)	Varying thickness (VT)	Percentage difference (%)
1	208.38	206.63	0.84
2	316.26	305.10	3.53
3	576.10	552.05	4.17
4	676.85	631.51	6.69
5	1116.5	961.73	13.85
6	1125.2	1051.1	6.58
7	1336.0	1085.7	18.71
8	1446.3	1462.2	-1.10
9	1698.7	1584.5	6.72
10	1845.2	1737.2	5.85

Table 2: Comparison of mode shapes

Modes	Mode shapes		
	Uniform thickness (UT)	Varying thickness (VT)	Remarks
1			Bending
2			Torsion

Modes	Mode shapes		
	Uniform thickness (UT)	Varying thickness (VT)	Remarks
3			Bending
4			Coupled bending-torsion
5			Bending
6			Coupled bending-torsion
7			Bending/ mixed
8			Torsion/ mixed
9			Torsional-flex

Modes	Mode shapes		
	Uniform thickness (UT)	Varying thickness (VT)	Remarks
10			Complex mixed

4.0 CONCLUSION

This study investigates the impact of thickness variation on modal behaviour, by highlighting that varying thickness profiles influence natural frequencies and mode shapes. While detailed numerical quantification of frequency changes with specific thickness reduction is beyond the scope of this study, the findings contribute to the understanding towards the effects of thickness variation on modal characteristics. Based on the discussion and results presented, it is apparent that thickness profile has a significant impact on the dynamic characteristics of the structure which influences the natural frequencies value and the mode shapes of the structure. Novel contributions are introduced by providing detailed analysis of the effects of varying thickness profiles on modal behaviour, focusing on natural frequencies and mode shapes. This work is able to provide significant information for designing structure with optimised performanced and material efficiency. The findings of this study can be summarised as follows:

i. The thickness variation of the VT plates significantly affects the natural frequencies and mode shapes. The VT plate shows generally lower natural frequencies compared to UT plate especially in intermediate modes with complex bending and twisting deformations.
ii. Middle modes (mode 4 to mode 7) display more sensitivity to thickness variation with noticeable percentage differences in natural frequencies. Local stiffness reductions in VT plates also cause the decrease in natural frequencies. This aspect needs to be taken into consideration for further structural design process.
iii. VT plate shows more complex and asymmetrical mode shapes compared to UT plate. This indicates the non-uniform stiffness distribution in VT plate accelerates the transition to higher order modes and causes asymmetry. This can affect applications that require uniform stress distribution and predictable vibrational behaviour.

For further research on this topic, other researchers can explore the interaction of thickness variation with other design parameters such as material properties, boundary conditions and others so that optimize structures for enhanced vibration performance can be suggested.

.ACKNOWLEDGEMENT

The authors are grateful to Universiti Malaysia Pahang Al-Sultan Abdullah (UMPSA) and the Automotive Engineering Centre (AEC) for all the given support.

REFERENCES

[1] P.R. Sarkar, A.S. Rahman, Finite-Difference Analysis of Stresses of a Non-Uniform Functionally Graded Material Circular Disk Rotating in the Magneto-Thermal Environment: An Equal Mass Study, *Proc. Inst. Mech. Eng. Part L J.* Mater. Des. Appl. 237 (2022) 301–316. https://doi.org/10.1177/14644207221111292.
[2] R. Madan, S. Bhowmick, A Numerical Solution to Thermo-mechanical Behavior of Temperature Dependent Rotating Functionally Graded Annulus Disks, *Aircr. Eng. Aerosp. Techno*l. 93 (2021) 733–744. https://doi.org/10.1108/aeat-01-2021-0012.
[3] H. Meng, W. Dou, Multi-Objective Optimization of Radome Performance With the Structure of Local Uniform Thickness, Ieice Electron. *Express. 5* (2008) 882–887. https://doi.org/10.1587/elex.5.882.
[4] Z. Yuan, L. Geng, N. Wang, T. Wu, W. Qi, Y. Dai, J. Huang, Topology Optimization Method of Stamping Structures Based on the Directional Density Field, *Materials* (Basel). 17 (2024) 656. https://doi.org/10.3390/ma17030656.
[5] H. Li, D. Wang, H. Zhang, X. Wang, Z. Qin, Z. Guan, Optimal design of vibro-impact resistant fiber reinforced composite plates with polyurea coating, *Compos. Struct.* 292 (2022) 115680.

https://doi.org/https://doi.org/10.1016/j.compstruct.2022.115680.

[6] R. Lal, R. Saini, Vibration analysis of functionally graded circular plates of variable thickness under thermal environment by generalized differential quadrature method, *J. Vib. Control.* 26 (2019) 73–87. https://doi.org/10.1177/1077546319876389.

[7] A. Deka, D. Datta, Optimum Annular Plane Fin Profile with Uniformly Varying Thickness, in: L. Vijayaraghavan, K.H. Reddy, S.M. Jameel Basha (Eds.), *Lect. Notes Mech. Eng.*, Springer Singapore, Singapore, 2020: pp. 427–435. https://doi.org/10.1007/978-981-32-9931-3_41.

[8] A. Baroutaji, A. Arjunan, M. Stanford, J. Robinson, A.G. Olabi, Deformation and Energy Absorption of Additively Manufactured Functionally Graded Thickness Thin-Walled Circular Tubes Under Lateral Crushing, *Eng. Struct*. 226 (2021) 111324. https://doi.org/10.1016/j.engstruct.2020.111324.

[9] J. Xu, Y.B. Li, X. Chen, D.Y. Ge, B.H. Liu, M. Zhu, T. Park, Automotive Windshield — Pedestrian Head Impact: Energy Absorption Capability of Interlayer Material, *Int. J. Automot. Technol.* 12 (2011) 687–695. https://doi.org/10.1007/s12239-011-0080-2.

[10] Y. Shu, J. Ren, W. Zhang, W. Liu, Gradient Thickness-Dependent Distribution of Residual Stress and Springback of Thin-Walled TC4 Titanium Alloy Sheet With Variable Thickness in Collaborative Manufacturing Process of Pre-Plastic Forming and Milling, (2024). https://doi.org/10.21203/rs.3.rs-4308626/v1.

[11] G. Yao, C.K. Mechefske, B.K. Rutt, Characterization of Vibration and Acoustic Noise in a Gradient-Coil Insert, Magn. Reson. Mater. Phys. Biol. Med. 17 (2004) 12–27. https://doi.org/10.1007/s10334-004-0041-0.

[12] N.A.Z.Z. Abdullah, M.N.A.M. Asri, M.S.M.S.M. Sani, Strategies of Finite Element Modeling for Spot Welded Joints and its Modal Correlation with Experimental Data, I*nt. J. Automot. Mech.* Eng. 19 (2022) 9543–9550. https://doi.org/10.15282/ijame.19.1.2022.17.0736.

[13] Y.N. Aydın, T.B. Korkut, O. Ozaydin, E. Armakan, G. Sarı, A. Goren, Numerical and Experimental Modal Analysis of Wheels of Solaris 10 Solar Car and Parametric Design of Lightweight EV Wheel, Deu Muhendis. *Fak. Fen Ve Muhendis*. 23 (2021) 689–699. https://doi.org/10.21205/deufmd.2021236829.

[14] N.A.Z. Abdullah, M.S.M. Fouzi, M.S.M. Sani, Computational modal analysis on finite element model of body-in-white structure and its correlation with experimental data, *Int. J. Automot. Mech.* Eng. 17 (2020) 7915–7926. https://doi.org/10.15282/ijame.17.2.2020.10.0591.

[15] S. Chai, S.W. Yang, Z.Q. Wang, Y.X. Hao, W. Zhang, Variable Stiffness and Free Vibration Analysis of Cylindrically Curved Plate with Variable Thickness Graphene Reinforced Porous Material, *J. Vib. Eng. Technol.* (2024). https://doi.org/10.1007/s42417-024-01451-8.

[16] E.B. Magrab, Thin Beams: Natural Frequencies and Mode Shapes, in: E.B. Magrab (Ed.), *Springer Nature Switzerland, Cham*, 2024: pp. 67–182. https://doi.org/10.1007/978-3-031-52102-7_3.

[17] K. He, W.D. Zhu, Modeling of Fillets in Thin-Walled Beams Using Shell/Plate and Beam Finite Elements, *J. Vib*. Acoust. 131 (2009) 0510021–05100216. https://doi.org/10.1115/1.3142879.

[18] A. Sinha, A New Approach to Compute Natural Frequencies and Mode Shapes of One-Dimensional Continuous Structures With Arbitrary Nonuniformities, *J. Comput. Nonlinear Dyn.* 15 (2020). https://doi.org/10.1115/1.4048360.