



Comparative Study of the Selection of the Best Contractor Using Fuzzy Weak Autocatalytic Set and Analytic Hierarchy Process

Siti Salwana Mamat

Centre of Foundation Studies, Universiti Teknologi MARA, Selangor Branch, Dengkil Campus, Selangor, Malaysia

sitalwana@uitm.edu.my

Zarith Sofiah Othman

Centre of Foundation Studies, Universiti Teknologi MARA, Selangor Branch, Dengkil Campus, Selangor, Malaysia

zarithsofiah@uitm.edu.my

Noraini Ahmad

Centre of Foundation Studies, Universiti Teknologi MARA, Selangor Branch, Dengkil Campus, Selangor, Malaysia

norainiahmad@uitm.edu.my

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ABSTRACT

Pairwise comparisons are commonly used in decision-making procedures, especially when Multiple Criteria Decision Making (MCDM) is involved. Effective techniques are needed to address the problem related to MCDM. Using the Fuzzy Weak Autocatalytic Set (FWACS) method, the pairwise comparisons of alternatives can be represented as a directed graph with fuzzy edges. The objectives of this study are: 1) to explore and apply the FWACS technique in the problem of contractor selection. 2) to compare the outcomes of FWACS with those obtained using the Analytic Hierarchy Process (AHP). To ensure that judgments made with FWACS are stable and consistent, a sensitivity analysis is carried out. The choices reached using FWACS are consistent and stable, thus comparable to AHP. However, FWACS distinguishes itself with its proficiency in handling ambiguity, particularly in the fuzzy framework. This study highlights the necessity of implementing comprehensive and systematic contractor selection criteria to enhance the efficiency and success of construction projects, thereby mitigating the risks associated with inadequate contractor performance.

Corresponding Author:

Siti Salwana Mamat

Centre of Foundation Studies, Universiti Teknologi MARA, Selangor Branch, Dengkil Campus, Selangor, Malaysia

email: sitalwana@uitm.edu.my

1. Introduction

Pairwise Comparison of alternatives is a common method used to solve multi-criteria decision-making (MCDM) problems. Each alternative is considered and evaluated to determine which is the best. A lot of techniques can be used in the field of MCDM, such as Simple Additive Method (SAW), Analytic Hierarchy Process (AHP), Technique for the Order of Preference by Similarity to an Ideal Solution (TOPSIS), Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), Elimination Choice Translating Reality (ELECTRE) [1]-[3], and Potential Method [4]. Among these tools, the Fuzzy Weak Autocatalytic Set (FWACS) stands out as a new tool for tackling MCDM problems [5].

The theoretical foundation of FWACS comes from the weak autocatalytic set (WACS), a variation of the autocatalytic set (ACS). Every vertex in ACS receives at least one incoming link from vertices in the same subgraph, forming a directed graph [6]. On the other hand, a weak autocatalytic set (WACS) presents a weak form of ACS, forming a non-loop subgraph inclusive of a vertex without incoming links [7]. This notion of WACS expands into the domain of fuzziness and presents a new



idea of FWACS. The integration between FWACS and the Potential Method (PM) produced an algorithm for ranking, named ranking by FWACS.

This study addresses a significant research gap by implementing the Fuzzy Weak Autocatalytic Set (FWACS) in a multi-criteria decision-making (MCDM) scenario, specifically in contractor selection, an area that has not been extensively explored with this method. The primary objective is to demonstrate the applicability and effectiveness of FWACS, expanding its theoretical foundation from the weak autocatalytic set (WACS) to include fuzziness, thereby offering a novel approach to MCDM problems. This paper contributes to the field by presenting a new algorithm, derived from integrating FWACS with the Potential Method, for ranking alternatives, and illustrating its practical application through a case study involving contractor selection [8].

This paper reports the related theoretical modelling and implementation. It is organized into four sections. The first preliminary section outlines essential concepts and definitions to describe the fuzzification process of WACS to FWACS. The second implantation section provides a detailed examination of the ranking procedure before concluding the summary of findings in the conclusions section.

2. Preliminary

This section expounds on various structures of WACS and FWACS, contextualized within the realm of decision-making. Furthermore, the section introduces the concept of ranking through FWACS.

2.1 Autocatalytic Set (ACS)

The notion of autocatalysis initially emerged within the field of chemistry to elucidate catalytic interactions between molecules [9]. This concept subsequently led to the formalization of the autocatalytic set (ACS) through a directed graph structure. The definition of an ACS is presented in Definition 1 and illustrated in Figure 1.

Definition 1: An ACS is a subgraph in which each vertex possesses a minimum of one incoming link originating from vertices within the same subgraph. This structural characteristic ensures that every vertex can potentially catalyze a reaction or process involving its neighbors, thereby maintaining the activity within the subgraph. The dynamics of ACS highlight the interdependence of elements in a network, making it a fundamental concept in understanding complex systems [6], [10].

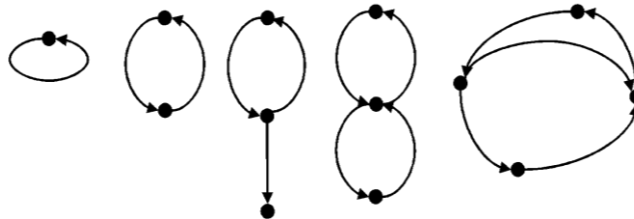


Figure 1. Several ACSs

This concept has also given rise to a modified graph structure called Weak Autocatalytic Set (WACS). The WACS allows for a more flexible interpretation of connectivity, accommodating vertices that do not necessarily receive input from every other vertex within the same subgraph, thus broadening the application of autocatalytic sets in complex systems modeling. The subsequent subsection provides detailed features of WACS.

2.2 Weak Autocatalytic Set (WACS)

Definition 2: A WACS is a non-loop subgraph that contains a vertex with no incoming link. Figure 2 shows the cases of several instances demonstrating the WACS [11].

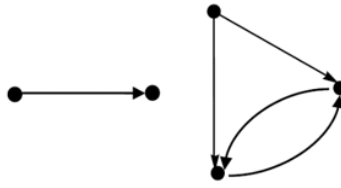


Figure 2. Some examples of WACS

The characteristics of a WACS are outlined as follows [12]:

Theorem 1: Every WACS inherently forms a weakly connected graph.

Theorem 2: Every WACS must have at least a path, which is not closed.

A fuzzy graph serves as a replication of a crisp graph [13], [14]. The subsequent theorem establishes that WACS can be considered a distinct instance of a fuzzy graph.

Theorem 3: Every WACS is a fuzzy graph.

The introduction of fuzzy graphs and WACS has led to the inception of a novel concept, denoted as FWACS. The introduction of FWACS is elaborated in the following subsection.

2.3 Fuzzy Weak Autocatalytic Set (FWACS)

The integration of a fuzzy graph and a WACS has led to the emergence of the concept known as FWACS [5]. The specific definition and detailed description of FWACS are elaborated in the following definition [12]:

Definition 3: A FWACS is a WACS such that every edge e_i has a membership value, $\mu(e_i) \in [0,1]$ for $e_i \in E$.

An example of a FWACS is presented in Figure 3. The edges exhibit varying "strengths," which are defined by their respective membership values. A higher membership value corresponds to a stronger connection between the two graph vertices. Consequently, distinct thicknesses and colours of an edge symbolize the strength of the connection between its associated vertices.

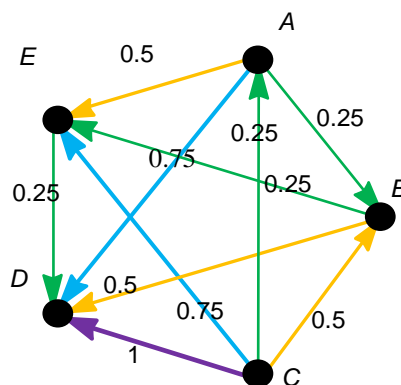


Figure 3. Illustration of a FWACS

2.3.1 Ranking by Fuzzy Weak Autocatalytic Set (FWACS)

This subsection introduces an algorithm for conducting rankings through FWACS. The inputs for this algorithm encompass edge directions and their corresponding values. Membership values of edges are established through pairwise comparisons [7], [11], which are then encapsulated within

matrix known as the flow matrix, denoted by F_μ . The direction of edges is outlined in an incidence matrix, designated as A . The ranking procedure with FWACS is detailed as follows:

Step 1: Construct a FWACS, denoted as $G = (V, F_\mu)$, for a given problem and determine the membership values for its edges. Let V denote a set of vertices and F_μ , represent the fuzzy flow matrix containing edge membership values.

Step 2: Create the incidence matrix, A . A $m \times n$ incidence matrix is given by

$$A_{\alpha,v} = \begin{cases} -1, & \text{if the edge } \alpha \text{ leaves } v \\ 1, & \text{if the edge } \alpha \text{ enter } v \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Step 3: Define Laplacian matrix, L , which is $L = A^T A$ with entries as:

$$L_{i,j} = \begin{cases} -1, & \text{if there is an edge } (i, j) \text{ or } (j, i), \\ \text{deg}(i), & \text{if } i = j, \\ 0, & \text{else.} \end{cases} \quad (2)$$

such that $\text{deg}(i)$ is the degree of vertex i .

Step 4: Calculate the flow difference. The flow difference component, $D_\mu = A^T F_\mu$ is computed as:

$$\begin{aligned} D_\mu &= \sum_{\alpha=1}^m A_{v,\alpha}^T F_\alpha \\ &= \sum_{\alpha \text{ enters } v} F_\alpha - \sum_{\alpha \text{ leaves } v} F_\alpha \end{aligned} \quad (3)$$

whereby D_μ is the difference between the total flow which enters v and the total flow which leaves v .

Step 5: Determine the potential, X , which is a solution to the Laplacian system:

$$LX = D_\mu \quad (4)$$

such that $\sum X_v = 0$ of its connected components.

Step 6: Check the degree of consistency, $\beta < 12^\circ$. The inconsistency measure is given by:

$$\text{Inc}(F_\mu) = \frac{\|F_\mu - AX\|_2}{\|AX\|_2} \quad (5)$$

where $\|\cdot\|_2$ denotes 2-norm and $\beta = \arctan(\text{Inc}(F_\mu))$ is the angle of inconsistency. The ranking is considered acceptable if $\beta < 12^\circ$. If the matrix is not consistent, then it needs to be revised [15].

Step 7 Calculate the weight, w . The following equation is used to obtain the weight.

$$w = \frac{a^X}{\|a^X\|_1} \quad (6)$$

where $\|\cdot\|_1$ represents 1-norm and parameter a is chosen to be 2 [4].

Step 8: Rank the objects based on their respective weights.

The ranking process using the FWACS approach is summarized in Figure 4, which shows the sequential steps involved in the process. Each step is represented as a distinct box within the flowchart, with arrows connecting the boxes to indicate the order of execution. The flowchart provides a high-level overview of the algorithm, making it easier to understand the progression of actions and calculations in the FWACS ranking process.

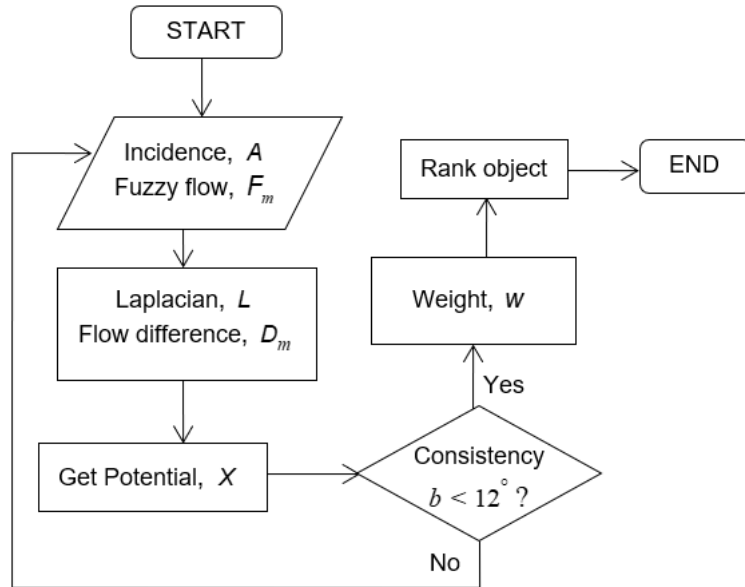


Figure 4. Flowchart illustrating the FWACS Ranking Process

The algorithm begins by constructing a FWACS representation (denoted as $G = (V, F_\mu)$), which serves as the foundation for subsequent calculations. This FWACS is established by incorporating both edge directions and their corresponding membership values. Following this, the incidence matrix (A) is formulated to characterize the edge orientations, thereby providing a clear understanding of the graph's structural connections.

With these foundational components in place, the algorithm proceeds through a series of mathematical operations. The Laplacian matrix (L) is defined and computed using the provided inputs, leading to the derivation of flow differences (D_μ) for each vertex within the graph. The potential (X) is then determined, involving a solution to the Laplacian system, which contributes to the overall ranking methodology.

To ensure the consistency of the ranking results, the algorithm evaluates the degree of inconsistency ($\text{Inc}(F_\mu)$) by calculating the ratio of the 2-norm of the Laplacian matrix result (LX) to the angle of inconsistency ($\beta = \arctan(\text{Inc}(F_\mu))$). The ranking outcomes are considered valid and consistent if this ratio meets the predetermined threshold.

Subsequently, the algorithm computes weights (w) for each object under consideration, involving the division of the 2-norm of flow differences (F_μ) by the 2-norm of the Laplacian matrix result (LX). These weights reflect the relative importance of each object within the decision-making context.

Finally, the algorithm concludes by ranking the objects based on their calculated weights, thereby providing an ordered list of preferences or choices. The comprehensive execution of these steps encapsulates the FWACS-based ranking process, ultimately contributing to well-informed decision-making. The algorithm's flow synergizes the intricate calculations and evaluations required for meaningful rankings within a structured and comprehensible framework, facilitating the practical implementation of FWACS in various decision-making scenarios.

Figure 5 outlines the step-by-step algorithm for conducting rankings through the FWACS approach. The algorithm is succinctly presented in a structured format, with each step numbered and described in detail. The figure serves as a clear reference to implement the FWACS ranking method, offering a comprehensive guide to follow throughout the process.

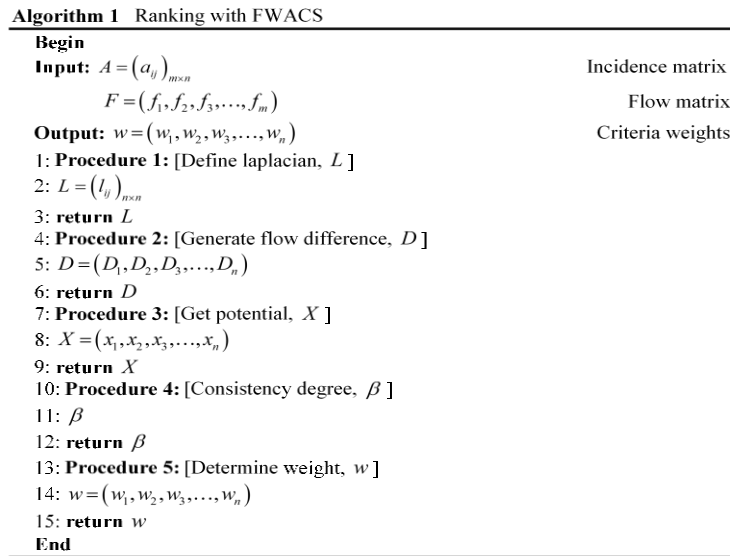


Figure 5. Algorithm for FWACS Ranking

The FWACS algorithm starts with an incidence matrix as input, and through a series of procedures, it aims to produce a set of criteria weights as output. Initially, the algorithm defines a Laplacian matrix, then generates flow differences, and subsequently computes the potential for each criterion. Finally, it determines the consistency degree and calculates the respective weights for the criteria, which are then returned as the final output of the algorithm. This structured approach allows for a comprehensive assessment and ranking of alternatives in a decision-making scenario. In the following section, the application of FWACS to a contractor selection problem is discussed, drawing on work by [8].

3. Implementation

3.1 Case Study Overview

Reference [8] are involved in a project focused on selecting a highly skilled and competent contractor. There are six criteria involved for contractor evaluation. The criteria are financial capability (FC), past performance (PP), contractor's past experience (PE), contractor's resources (CR), ongoing workload (CW), and safety performance (SP). Figure 6 depicts the hierarchical framework that outlines the hierarchy of criteria for selecting contractors. In their project, the contractors are represented as Bidder 1, Bidder 2, and Bidder 3, identified by the symbols A, B, and C, respectively. This case study serves as an application for FWACS ranking, offering a practical scenario for its real-world implementation.

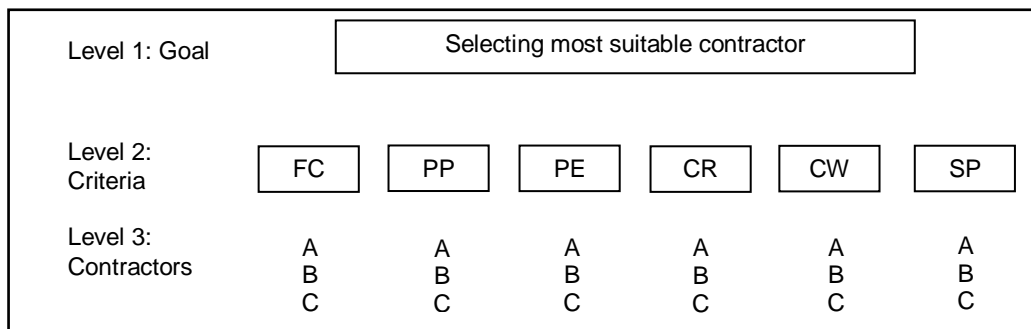


Figure 6. The hierarchical structure of contractor selection

In order to choose the best applicant, the six criteria are first analyzed and then the three contractors of each criterion are assessed.

3.2 Determination of Criteria Weight

The first step is synthesizing the criteria for contractors. This is achieved through matrix comparisons of the criteria, as detailed in Table 2. The FWACS representation for these criteria is depicted in Figure 7. Notably, the arrows pointing towards the criterion PP indicate its higher preference, while the arrows pointing away from CW suggest its relatively lower preference.

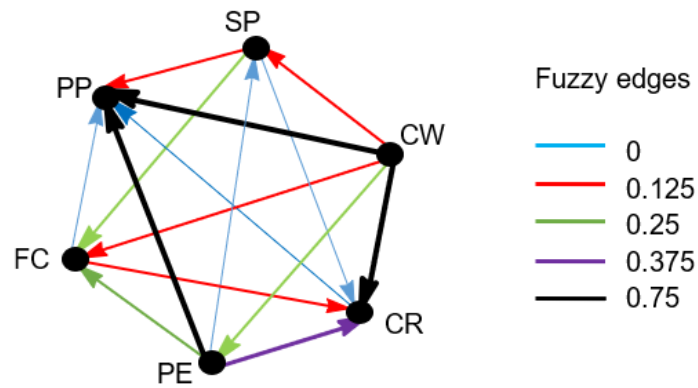


Figure 7. The FWACS for criteria

The graph is subsequently converted into incidence and flow matrices, denoted as A and F_{μ} respectively. The incidence matrix, A signifies the direction of edges within the graph illustrated in Figure 7. The flow matrix, F_{μ} encapsulates the fuzzy values attributed to the edges. These edge values undergo a linear transformation into fuzzy membership values, as detailed in Table 1.

Table 1. Fuzzy Membership Values

Likert scale	Definition	AHP	FWACS
1	Equal preferred	1	0
2	Equally to moderately	2	0.125
3	Moderately preferred	3	0.25
4	Moderately to strongly	4	0.375
5	Strongly preferred	5	0.5
6	Strongly to very strongly	6	0.625
7	Very strongly preferred	7	0.75
8	Very strongly to extremely	8	0.875
9	Extremely preferred	9	1

The incidence, fuzzy flow and flow difference matrices corresponding to the graph depicted in Figure 7 are provided as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \mathbf{F}_\mu = \begin{bmatrix} 0 \\ 0.25 \\ 0.125 \\ 0.125 \\ 0.25 \\ 0.75 \\ 0 \\ 0.75 \\ 0.125 \\ 0.375 \\ 0.25 \\ 0 \\ 0.75 \\ 0 \\ 0.125 \end{bmatrix}, \text{ and } \mathbf{D}_\mu = \begin{bmatrix} 0.5 \\ 1.625 \\ -1.125 \\ 1.25 \\ -2 \\ -0.25 \end{bmatrix}$$

Next is to find the potential values, X using equation (4). The weights are. The potential values are calculated as follows:

$$X = L^{-1}D_\mu = \begin{bmatrix} 1/9 & -1/18 & -1/18 & -1/18 & -1/18 & -1/18 \\ -1/18 & 1/9 & -1/18 & -1/18 & -1/18 & -1/18 \\ -1/18 & -1/18 & 1/9 & -1/18 & -1/18 & -1/18 \\ -1/18 & -1/18 & -1/18 & 1/9 & -1/18 & -1/18 \\ -1/18 & -1/18 & -1/18 & -1/18 & 1/9 & -1/18 \\ -1/18 & -1/18 & -1/18 & -1/18 & -1/18 & 1/9 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1.625 \\ -1.125 \\ 1.25 \\ -2 \\ -0.25 \end{bmatrix} = \begin{bmatrix} 0.0833 \\ 0.2708 \\ -0.1875 \\ 0.2083 \\ -0.3333 \\ -0.0416 \end{bmatrix}$$

The corresponding weights are obtained through equation (6), and the calculation is outlined as follows.

$$w_{FC} = \frac{2^{0.0833}}{6.0644} = 0.175$$

$$w_{PP} = \frac{2^{0.2708}}{6.0644} = 0.199$$

$$w_{PE} = \frac{2^{-0.1875}}{6.0644} = 0.145$$

$$w_{CR} = \frac{2^{0.2083}}{6.0644} = 0.191$$

$$w_{CW} = \frac{2^{-0.3333}}{6.0644} = 0.131$$

$$w_{SP} = \frac{2^{-0.0416}}{6.0644} = 0.160$$

The weights are listed in Table 2, comparing those from AHP with those from FWACS. In this comparison, PP is at the highest rank, while CW is at the lowest.

Table 2. Comparison Matrix and Criteria Weightage

Criteria a	FC	PP	PE	CR	CW	SP	AHP		FWACS	
							weight	Rank	weight	Rank
FC	1	1	3	1/2	2	3	0.192	3	0.175	3
PP	1	1	7	1	7	2	0.279	1	0.199	1
PE	1/3	1/7	1	1/4	3	1	0.078	5	0.145	5
CR	2	1	4	1	7	1	0.264	2	0.191	2
CW	1/2	1/7	1/3	1/7	1	1/2	0.048	6	0.131	6
SP	1/3	1/2	1	1	2	1	0.118	4	0.160	4

3.2.1 Contractors' Weightage Based on PP

With three contractors under consideration for the final decision, the subsequent step is to assess and compare these contractors in relation to each criterion. This section only addresses how contractors are evaluated using the criterion PP criterion. Table 3 presents a comparison matrix of contractors based on PP to facilitate the evaluation process. Subsequently, this matrix is transformed into the FWACS representation. Figure 8 illustrates the resulting FWACS structure along with the corresponding flow values.

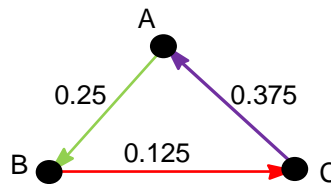


Figure 8. The FWACS contractors with respect to PP

The information related to the graph in Figure 8 is then converted into incidence and flow matrices, denoted as **A** and **F**, respectively. The matrices are presented as follows:

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } \mathbf{F} = \begin{bmatrix} 0.25 \\ 0.375 \\ 0.125 \end{bmatrix}$$

Table 3 displays the contractor weights for FWACS in addition to the weight produced by [8] using AHP. The least preferred option is assigned to Contractor C, whereas Contractors A and B are given equal weights in the FWACS analysis.

Table 3. The weights for contractors with respect to PP

PP	A	B	C	AHP		FWACS	
				weight	Rank	weight	Rank
A	1	1/3	4	0.354	2	0.343	1
B	3	1	1/2	0.366	1	0.343	1
C	1/4	2	1	0.280	3	0.314	2

3.2.2 Global Weights for Contractors

The next stage is to calculate the weights for contractors based on the following criteria: FS, PE, CR, CW, and SP. These global weights serve as a basis for identifying the most suitable contractors. The global weights of AHP are reevaluated, and compared with the weights derived by

FWACS, which are combined in Table 4. It signifies that the contractor selection determined by FWACS aligns with the order A > C > B, which concurs with the AHP results.

Table 4: Global Weights Comparison

Contractors	FC	PP	PE	CR	CW	SP	FWACS		AHP	
	0.175	0.199	0.145	0.191	0.131	0.160	weight	Rank	weight	Rank
A	0.383	0.343	0.314	0.313	0.362	0.428	0.288	1	0.396	1
B	0.304	0.343	0.363	0.304	0.287	0.339	0.270	3	0.299	3
C	0.313	0.314	0.323	0.383	0.351	0.233	0.283	2	0.306	2

3.3 Sensitivity Analysis

Sensitivity analyses were carried out to investigate how changes in the relative synthesis values of each criterion would affect the overall priority of alternatives once the first solution with the specified weights of criteria had been obtained [16]. Sensitivity analysis provides important information about how adjustments to the weights of particular criteria can affect the way alternatives are prioritized and ranked overall. Using Expert Choice Software, the performance of contractors is shown on the graph in Figure 9 across all criteria. Contractor A is ranked as the top performer, followed by Contractor C, and Contractor B is ranked as the least preferred, according to the first sensitivity analysis, which verifies the results.

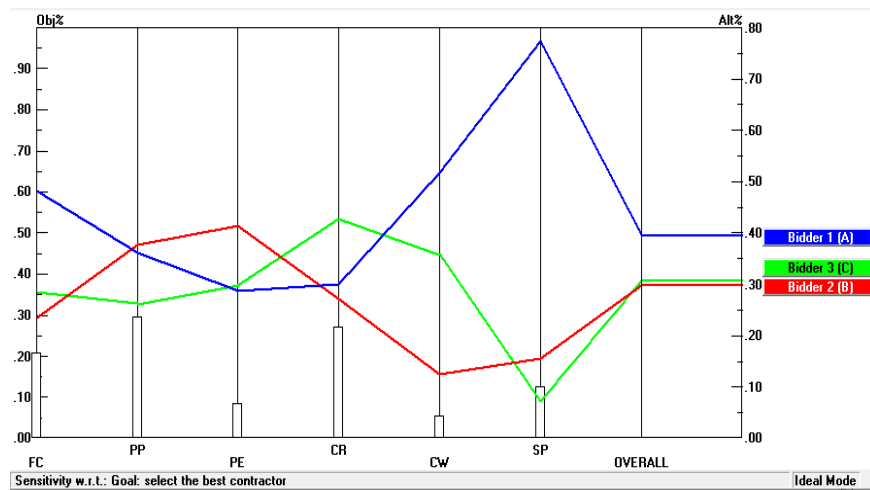


Figure 9. Sensitivity Analysis of Contractor Performance

Sensitivity analysis can be performed by increasing or decreasing the weight of individual criteria, hence, the resulting changes in the weights and the ranking of the alternatives can be observed [17]. The analysis focuses on the sensitivity of contractor performance as the financial capability (FC) is increased to 30%. Interestingly, this increase in FC fails to impart any noticeable impact, and as a result, the ranking remains unchanged, as shown in Figure 10.

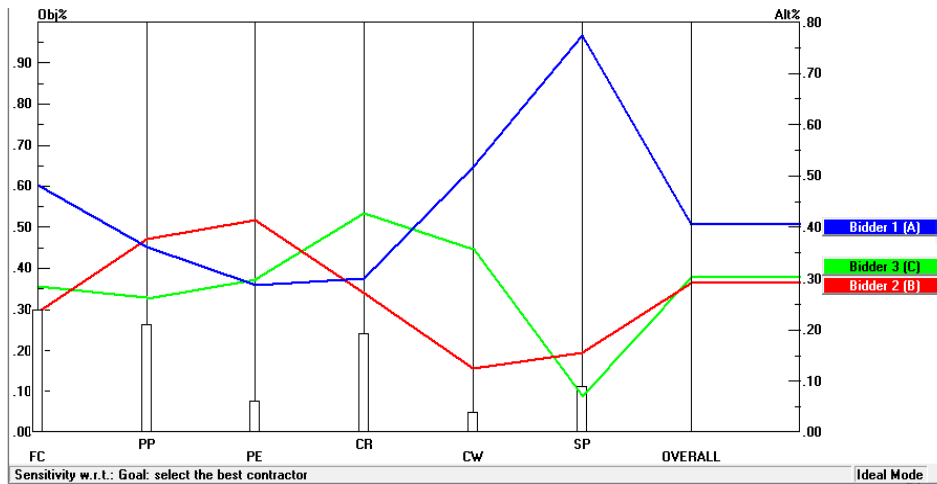


Figure 10. Sensitivity Analysis of Solution Set Performance with 30% increase in FC

The analysis of contractor performance sensitivity is extended to scenarios where the criterion for past performance (PP) is decreased to 10%. Importantly, the decrease in PP does not lead to any changes in the ranking, as illustrated in Figure 11.

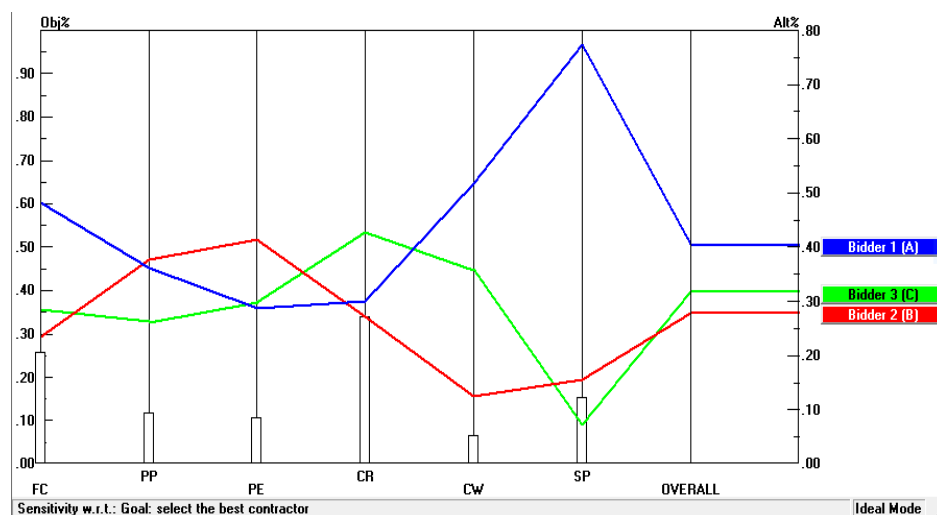


Figure 11. Sensitivity Analysis of Contractor Performance with 10% decrease in PP

Based on the sensitivity analysis, the results confirm that the final choice maintains a high degree of reliability and consistency. In conclusion, sensitivity analysis investigates the ways in which several factors influence a decision's outcome. This helps ascertain how flexible the choice is in response to new information. Consequently, if sensitivity analysis shows that the decision is reliable, it increases confidence in it.

4. Conclusion

This study examines the effectiveness of the FWACS in the context of contractor selection by comparing its results with those obtained using the AHP. This investigation confirms the importance and adaptability of FWACS by revealing a thorough grasp of its advantages in decision-making. This study provides evidence for the significance and flexibility of FWACS by demonstrating a comprehensive understanding of its benefits for decision-making. The evaluation of contractor selection provided insight into how FWACS can manage the challenges associated with the aspect of decision-making. Because of its ability to comprehend and assess complicated sets of criteria,

FWACS is a helpful tool for assessing possibilities in a complex situation. Its hierarchical criterion description and subsequent synthesis of data lead to well-informed conclusions.

An essential validation step is provided by the comparison with AHP, which shows that FWACS is not only comparable to well-established approaches but also capable of managing uncertainties. FWACS has an advantage in difficult decision situations since it can effectively manage uncertainty, especially in the fuzzy framework. In essence, this study shows that FWACS is a useful technique that can result in solid and trustworthy solutions in MCDM scenarios. By showcasing its alignment with established approaches like AHP and its adaptability in uncertain environments, FWACS emerges as a method to improve the quality and confidence of decisions in the context of contractor selection and beyond.

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Conflict of Interest



The authors affirm that there are no conflicts of interest related to the subject matter or materials discussed in this manuscript.

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Biography

Picture	Biography	Authorship contribution
	Siti Salwana Mamat is a senior lecturer at the Centre of Foundation Studies, Universiti Teknologi MARA, Malaysia. She has an academic background in mathematics, having obtained her first degree in mathematics from the University Teknologi MARA (UiTM). She further pursued her studies and earned a master's degree in mathematics from Universiti Sains Malaysia (USM). Her passion for mathematics led her to pursue a Doctor of Philosophy degree in mathematics from Universiti Teknologi Malaysia (UTM), where she successfully completed her research and was awarded the doctoral degree. Throughout her academic journey, she has specialized in the fields of graph theory and multi-criteria decision-making.	Siti Salwana Mamat conceived and planned the experiments. She carried out the experiments and carried out the simulations.
	Zarith Sofiah Othman is a senior lecturer at the Centre of Foundation Studies, UiTM Cawangan Selangor, Kampus Dengkil. Currently, she is pursuing her Ph.D. at the Universiti Malaya in the field of boundary layers, with an expected completion date in 2024. While her primary research focus is in the area of boundary layers, she also engages in research related to teaching and learning, mathematics in education, and applied mathematics. She has been involved in research activities since 2016 and continues her research endeavors to the present.	Zarith Sofiah Othman contributed to the interpretation of the results and took the lead in writing the manuscript.
	Noraini Ahmad obtained her first degree in Mathematics Management from the University Teknologi MARA, Malaysia in 2010. She received a master's degree from Universiti Teknologi MARA, Malaysia in Industrial Mathematics. Currently, she is a lecturer at Centre of Foundation Studies, Universiti Teknologi MARA, Malaysia. Her research interests are Linear Algebra, Multi-criteria decision-making, and Fuzzy Theory	Noraini Ahmad conceived and planned the experiments.