INTRODUCTION TO STATISTICS (STA104)

INTRODUCTION TO PROBABILITY

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Preface

In the name of Allah, the Almighty who give us the enlightenment, the truth, the knowledge and with regards to Prophet Muhammad (peace be upon him) for guiding us to the straight path. We thank to Allah for giving us guidance and strength to write this e-book.

This e-book consists of five sections which starts with introduction. In Introduction section, some definitions are stated to give an information to the students. The next sections that follow are set theory, rules of probability, counting rule, and tree diagram and Bayes' theorem. In each section, some examples and exercises are given to give the students better understanding.

We hope that this e-book will meet the requirements and the expectations of all the diploma students who take Introduction to Statistics course.

LEARNING OBJECTIVES

By the end of this topic students should be able to :

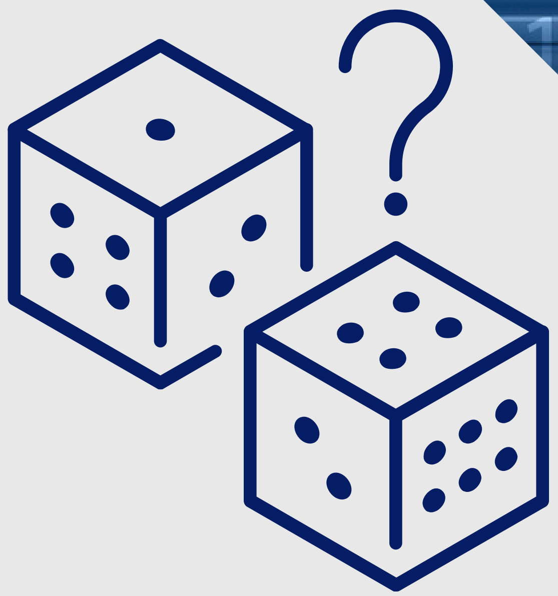
1. Understand the concept of additional and multiplication rules in probability.
2. Understand the concepts of counting rules, permutation and combination
3. Solve the problem involve counting rules, permutation and combination
4. Construct tree diagram based on the information given.
5. Solve the conditional probability problem using Bayes' theorem.

01 INTRODUCTION

INTRODUCTION TO PROBABILITY

WHAT IS PROBABILITY ?

- Probability is an analysis of the likelihood that an event will happen.
- It is about the chance and opportunity that some events will happen.



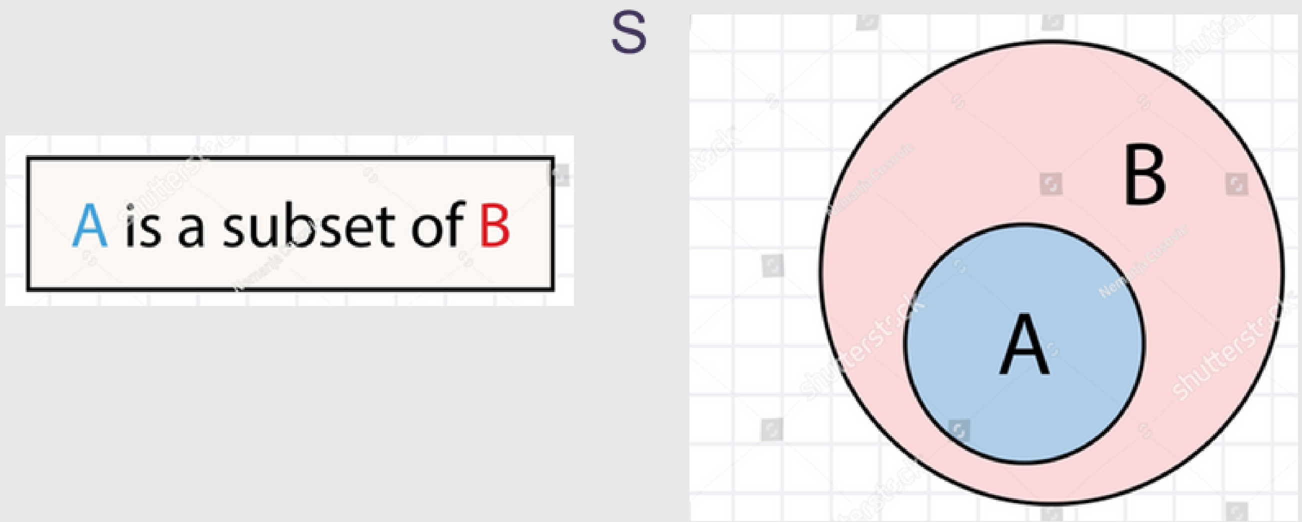
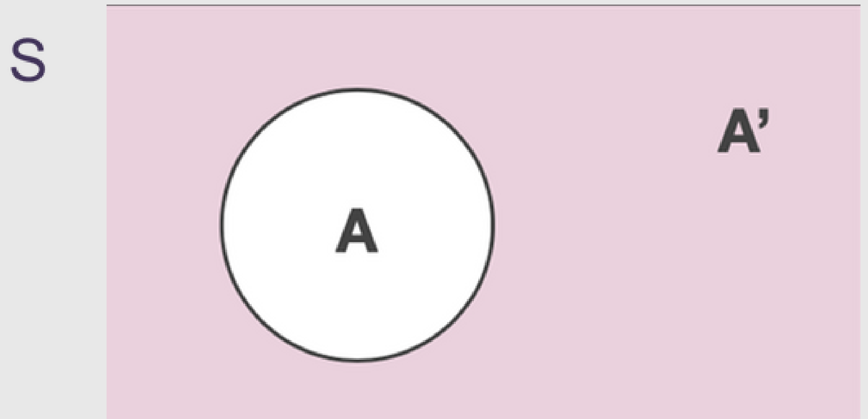
02 SET THEORY

INTRODUCTION TO PROBABILITY

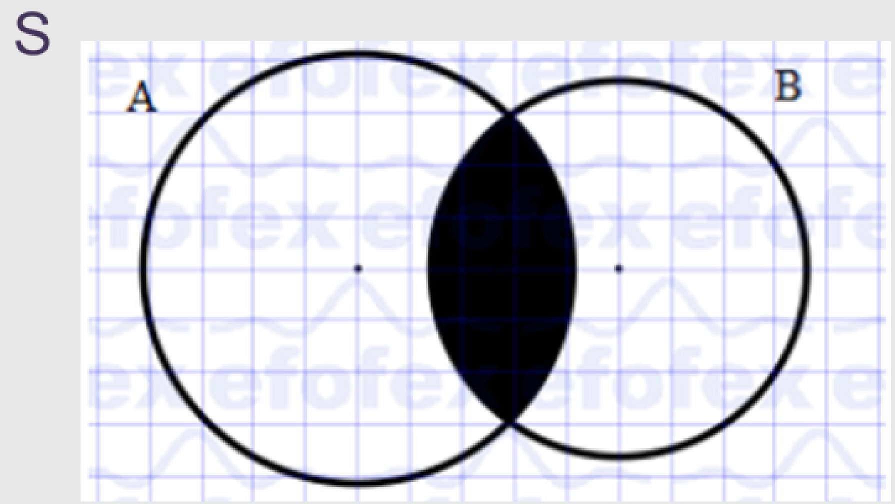
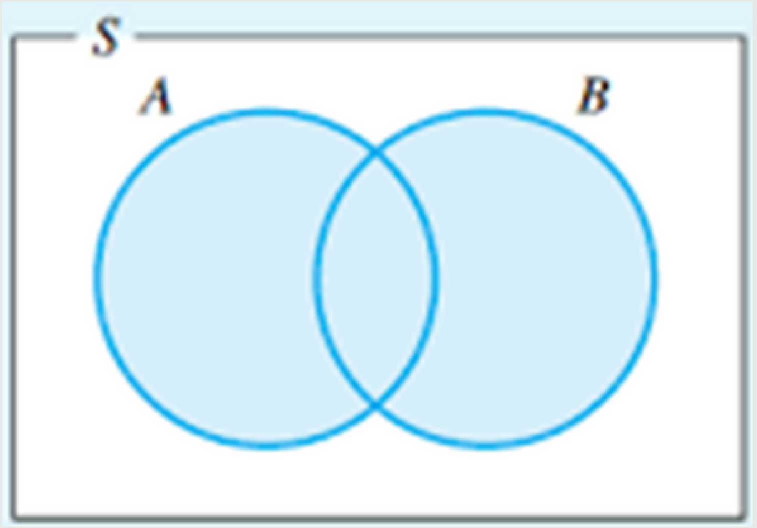
SET THEORY

	Definition	Example
Experiment	A process that generates a set of data	We roll a fair die.
Sample Space, S	A set of all possible outcomes. The possible outcomes is also known as the element	The possible outcomes is one, two, three, four, five or six. S = {1, 2, 3, 4, 5, 6}
Event, A	A set of outcomes or a subset of the sample space.	Let A be the event of getting an even number. There are 3 possible outcomes in this event. A = {2, 4, 6}
Number of elements / possible outcomes	-	There are six elements in the sample space, n(S)=6. There are three elements in A, n(A)=3.

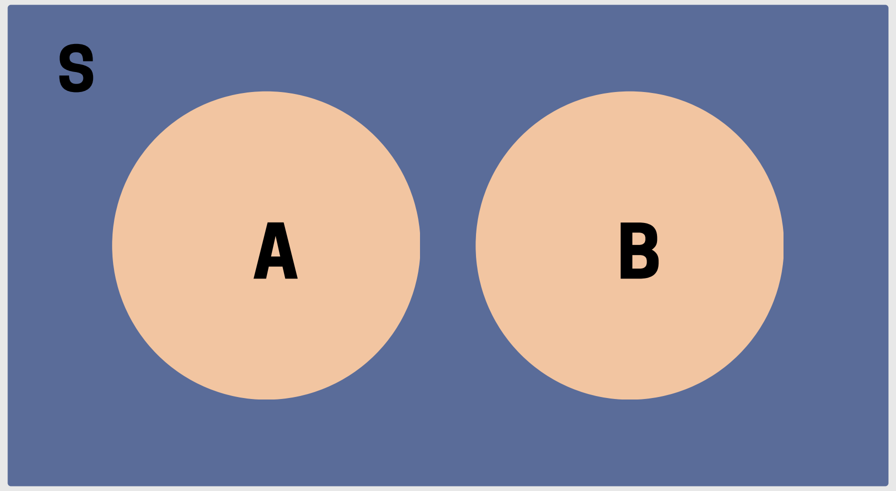
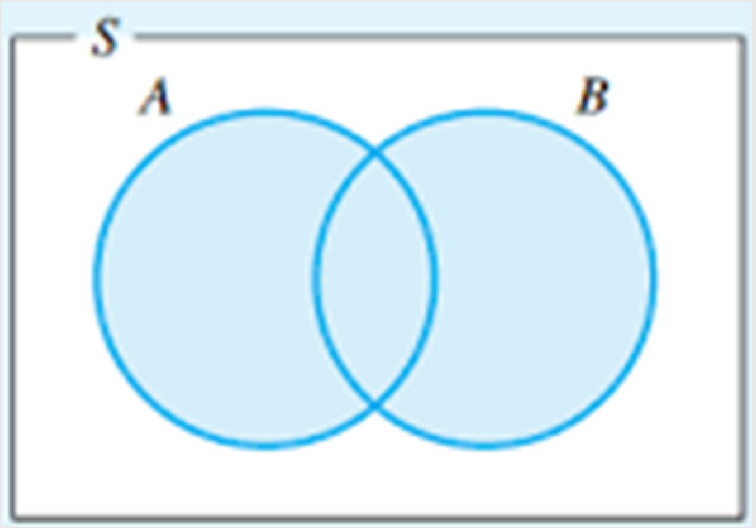
SET THEORY

	Definition	Illustration
Empty Set, {}	A set with no element	
Subset	A set A is subset of another set B if all the elements in A are also in B such A⊂ B.	
Complement of an event	The compliment of event A with respect to S is denoted by A’.	

SET THEORY

	Definition	Illustration
Intersection of two events	The intersection of A and B, written as A ∩ B, whose outcomes belong to both A and B	A ∩ B is shaded 
Union of two events	Union of A and B are written as A U B, event whose outcomes belong to either A or B or both.	A U B is shaded 

SET THEORY

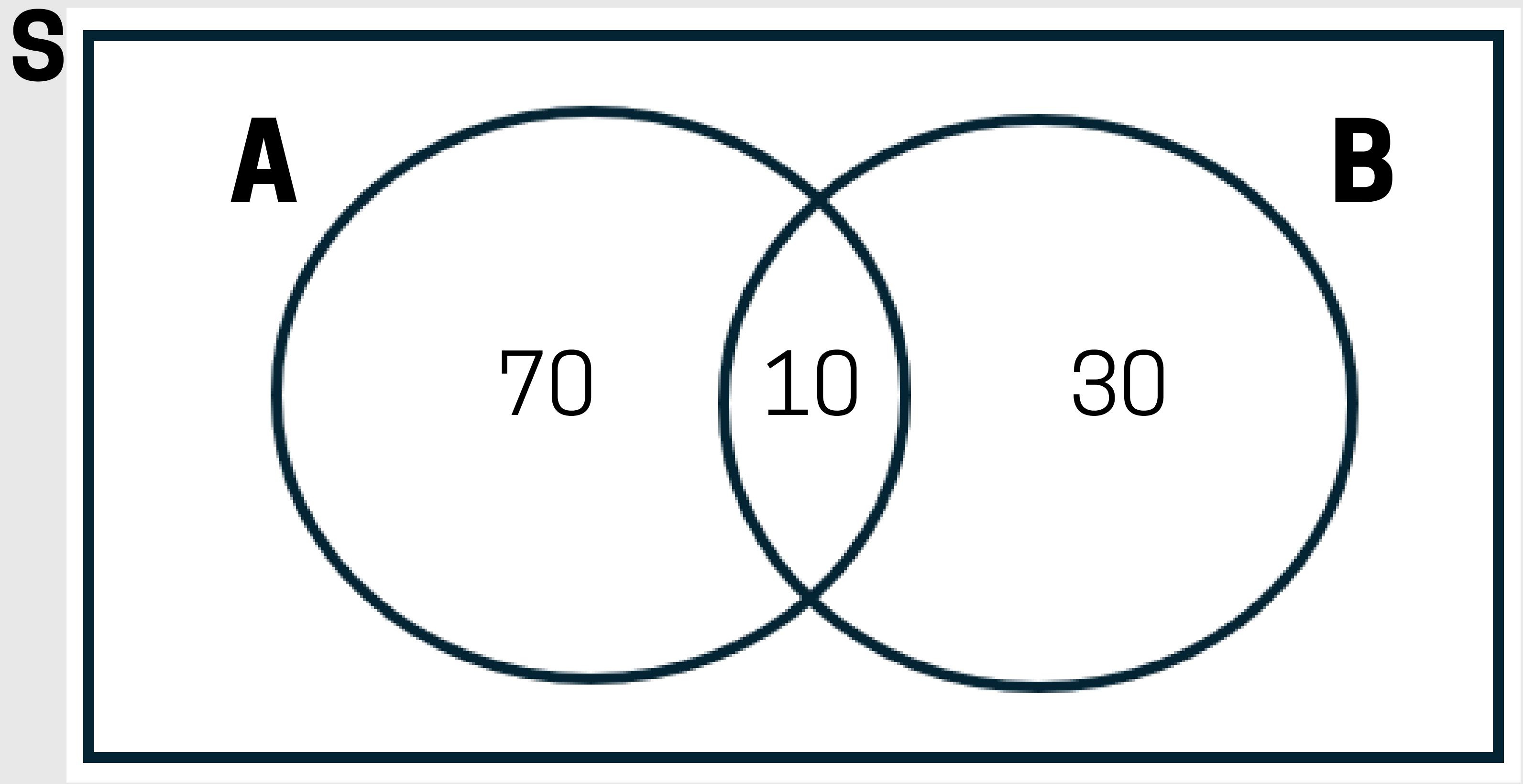
	Definition	Illustration
Mutually exclusive events	Events A and B are said to be mutually exclusive if they cannot occur at the same time	
Non-Mutually exclusive events	Events A and B are said to be mutually exclusive if they can occur at the same time	

EXAMPLE 2.1

A researcher conducted a survey to find out where people did their holiday shopping. Out of a group of 110 randomly selected shoppers, 70 said that they shopped exclusively at the local mall, 30 said they shopped exclusively in the downtown area and 10 said that they shopped both at the local mall and in downtown area. Illustrate the events in a Venn Diagram.

EXAMPLE 2.1 (SOLUTION)

A researcher conducted a survey to find out where people did their holiday shopping. Out of a group of 110 randomly selected shoppers, 70 said that they shopped exclusively at the local mall, 30 said they shopped exclusively in the downtown area and 10 said that they shopped both at the local mall and in downtown area. Illustrate the events in a Venn Diagram.



EXAMPLE 2.2

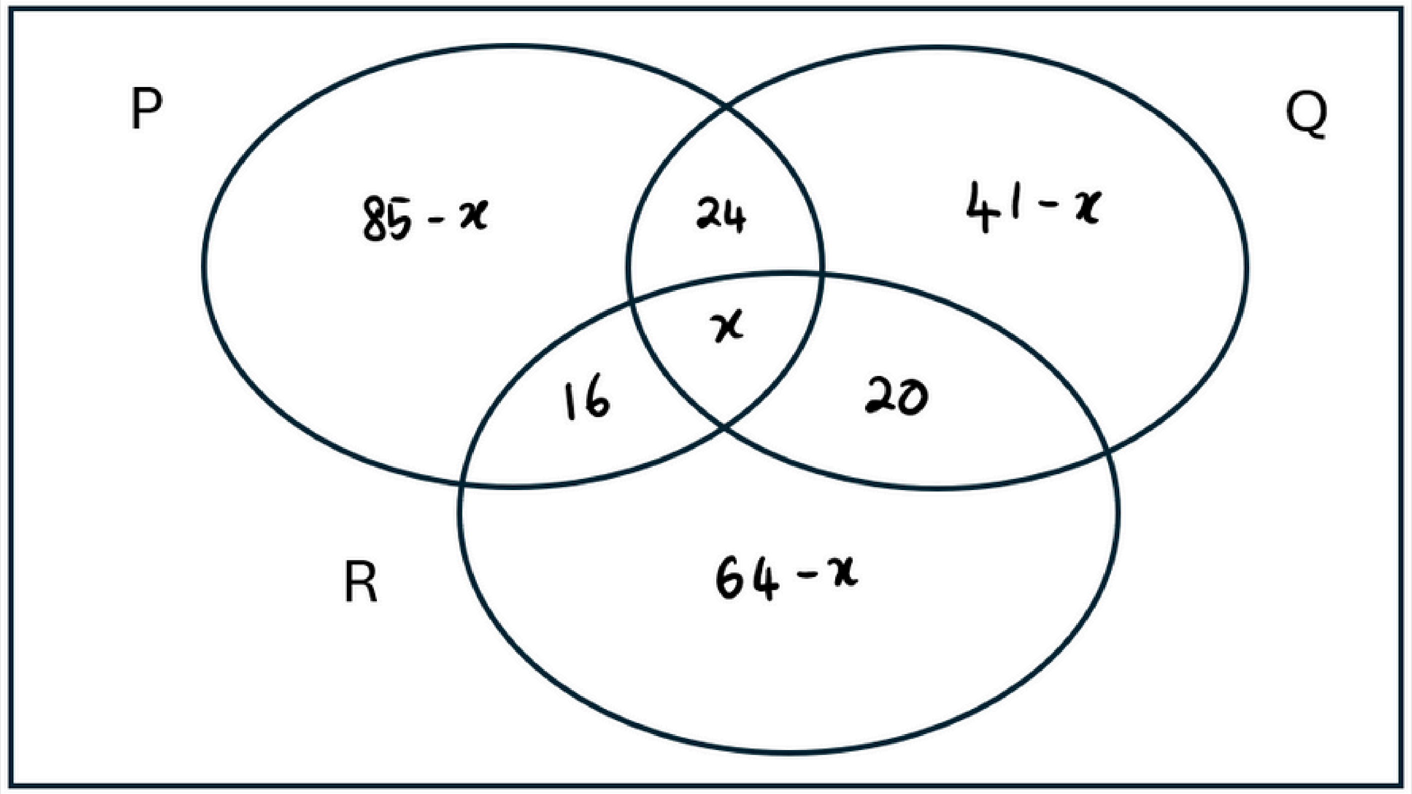
A health product consists of 3 types of ingredients namely P, Q, and R. A defective product can be due to the contamination in one, two or three ingredients. Out of 210 defective products, it was found that 125 was due to the contamination of P, 85 due to Q, and 100 due to R. From the defective products, it was found that 60 has exactly two contaminated ingredients of which 24 was contaminated in P and Q only ang 16 was contaminated in P and R only.

a) Draw a Venn diagram to represent the above data.
b) Find the number of defective products that are due to the contamination of all three ingredients.
c) Find the number of defective products that contains exactly one ingredient that is contaminated.

EXAMPLE 2.2 (SOLUTION)

a) $P = 125, Q = 35, R = 100$

$P \cap Q = 24, \quad P \cap R = 16, \quad Q \cap R = 20$



b) $(85 - x) + 24 + (41 - x) + 16 + x + 20 + (64 - x) = 210$

$-2x + 250 = 210$

$-2x = -40$

$x = 20$

c) $65 + 21 + 44 = 130$

PROBABILITY OF EVENT

Let A be an event defined over S. The probability of event A is the summation of the probabilities of all the sample points in A. It is denoted by P (A).

$$P(A) = \frac{n(A)}{n(S)}$$

$P(A)$ =Probability of an event A
$n(A)$ = number of elements in event A
$n(S)$ = number of elements in sample space S

PROPERTIES OF PROBABILITY

Each event A in the event space has a probability *P(A)* satisfying

$$0 \leq P(A) \leq 1$$

If an event A is certain to occur, then

$$P(A) = 1$$

If an event A is impossible to occur, then

$$P(A) = 0$$

The compliment of event A is denoted by

$$\bar{A}$$

and its probability is denoted by

$$P(\bar{A})$$

The sum of probabilities of $P(A)$ and $P(\bar{A})$ must equal to 1, that is

$$P(A) + P(\bar{A}) = 1$$

EXAMPLE 2.3

Consider an experiment of tossing two coins simulteneously.

a) List all possible outcomes of the given experiment

b) Find the probability of getting one tail.

b) Find the probability of getting two head.

EXAMPLE 2.3 (SOLUTION)

Consider an experiment of tossing two coins simulteneously.

a) List all possible outcomes of the given experiment

S = {HT,HH,TT,TH}

b) Find the probability of getting one tail.

Let A = Getting one tail $$\boldsymbol{P(A) = \frac{2}{4} = \frac{1}{2}}$$

b) Find the probability of getting two head.

Let A = Getting two head $$\boldsymbol{P(A) = \frac{1}{4}}$$

03 RULES OF PROBABILITY

INTRODUCTION TO PROBABILITY

ADDITION RULE

MUTUALLY EXCLUSIVE EVENTS

- Events A and B are said to be mutually exclusive if they **cannot occur at the same time.**
- For example, let say if we have to travel to a place and there are only two choices, either by airplane or train. It is not possible to go by airplane and taxi at the same time.
- Hence, the event of going by airplane and going by train are mutually exclusive.

To find the probability of event A or event B occurring :

$$P(A \cup B) = P(A) + P(B)$$

- Mutually exclusive also indicates that $P(A \cap B) = 0$.

ADDITION RULE

NON-MUTUALLY EXCLUSIVE EVENTS

- Events A and event B are said to be non-mutually exclusive if there is possibility that event A and event B **will happen at the same time**.
- For example, all students are allowed to take either Accounting or Statistics or both subjects this semester. This will results in some of the students taking either Accounting or Statistics only, and some taking both Accounting and Statistics this semester.
- Hence, the event of taking Accounting and Statistics are non-mutually exclusive.

ADDITION RULE

NON-MUTUALLY EXCLUSIVE EVENTS

To find the probability of either event A or event B or both events will occur :

$$P(A \cup B) = P(A) + P(B) - P(A \; and \; B)$$

OR

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Non- Mutually exclusive also indicates that $P(A \cap B) \neq 0$.

EXAMPLE 3.1

In a business course in a college, 95% of the students passed Account, 90% of the students passed Statistics, and 85% passed both Account and Statistics. A student is selected at random.

a) What is the probability that the student passed Account or Statistics?

b) What is the probability that the student passed neither Account nor Statistics?

EXAMPLE 3.1 (SOLUTION)

Let A = Student passed Account

S = Student passed Statistics

STEP 1: Extract all the information from the question

$P(A) = 0.85$, $P(S) = 0.75$, $P(A \cap S)=0.70$

A and B are **non-mutually exclusive**.
Recall the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

a) $P(A \cup S) = P(A) + P(S) - P(A \cap S)$

$= 0.85 + 0.75 - 0.70$

$= 0.9$

b) $P(A \cup S)' = 1 - P(A \cup S)$

$= 1 - 0.9$

$= 0.1$

EXAMPLE 3.2

Let x = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

If A is 'an odd number', and B is 'a multiple of three', find **P(A U B)**.

EXAMPLE 3.2 (SOLUTION)

Let x = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
If A is 'an odd number', and B is 'a multiple of three', find **P(A U B)**.

A and B are **non-mutually exclusive**.
Recall the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

STEP 1: List out all the possible outcomes of event A and event B.

A = {1, 3, 5, 7, 9} B = {3, 6, 9}

STEP 2: Find all the required information to be filled in the formula.
List out all the possible outcomes of A ∩ B .

$$A \cap B = \{3, 9\}$$

$$P(A) = \frac{5}{10} \quad P(B) = \frac{3}{10} \quad P(A \cap B) = \frac{2}{10}$$

STEP 3: Substitute into the formula and calculate.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{10} + \frac{3}{10} - \frac{2}{10}$$

$$= \frac{6}{10} = \frac{3}{5}$$

EXAMPLE 3.3

The probability that candidate A will pass a certain job interview is 0.3 while the probability that candidate B will pass the job interview is 0.4. The probability that both A and B will pass the job interview is 0.05. What is the probability that **at least one** of these candidates will pass the job interview?

EXAMPLE 3.3 (SOLUTION)

The probability that candidate A will pass a certain job interview is 0.3 while the probability that candidate B will pass the job interview is 0.4. The probability that both A and B will pass the job interview is 0.05. What is the probability that at least one of these candidates will pass the job interview?

> A and B are **non-mutually exclusive**.
> Recall the formula
>
> $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The **keyword is 'at least one'**. Thus, we need to find the probability that either A or B will pass the job interview.

Let P(A) = probability that candidate A will pass the job interview,

P(B) = probability that candidate B will pass the job interview, and

$P(A \cap B)$ = probability that both candidate A and B will pass the job interview

STEP 1: Extract all the information given in the question.

$P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B) = 0.05$

STEP 2: Substitute into the formula and calculate.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.3 + 0.4 - 0.05$$
$$= 0.65$$

EXAMPLE 3.4

The town of DuniaBaru has two ambulance services; the Kejar service and a Selamat service. In an emergency, the probability that the Kejar service responds is 0.7, whereas the probability that the Selamat service responds is 0.5, and the probability that either of both services respond is 0.65. Find the probability that both services will responds to an emergency.

EXAMPLE 3.4 (SOLUTION)

The town of DuniaBaru has two ambulance services; the Kejar service and a Selamat service. In an emergency, the probability that the Kejar service responds is 0.7, whereas the probability that the Selamat service responds is 0.5, and the probability that either of both services respond is 0.65. Find the probability that both services will responds to an emergency.

A and B are **non-mutually exclusive**.
Recall the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The **keyword is 'both'**. Thus, we need to find the probability that both services will responds to an emergency.

Let P(K) = probability that Kejar service will respond,
P(S) = probability that Selamat service will respond, and
P(AUB) = probability that both Kejar and Selamat service will respond

STEP 1: Extract all the information given in the question.

P(K) = 0.7, P(S) = 0.5, P(AUB) = 0.65

STEP 2: Substitute into the formula and calculate.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.65 = 0.7 + 0.5 - P(A \cap B)$$

$$P(A \cap B) = 0.55$$

MULTIPLICATION RULE

INDEPENDENT EVENT

Two events are independent when the occurrence or non-occurrence of one event will not affect the probability of the occurrence of the other event.

For example, a fair coin is tossed twice. What is the probability of obtaining a head in the second toss?

The answer is the probability of getting head in the second toss is not affected by the result of the first toss. Thus, probability of getting head in the second toss is still 1/2.

MULTIPLICATION RULE

INDEPENDENT EVENT

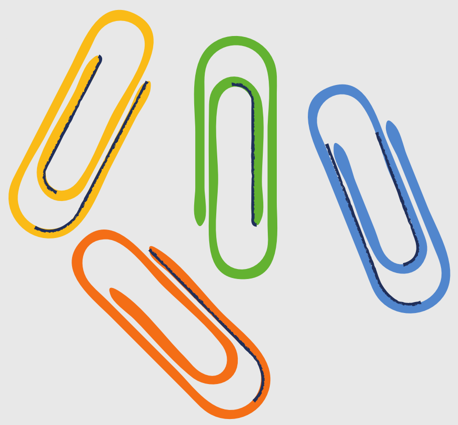
The multiplication rules that applied to an independent events are:

$$P(A \cap B) = P(A) \times P(B)$$

where A and B are two independent events.

EXAMPLE 3.5

A drawer contains 3 red paper clips, 4 green paper clips, and 5 blue paper clips. One paper clip is taken from the drawer and then replaced. Another paper clip is taken from the drawer. What is the probability that the first paper clip is red and the second paper clip is blue?



EXAMPLE 3.5 (SOLUTION)

The keyword is **replaced**. This indicates that the number of paper clips in the drawer does not change from the first event to the second event.

Let P(R) = probability that red paper clip is picked,

P(G) = probability that green paper clip is picked, and

P(B) = probability that blue paper clip is picked

R, G and B are **independent**.
Recall the formula

$$\boldsymbol{P(A \cap B) = P(A) \times P(B)}$$

STEP 1: Extract all the information given in the question.

$$P(R) = \frac{3}{12}, P(G) = \frac{4}{12}, P(B) = \frac{5}{12}$$

STEP 2: Substitute into the formula and calculate.

$$P(R \cap B) = P(R) \times P(B)$$

$$= \frac{3}{12} \times \frac{5}{12}$$

$$= \frac{5}{48}$$

EXAMPLE 3.6

Fitri travels from Johor Bahru to Terengganu via Kuala Lumpur by bus. He takes Redline Bus Service from Johor Bahru to Kuala Lumpur, and Blueway Bus Service from Kuala Lumpur to Terengganu. The probability that Redline Bus Service arrives safely in Jeddah is 0.92, and the probability that Blueway Bus Service arrives safely in Paris is 0.95. Find the probability that:

a) Fitri arrives safely in Kuala Lumpur and Terengganu.

b) Fitri arrives safely in Kuala Lumpur, but has difficulty in Terengganu.

c) Fitri has difficulty arriving in Kuala Lumpur and Terengganu.

EXAMPLE 3.6 (SOLUTION)

Let P(K) = probability that Fitri will arrive at Kuala Lumpur using Redline Bus Service, and

P(T) = probability that Fitri will arrive at Terengganu using Blueline Bus Service,

K and T are **independent**. Recall the formula

$$\boldsymbol{P(A \cap B) = P(A) \times P(B)}$$

STEP 1: Extract all the information given in the question.

P(K) = 0.92, P(T) = 0.95.

STEP 2: Substitute into the formula and calculate.

a) $P(K \cap T) = P(K) \times P(T)$

$$= 0.92 \times 0.98$$

$$= 0.874$$

b) $P(K \cap T') = P(K) \times P(T')$

$$= 0.92 \times (1 - 0.95)$$

$$= 0.046$$

c) $P(K \cap T)' = P(K)' \times P(T)'$

$$= (1 - 0.92) \times (1 - 0.95)$$

$$= 0.004$$

MULTIPLICATION RULE

DEPENDENT EVENT

Two events are dependent when the occurrence or non-occurrence of one event will affect the probability of the occurrence of the other event.

→

The probability of the occurrence of the second event depends upon the occurrence or non-occurrence of the first event.

→

For example, consider a bag contains 2 red balls and 5 blue balls. One ball is withdrawn from the bag and **not replaced**. A second ball is then withdrawn. Find the probability that the second ball is red.

MULTIPLICATION RULE

DEPENDENT EVENT

Let two events A and B are dependent events. **P(B|A)** is the probability that B occurs **given that A occurs**. Meanwhile **P(A|B)** is the probability that A occurs **given that B occurs**.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A occur first

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

B occur first

Note that P(A∩B) = P(B∩A). When P(B|A) = P(B), then A and B are independent.

EXAMPLE 3.7

The events A and B are such that $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(B|A) = \frac{1}{3}$. Find:

a) $P(A \cap B)$
b) $P(A \cup B)$
c) $P(A|B)$

EXAMPLE 3.7 (SOLUTION)

STEP 1: Check either A and B are independent or dependent.

Since P(B|A) ≠ P(B), so A and B are dependent

A and B are **dependent**.
Recall the formula

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

a)

$$P(A \cap B) = P(A) \times P(B|A)$$

$$= \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

Since P(A∩B) ≠ 0, so A and B are **non-mutually exclusive**

b)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{5} + \frac{3}{10} - \frac{2}{15} = \frac{17}{30}$$

c)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2}{15} \div \frac{3}{10} = \frac{4}{9}$$

EXAMPLE 3.8

Given that $P(A \cup B) = \frac{3}{4}$, $P(A') = \frac{2}{3}$, and $P(A \cap B) = \frac{1}{4}$. Find

a) $P(A)$
b) $P(B)$
c) $P(A|B)$
d) Are A and B independent events?

EXAMPLE 3.8 (SOLUTION)

STEP 1: Check either A and B are mutually exclusive or not.

Since P(A∩B) ≠ 0, so A and B are mutually exclusive

a)

$$P(A) + P(A') = 1$$

$$P(A) = 1 - P(A')$$

$$P(A) = 1 - \frac{1}{3} = \frac{2}{3}$$

b)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$$

$$P(B) = \frac{2}{3}$$

c)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$$

d) To check if A and B are independent

$$P(A|B) = P(A)$$

$$\frac{3}{8} \neq \frac{1}{3}$$

Hence, A and B are dependent

EXERCISES

EXERCISE 1

A record from a hospital shows that 12% of the patients are admitted for a cancer treatment, 16% is admitted for stomach treatment and 2 % receives both cancer and stomach treatments. Illustrates the events in a Venn diagram.

QUICK TIPS

Convert the percentage to decimal

EXERCISE 2

If C and D are two independent events and P(C) = 0.25 and P(D) = 0.35, find

a) $P(C \cup D)$

b) $P(C \mid D)$

QUICK TIPS

The keyword is independent. Hence, $P(C \cap D) \neq 0$. So, C and D are non-mutually exclusive event

ANSWER

(a) 0.5125
(b) 0.25

EXERCISE 3

Two events R and T are defined in a sample space. It was given that P(R) = 0.3 and P(T) = 0.4 and $P(R \cap T) = 0.02$. Find

a) $P(R \cup T)$

b) $P(R' \cap T')$

c) Are R' and T' are independent events? Why?

ANSWER

(a) 0.68
(b) 0.32
(c) R' and T' are dependent

QUICK TIPS

Since $P(R \cap T) \neq 0$. So, R and T are non-mutually exclusive event

EXERCISE 4

If $P(A) = 0.35$, $P(B) = 0.66$ and $P(A \cap B) = 0.231$. Verify whether

a) Events A and B are mutually exclusive.

b) Events A and B are independent.

ANSWER

(a) Non-mutually exclusive
(b) Independent

QUICK TIPS

Check the intersection between A and B. Check the independent by using independent event formula

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04 COUNTING RULE

INTRODUCTION TO PROBABILITY

COUNTING RULE

We need to count in determining the number of outcomes that occur for a particular experiments.

SCAN ME

Watch counting rule video

4 types of Counting rules

1

If a task can be done in **m ways** and another task can be done in **n ways.**
Formula : **m x n to accomplish 2 tasks.**

2

If a task has **r** possibilities to be accomplish in **same task for n times**
Formula : r^n

3

Number of ways to arrange **n different objects**
Formula : **n!**

4

Number of ways to arrange $r_1, r_2, \ldots, r_n$ different objects
Formula : $\frac{n!}{r_1! r_2! \ldots r_n!}$

EXAMPLE 4.1

1

If a task can be done in **m ways** and another task can be done in **n ways.**
Formula : **m x n to accomplish 2 tasks.**

Anis has 5 dresses and 7 scafts that can be choose to wear for a day. How many different outfits can she wear for a day?

SOLUTION:

Apply m x n counting rule we obtain total outfits
= 5 x 7 = 35 outfits

EXAMPLE 4.2

2

If a task has **r** possibilities to be accomplish in **same task for n times**
Formula : r^n

A dice is tossed three times. Find the number of possible outcomes for this experiment.

SOLUTION:

Apply r^n , possible outcome, r = 6 {1,2,3,4,5,6}. n = 3
we obtain

$$6 \times 6 \times 6 = 6^3 = 216$$

EXAMPLE 4.3

3

Number of ways to arrange **n different objects**

Formula : **n!**

In how many ways can 4 letters A, B, C and D can be arranged?

SOLUTION:

Apply n!

we obtain

= 4! = 4 x 3 x 2 x 1= 24

EXAMPLE 4.4

4 Number of ways to arrange

$$r_1, r_2, \ldots, r_n$$

different objects

Formula : $\dfrac{n!}{r_1!r_2!\ldots r_n!}$

In how many ways can the word "BOOKS" be arranged?

SOLUTION:

Apply $\dfrac{n!}{r_1!r_2!\ldots r_n!}$

we obtain

$$\frac{5!}{2!1!1!1!} = 60$$

PERMUTATION

Permutation is an arrangement of a set of a objects in which order of arrangement is important to taken into consideration.

$$nPr = \frac{n!}{(n-r)!}$$

SCAN ME

Watch permutation & combination video

EXAMPLE 4.5

$$nPr = \frac{n!}{(n-r)!}$$

Find the number different arrangement to form 3 letters from vowel letters?

SOLUTION:

vowel letters : A E I O U , n=5 , r = 3

Apply

$$nPr = \frac{n!}{(n-r)!}$$

we obtain

$$5P3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

EXAMPLE 4.6

$$nPr = \frac{n!}{(n-r)!}$$

Find the number different committees consist of a chairman, secretary and treasury from 12 members.

SOLUTION:

n=12, r = 3

Apply

$$nPr = \frac{n!}{(n-r)!}$$

we obtain

$$12P3 = \frac{12!}{(12-3)!} = \frac{12!}{9!} = 1320$$

COMBINATION

Combination is an arrangement of a set of a objects in which order of arrangement is not important or not taken into consideration.

$$nCr = \frac{n!}{r!\,(n-r)!}$$

SCAN ME

Desmos calculator for combination

EXAMPLE 4.7

$$nCr = \frac{n!}{r!\,(n-r)!}$$

Find the number of combinations consist of three letters that can formed from the vowel letters ?

SOLUTION:

vowel letters : A E I O U , n=5 , r = 3

Apply

$$nCr = \frac{n!}{r!\,(n-r)!}$$

we obtain

$$5C3 = \frac{5!}{3!\,(5-3)!} = \frac{5!}{3!2!} = 10$$

EXAMPLE 4.8

$$nCr = \frac{n!}{r!\,(n-r)!}$$

Find the number of committees consist of 4 committees that can be formed from 12 persons ?

SOLUTION:

Apply

$$nCr = \frac{n!}{r!\,(n-r)!}$$

we obtain

$$12C4 = \frac{12!}{4!\,(12-4)!} = \frac{12!}{4!8!} = 495$$

EXAMPLE 4.9

Find the number of committees that can be formed from 6 men and 4 women if the committees of 5 persons must consists of

a) exactly 5 persons
b) 3 men and 2 women
c) at most 3 women
d) at least 3 men

SOLUTION:

a) exactly 5 persons

Apply

$$nCr = \frac{n!}{r!\,(n-r)!}$$

we obtain

$$10C5 = \frac{10!}{5!\,(10-5)!} = \frac{10!}{5!5!} = 252$$

EXAMPLE 4.9 (CONT.)

Find the number of committees that can be formed from 6 men and 4 women if the committees of 5 persons must consists of

a) exactly 5 persons
b) 3 men and 2 women
c) at most 3 women
d) at least 3 men

SOLUTION:

b) 3 men and 2 women

Apply

$$nCr = \frac{n!}{r!\,(n-r)!}$$

we obtain

$$6C3 \times 4C2 = \frac{6!}{3!\,(6-3)!} \times \frac{4!}{2!\,(4-2)!}$$

$$\frac{6!}{3!3!} \times \frac{4!}{2!2!} = 120$$

EXAMPLE 4.9 (CONT.)

Find the number of committees that can be formed from 6 men and 4 women if the committees of 5 persons must consists of

a) exactly 5 persons
b) 3 men and 2 women
c) at most 3 women
d) at least 3 men

SOLUTION:

c) at most 3 women

Apply

$$nCr = \frac{n!}{r!\,(n-r)!}$$

we obtain

$$= (6C2 \times 4C3) + (6C3 \times 4C2) + (6C4 \times 4C1)$$

$$= 60 + 120 + 60 = 240$$

EXAMPLE 4.9 (CONT.)

Find the number of committees that can be formed from 6 men and 4 women if the committees of 5 persons must consists of

a) exactly 5 persons
b) 3 men and 2 women
c) at most 3 women
d) at least 3 men

SOLUTION:

d) at least 3 men

Apply

$$nCr = \frac{n!}{r!\,(n-r)!}$$

we obtain

$$= (6C3 \times 4C2) + (6C4 \times 4C1) + (6C5 \times 4C0)$$

$$= 120 + 60 + 6 = 186$$

EXAMPLE 4.10

How many four-digit numbers can be formed from numbers 2, 3, 4, 5 and 6 if

a) repetition is allowed
b) repetition is not allowed

SOLUTION:

a) repetition is allowed

Draw :

(2,3,4,5,6) (2,3,4,5,6) (2,3,4,5,6) (2,3,4,5,6)

we obtain

$$= 5 \times 5 \times 5 \times 5 = 625$$

b) repetition is not allowed

Draw :

(2,3,4,5,6) (~~2~~,3,4,5,6) (~~2~~,~~3~~,4,5,6) (~~2~~,~~3~~,~~4~~,5,6)

we obtain

$$= 5 \times 4 \times 3 \times 2 = 120$$

EXERCISE 5

How many 3-digit numbers can be formed from numbers 1, 3, 4, 5 ,6 and 9 if

a) repetition is allowed
b) repetition is not allowed

SOLUTION:

answer : 216, 120

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05 TREE DIAGRAM & BAYES THEOREM

INTRODUCTION TO PROBABILITY

TREE DIAGRAM

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A tree diagram is used to display the outcomes of an experiment which consists of a series of activities. This ensures that all possibilities are considered.

Characteristics of tree diagram:

1. Each path of the tree represents the sequence of event.
2. Write the conditional probability of the event given all events on branches leading to it.
3. The probability on any node of the tree is obtained by multiplying the probabilities on the branches leading to the node, and equals the probability of the intersection of the event leading to it.
4. Probability on the terminal node must add up to 1.

Tree diagram introduction video

SCAN HERE

Source: Lau, T.K., Phang, Y.N., & Awang, Z. (2022). Statistics for UiTM. Sixth Ed. SJ Learning.

EXAMPLE 5.1

Let H be Head

T be Tail

If a coin is **tossed twice**, the possible outcomes are

$$\{(H \cap H), (H \cap T), (T \cap H), or\ (T \cap T)\}$$

More examples

SCAN HERE

Tree diagram to display outcomes from tossing a coin

First toss	Second toss	Possible outcomes	Probability
0.5 H	0.5 H	$(H \cap H)$	0.5 x 0.5 = 0.25
	0.5 T	$(H \cap T)$	0.5 x 0.5 = 0.25
0.5 T	0.5 H	$(T \cap H)$	0.5 x 0.5 = 0.25
	0.5 T	$(T \cap T)$	0.5 x 0.5 = 0.25

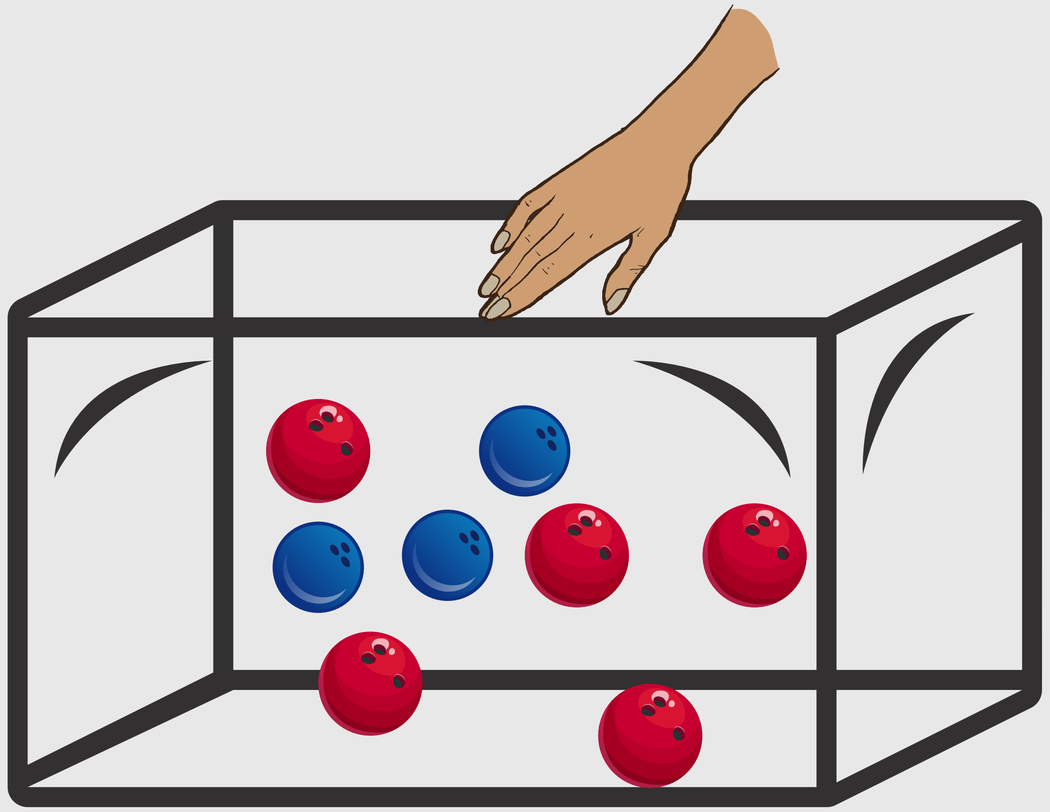
EXAMPLE 5.2

A bag contains of five red balls and three blue balls. Three balls are drawn one by one.

(a) With replacement. What is the outcome of at least two blue balls will be selected. Then, find the probability of the outcome.

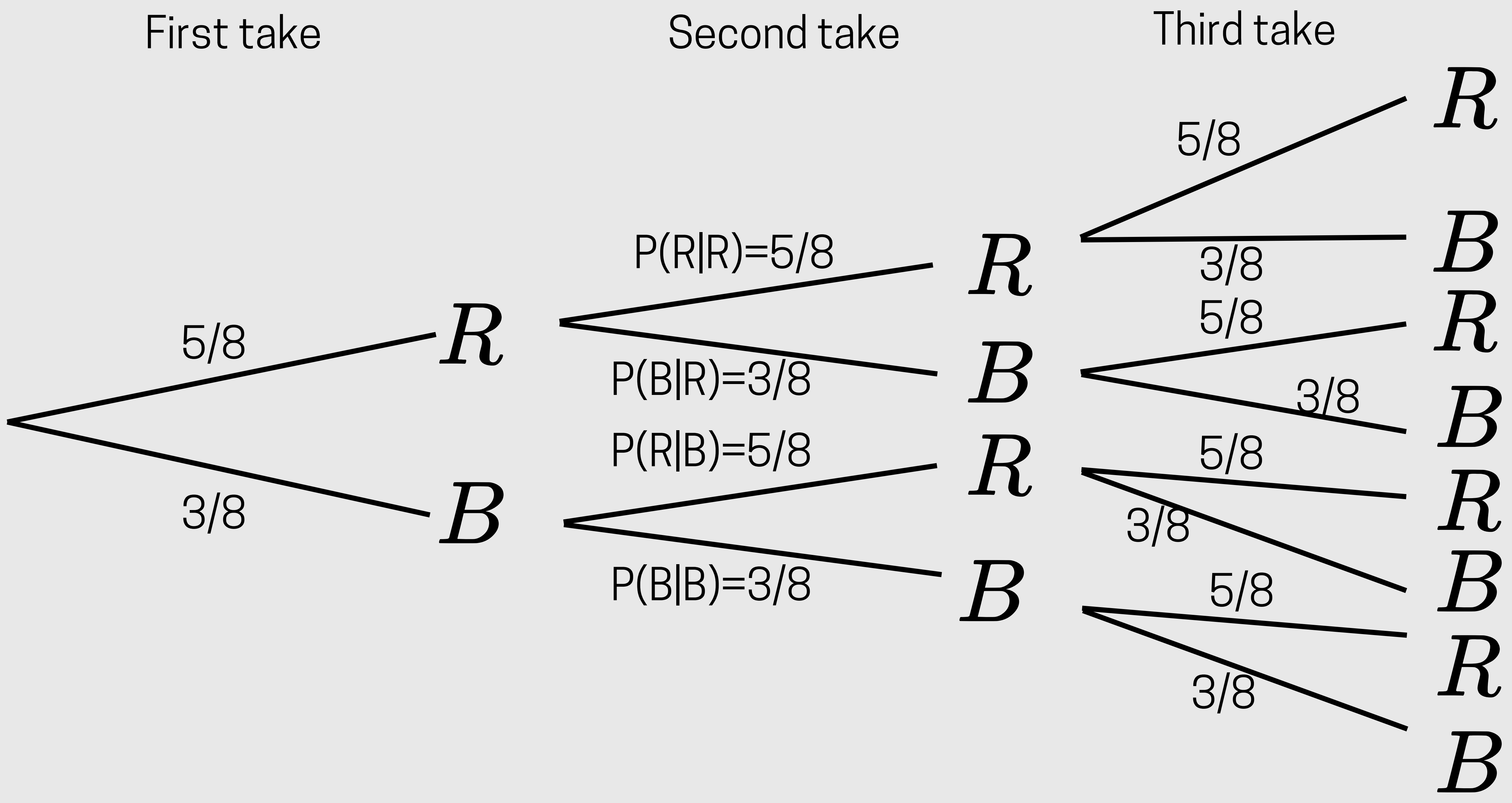
(b) Without replacement. What is the outcome of getting the first ball blue and the third ball red? Then, find the probability of the outcome.

Take a ball. Repeat three times.



EXAMPLE 5.2 (SOLUTION)

(a) Let R be the red ball and B be the blue ball.



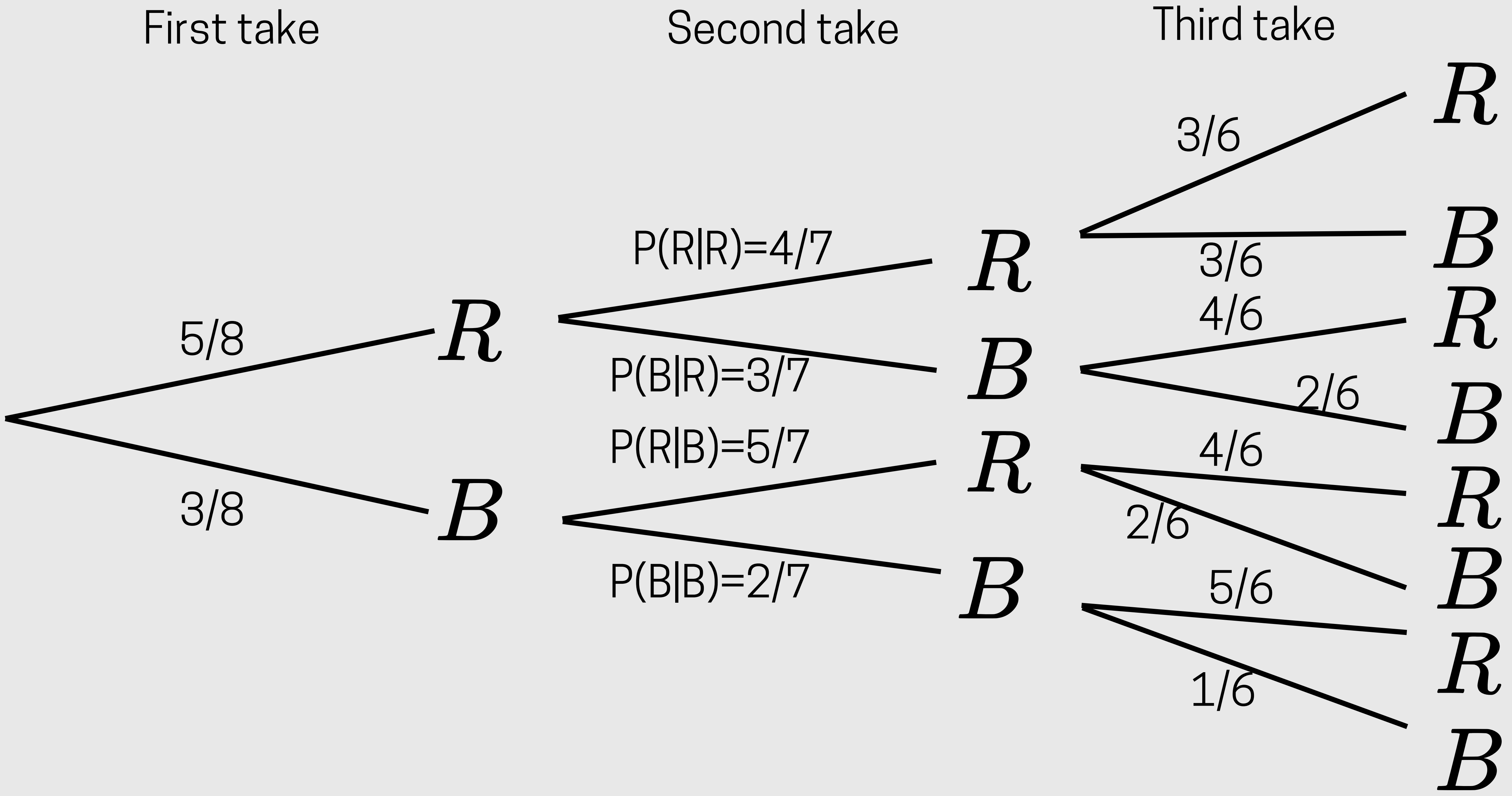
Outcomes	Probability
RRR	125/512
RRB	75/512
RBR	75/512
RBB	45/512
BRR	75/512
BRB	45/512
BBR	45/512
BBB	27/512

Outcomes= {RBB, BRB, BBR, BBB}

Probability $= \frac{45}{512} + \frac{45}{512} + \frac{45}{512} + \frac{27}{512} = 0.3164$

EXAMPLE 5.2 (SOLUTION)

(b) Let R be the red ball and B be the blue ball.



Outcomes	Probability
RRR	5/28
RRB	5/28
RBR	5/28
RBB	5/56
BRR	5/28
BRB	5/56
BBR	5/56
BBB	1/56

Outcomes= {BRR, BBR}

Probability $= \frac{5}{28} + \frac{5}{56} = \frac{15}{56}$

BAYES' THEOREM

Bayes' theorem for conditional probability was developed by the mathematician Reverend Thomas Bayes.
The Bayes' formula:

$$P(A_i|B) = \frac{P(A_i) \times P(B|A_i|)}{\sum_j [P(A_j) \times P(B|A_j|)]}$$

where $A_1, \ldots, A_n$ is an all-inclusive set of possible outcomes, given that event B occurs.

Source: Lau, T.K., Phang, Y.N., & Awang, Z. (2022). Statistics for UiTM. Sixth Ed. SJ Learning.

Watch to have better understanding on Bayes' theorem

SCAN HERE

EXAMPLE 5.3

Three local pharmaceutical wholesalers W,S and R are competing for a contract to supply medicines to a hospital. The probabilities that pharmaceutical wholesaler, W, S and R will win the contract are 0.34, 0.16 and 0.50, respectively. If pharmaceutical wholesaler, W,S and R win the contract, the probabilities that they will make profits are 0.69, 0.82 and 0.45, respectively.

(a) Draw a tree diagram for the above information.

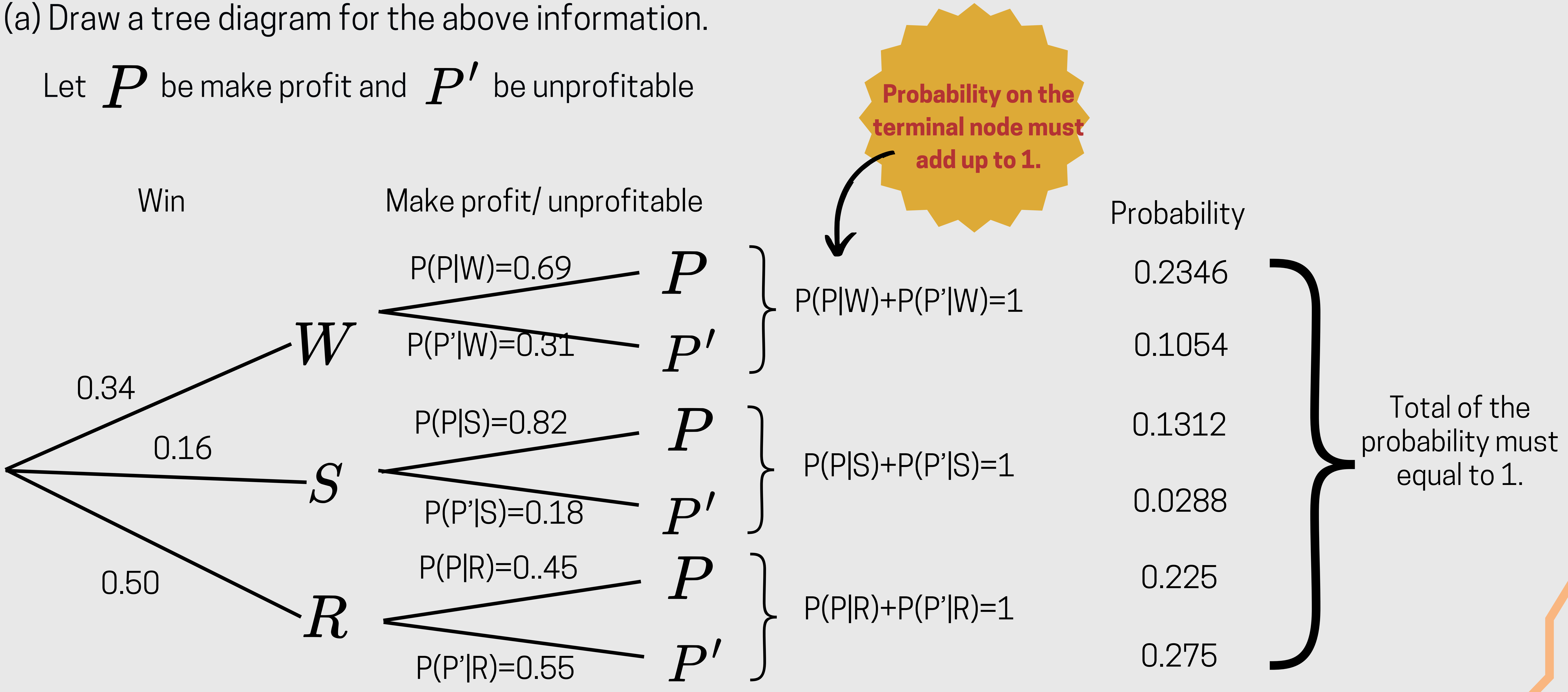
(b) What is the probability that the pharmaceutical wholesaler will make profit?

(c) If the contract is found to be unprofitable, find the probability that the contract was given to pharmaceutical wholesaler S.

EXAMPLE 5.3 (SOLUTION)

(a) Draw a tree diagram for the above information.

Let P be make profit and P' be unprofitable



EXAMPLE 5.3 (SOLUTION)

(b) What is the probability that the pharmaceutical wholesaler will make profit?

Consider the probability of each wholesaler win the contract and make profit.

$$
\begin{aligned}
P(P) &= P(W \cap P) + P(S \cap P) + P(R \cap P) \\
&= P(W)P(P|W) + P(S)P(P|S) + P(R)P(P|R) \\
&= 0.34(0.69) + 0.16(0.82) + 0.50(0.45) \\
&= 0.5908
\end{aligned}
$$

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EXAMPLE 5.3 (SOLUTION)

Use Bayes' Theorem

(c) If the contract is found to be unprofitable, find the probability that the contract was given to pharmaceutical wholesaler S.

$$P(S|P') = \frac{P(S)P(P'|S)}{P(W)P(P'|W) + P(S)P(P'|S) + P(R)P(P'|R)}$$

Total all unprofitable outcomes.

$$= \frac{0.16(0.18)}{0.34(0.31) + 0.16(0.18) + 0.50(0.55)}$$

$$= \frac{24}{341}$$

Probability must be within 0 until 1.

EXERCISES

EXERCISE 6

In company JOJO JAYA, 80% of the project are handled by Hasni while the rest by Ariaty. The probability that a project handled by Hasni is completed on time is 90%. The probability that a project handled by Ariaty is not completed on time is 30%.

(a) Draw the tree diagram of the above information.

(b) What is the probability that particular project is completed on time?

(c) If a project is not completed on time, what is the probability that is handled by Ariaty?

QUICK TIPS

Use Bayes' Theorem for (C). Question that involved conditional probabilities

ANSWER

(b) 0.86
(c) 0.429

EXERCISE 7

In a particular country, airport A handles 50% of all airline traffic, airport B and C handle 30% and 20%, respectively. The detection rates for drug smuggling at three airports are 0.9, 0.8 and 0.85, respectively.

(a) Draw the tree diagram of the above situation.

(b) What is the probability that drug will be detected?

(c) If a passenger at one of the airports is found to have drug in his luggage, what is the probability that he is boarding from airport B?

QUICK TIPS

1. Read the information carefully.
2. Identify the number of situation to decide the tree path and branches.
3. Write the probability on the path.
4. Make sure each terminal node is total to 1.

ANSWER

(b) 0.86
(c) 12/43

REFFERENCES

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