

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**BANACH CONTRACTION METHOD FOR SOLVING PARTIAL
DIFFERENTIAL EQUATION PROBLEM**

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

Most numerical methods necessitate complicated calculations and extensive computational work. This is because the quantity of information to be analyzed by these methods is huge and causing the computations of these approaches consume an excessive amount of computer memory. Today, people are interested in discovering the best methods for Partial Differential Equation (PDE). In this research, we purposed a method named Banach Contraction Method (BCM) for solving PDEs specifically nonlinear PDEs. The PDE is an equation that involve two or more independent variables. There are many methods that exist for solving partial differential equations such as Sumudu Decomposition Method (SDM), Homotopy Perturbation Method (HPM), Laplace Decomposition Method (LDM), Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM). However, for this research, BCM will be tested against VIM on the ability in solving problems with four different equations which are KdV equation, K (2, 2) equation, Burgers equation, and cubic Boussinesq equation. Therefore, this paper presents research finding that have been conducted with the purpose to achieve two objectives. First objective is to solve PDE problems by using BCM. Second objective is to observe the effectiveness of BCM by comparing with VIM in solving nonlinear PDE problems. In this research, the calculation for BCM and others were performed by using the software Maple 2022 to make the calculations easier and to simplify the difficult results. In terms of the graph, the results obtained by the software are used to create the graphs. As a result, exact solutions are obtained by applying VIM to the nonlinear PDEs meanwhile by applying BCM, the successive approximations for KdV equation, K (2, 2) equation, and Burgers equation up to $u_4(x, t)$ are obtained. However, BCM for cubic Boussinesq equation are only up to $u_2(x, t)$ due to the long duration required for calculation. We also provide the absolute error for each equation by using various values of t . From the results, when the value of t is higher, the value of absolute error also becomes higher. Therefore, we prove that the BCM is comparable to VIM. We suggest that future research should provide more new ways to solve problems. In the future, the increasing technology can help us find better and easier ways to solve the problems.