

**THE EFFECT OF RUNGE-KUTTA IN ADAMS-BASHFORTH  
METHOD FOR SOLVING FIRST ORDER ORDINARY  
DIFFERENTIAL EQUATION**

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## ABSTRACT

Numerical problems can be solved using the multi-step method. Adams-Bashforth method is one of the multi-step methods that are often used together with Runge-Kutta as a starter to the multi-step. The objective of this study is to compare the numerical method using Adams-Bashforth method with theoretical method using linear differential equation, separable differential equation and Bernoulli's differential equation and to find the accuracy between these two methods, to determine which order of Runge-Kutta are more accurate in AB2, AB3 and AB4. Hence, each order of Adams-Bashforth has been applied with RK3, RK4 and RK5 and the results has been compared with the exact solution in order to find the error. The Graphical User Interface (GUI) has been used to solve this problem. The highest accuracy in this study reached at  $h=0.01$ . Then, the best order of Runge-Kutta AB2 and AB4 is when applied together with RK4, while the best order of Runge-Kutta for AB3 is when applied together with RK3. The findings of this study, to determine which step of Adams-Bashforth more accurate to RK2, RK3, RK4 and the results of this study can be used in any fields of study.

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