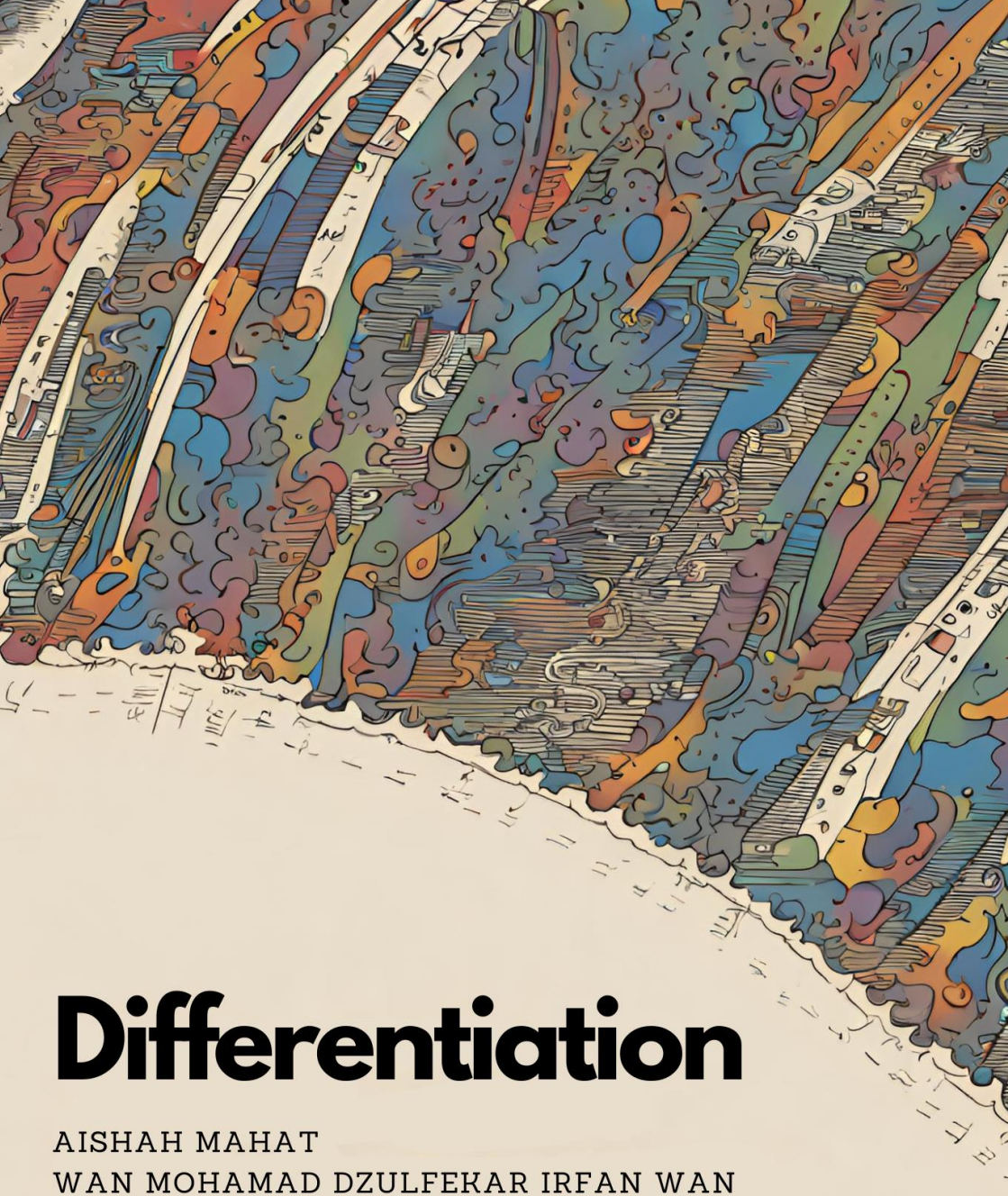

Differentiation

AISHAH MAHAT
WAN MOHAMAD DZULFEKAR IRFAN WAN MOHD DZULKARNAIN,
MUHAMMAD FAIZ ADZEEM MOHD FADLY,

Differentiation

AISHAH MAHAT
WAN MOHAMAD DZULFEKAR IRFAN WAN MOHD DZULKARNAIN,
MUHAMMAD FAIZ ADZEEM MOHD FADLY,

CONTENTS

DISCLAIMER

Author

Aishah Mahat

Wan Mohamad Dzulfekar Irfan Bin Wan Mohd Dzulkarnain

Muhammad Faiz Adzeem Bin Mohd Fadly

Editor & Illustrator

Aishah Mahat

Wan Mohamad Dzulfekar Irfan Bin Wan Mohd Dzulkarnain

Mohd Faiz Adzeem Bin Mohd Fadly

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PREFACE

This e-book, Differentiation, aimed to help students in mathematics. Our target audience for this module are students who take foundation courses. This e-book includes many examples of varying types of questions on the topic of differentiation, which would help students to become more familiar with differentiation questions. Furthermore, solutions for these questions are provided, which would also help students learn how to solve familiar questions.

DERIVATIVES OF

Sums & Differences

QUESTIONS AND SOLUTIONS

2) $f(x) = 4x^3 - 6x^2 + 2x$

3) $f(x) = 3x + \dfrac{1}{x}$

4) $f(x) = 5x^3 - 8x^2$

5) $f(x) = \sqrt{x}$

Solution

$$\begin{aligned} f'(x) &= 2\left(3x^{2-`}\right) + 5x^{1-1}1) \\ &= 6x + 5 \end{aligned}$$

$$\begin{aligned} f'(x) &= 3\left(4x^{3-1}\right) - 2\left(6x^{2-1}\right) + 2x^{1-1}2) \\ &= 12x^2 - 12x + 2 \end{aligned}$$

$$\begin{aligned} f'(x) &= 3x^{1-1} + \left(-1x^{(-1-1)}\right)3) \\ &= 3 - x^{-2} \end{aligned}$$

$$\begin{aligned} f'(x) &= 3\left(5x^{3-1}\right) - 2\left(8x^{2-1)}\right)4) \\ &= 15x^2 - 16x \end{aligned}$$

5) $$\begin{aligned} f'(x) &= \frac{1}{2}x^{\frac{1}{2}-1} \\ &= \frac{1}{2}x^{-\frac{1}{2}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

2.Differentiate each of the following with respect to x.

$f(x) = x^4 - 3x^2$ 1)

$f(x) = -7x^4 + x$ 2)

$f(x) = 4x^3 - 2x^2 + 5x$ 3)

$f(x) = \frac{1}{2}x^2 + 3x + 1$ 4)

5) $f(x) = \sqrt{x} + 2x^2 + 3$

Solution

$$f'(x) = 4x^{4-1} - 2\left(3x^{2-1}\right)$$ 1)
$$= 4x^3 - 6x$$

$$f'(x) = 4\left(-7x^{4-1}\right) + x^{1-1}$$ 2)
$$= -28x^3 + 1$$

$$f'(x) = 3\left(4x^{3-1}\right) - 2\left(2x^{2-1}\right) + 5x^{1-1}$$ 3)
$$= 12x^2 - 4x + 5$$

4) $f'(x) = 2\left(\frac{1}{2}x^{(2-1)}\right) + 3x^{1-1}$
$$= x + 3$$

5) $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} + 2\left(2x^{2-1}\right)$
$$= \frac{1}{2}x^{-\frac{1}{2}} + 4x$$
$$= \frac{1}{2\sqrt{x}} + 4x$$

3.Differentiate each of the following with respect to x.

1) $f(x) = 2x^5 - x^4$

2) $f(x) = \frac{4}{x} + \sqrt{x}$

3) $f(x) = 3x^9 - x^3$

4) $f(x) = 3x^5 + 9x$

5) $f(x) = 6x^3 + \frac{1}{x}$

Solution

1) $$\begin{aligned} f'(x) &= 5\left(2x^{5-1}\right) - 4x^{4-1} \\ &= 10x^4 - 4x^3 \end{aligned}$$

2) $$\begin{aligned} f'(x) &= -1\left(4x^{-1-1}\right) + \frac{1}{2}x^{\frac{1}{2}-1} \\ &= -\frac{4x}{x^2} + \frac{1}{2\sqrt{x}} \end{aligned}$$

3) $$\begin{aligned} f'(x) &= 9\left(3x^{9-1}\right) + 3x^{3-1} \\ &= 27x^8 + 3x^2 \end{aligned}$$

4) $$\begin{aligned} f'(x) &= 5\left(3x^{5-1}\right) + 9x^{1-1} \\ &= 15x^4 + 9 \end{aligned}$$

5) $$\begin{aligned} f'(x) &= 3\left(6x^{3-1}\right) + \left(-1x^{(-1-1)}\right) \\ &= 18x^2 - x^{-2} \end{aligned}$$

4.Differentiate each of the following with respect to x.

$f(x) = 23x - 4x^4$ 1)
$f(x) = \sqrt[4]{x}$ 2)
$f(x) = x^{-7} - 8$ 3)
4) $f(x) = 1 - 6x^{\frac{5}{2}}$
5) $f(x) = x^4 + 2x^{\frac{5}{2}}$

Solution

$$f'(x) = 23x^{1-1} - 4\left(4x^{4-1}\right) \quad 1)$$
$$= 23 - 16x^3$$

2) $$f'(x) = \frac{1}{4}x^{\frac{1}{4}-1}$$
$$= \frac{1}{4}x^{-\frac{3}{4}}$$

$$f'(x) = -7\left(x^{-7-1}\right) \quad 3)$$
$$= -7x^{-8}$$

$$f'(x) = \frac{5}{2}\left(-6x^{\frac{5}{2}-1}\right) \quad 4)$$
$$= -15x^{\frac{3}{2}}$$

5) $$f'(x) = 4x^{4-1} + \frac{5}{2}\left(2x^{\frac{5}{2}-1}\right)$$
$$= 4x^3 + 5x^{\frac{3}{2}}$$

5.Differentiate each of the following with respect to x.

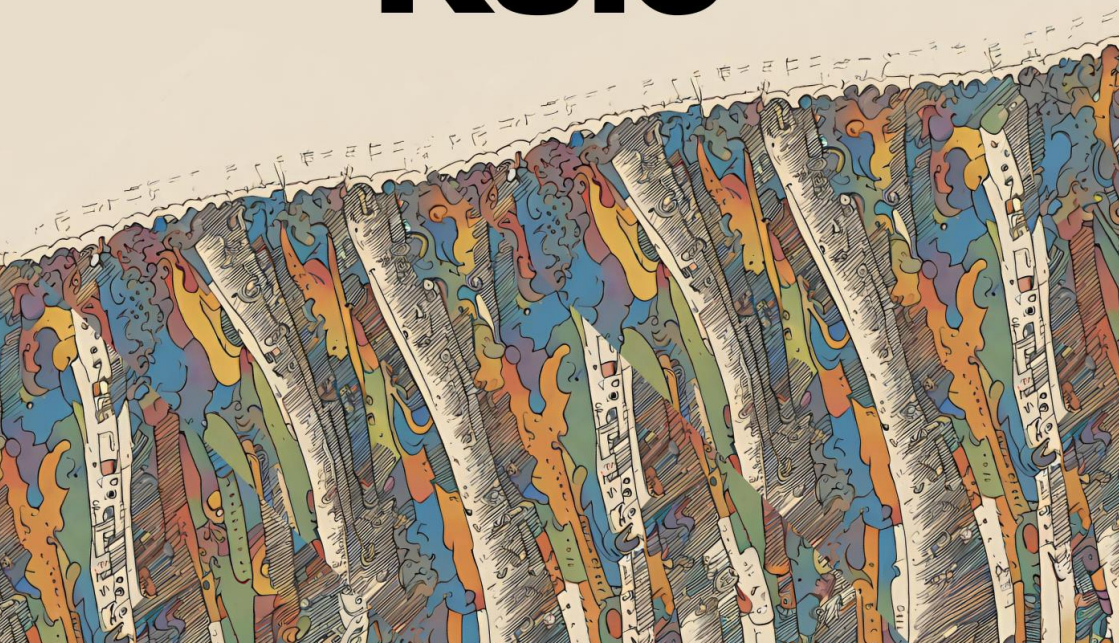
1) $f(x) = \frac{1}{2}x^2 - 4x^{-\frac{3}{2}}$

Solution

1) $f'(x) = 2\left(\frac{1}{2}x^{2-1}\right) - \left(-\frac{3}{2}4x^{-\frac{3}{2}-1}\right)$

$= x + 6x^{-\frac{5}{2}}$

Product Rule



QUESTIONS AND SOLUTIONS

Theorem

Let y = uv, where u and v are two differentiable functions,then

$f'(x) = uv' + vu'$

Differentiate each of the following in respect to x.

$f(x) = (2x-1)(4x+3)$ 1)

$f(x) = 4x^3(2-3x)$ 2)

Solution

$f(x) = (2x-1)(4x+3)$ 1)

$u = 2x-1 \quad v = 4x+3$

$u' = 2 \qquad v' = 4$

$$\begin{aligned} f'(x) &= uv' + vu' \\ &= (2x-1)(4) + (4x+3)(2) \\ &= 8x - 4 + 8x + 6 \\ &= 16x + 2 \end{aligned}$$

$f(x) = 4x^3(2-3x)$ 2)

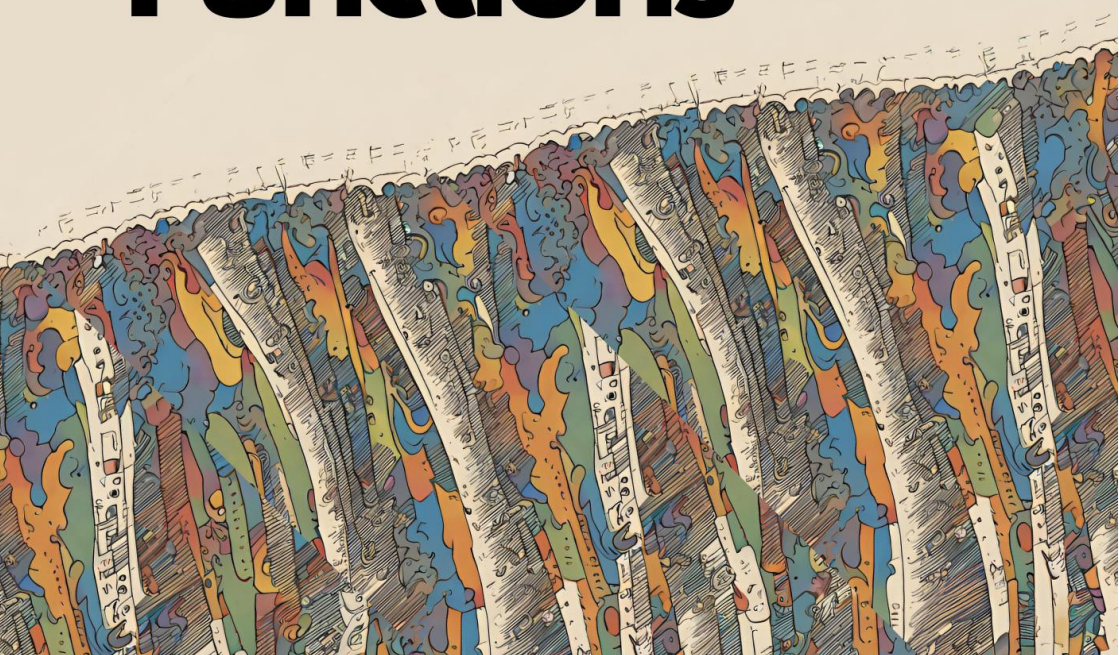
$u = 4x^3 \qquad v = 2-3x$

$u' = 8x^2 \qquad v' = -3$

$$\begin{aligned} f'(x) &= uv' + vu' \\ &= (4x)(-3) + (2-3x)\left(8x^2\right) \\ &= -12x + 16x^2 - 24x^3 \end{aligned}$$

DERIVATIVES OF

Trigonometry Functions



QUESTIONS AND SOLUTIONS

1.Find the derivatives of the following functions.

1) $f(x) = \sin x$

2) $f(x) = \cos 2x$

3) $f(x) = \sin 2x$

4) $f(x) = \sin(2x) + 4x$

Solution

$$f'(x) = \cos x \left(\frac{d}{dx} x \right) \text{1)}$$
$$= \cos x$$
$$f'(x) = -\sin 2x \left(\frac{d}{dx} 2x \right) \text{2)}$$
$$= -2 \sin 2x$$
$$f'(x) = \cos 2x \left(\frac{d}{dx} 2x \right) \text{3)}$$
$$= 2 \cos 2x$$
$$f'(x) = \cos 2x \left(\frac{d}{dx} 2x \right) + 4x^{1-1} \text{4)}$$
$$= 2 \cos 2x \ + 4$$

2.Find the derivatives of the following functions.

1) $f(x) = \cos^2 x$

2) $f(x) = 3\cos x - 2\sin 2x$

3) $f(x) = \tan 2x$

4) $f(x) = \tan^2 x$

Solution

1) $$\begin{aligned} f'(x) &= 2\cos^{2-1} x\left(\frac{d}{dx}\cos x\right) \\ &= 2\cos x(-\sin x) \\ &= -2\cos x \sin x \end{aligned}$$

2) $$\begin{aligned} f'(x) &= -3\sin x - 2\sin 2x(2) \\ &= -3\sin x - 4\sin 2x \end{aligned}$$

3) $$\begin{aligned} f'(x) &= \sec^2 2x\left(\frac{d}{dx}2x\right) \\ &= 2\sec^2 2x \end{aligned}$$

4) $$\begin{aligned} f'(x) &= 2\tan^{2-1} x\left(\frac{d}{dx}\tan x\right) \\ &= 2\tan x \sec^2 x \end{aligned}$$

3.Find the derivatives of the following functions.

a) $f(x) = \cos 3x$

b) $f(x) = \tan 4x$

c) $f(x) = \sin^2 x + \cos^2 x$

d) $f(x) = \tan x + \cot x$

e) $f(x) = \sin 2x + \cos 3x$

Solution

$$\begin{aligned} f'(x) &= \cos 3x(3) \\ &= 3\cos 3x \end{aligned}$$ a)

$$\begin{aligned} f'(x) &= \sec^2 4x(4) \\ &= 4\sec^2 4x \end{aligned}$$ b)

$$\begin{aligned} f'(x) &= 2\sin^{2-1} x(\cos x) + \left(2\cos^{2-1} x(-\sin x)\right) \\ &= 2\sin x\cos x - 2\cos x\sin x \\ &= 0 \end{aligned}$$ c)

$$f'(x) = \sec^2 x - \csc^2 x$$ d)

$$\begin{aligned} f'(x) &= \cos 2x(2) + (-\sin 3x(3)) \\ &= 2\cos 2x - 3\sin 3x \end{aligned}$$ e)

4.Find the derivatives of the following functions.

a) $f(x) = \tan 5x$

b) $f(x) = \sin 3x$

c) $f(x) = \cos 2x$

d) $f(x) = \tan x \cot x$

Solution

a)
$$\begin{aligned} f'(x) &= \sec^2 5x(5) \\ &= 5\sec^2 5x \end{aligned}$$

b)
$$\begin{aligned} f'(x) &= \cos 3x(3) \\ &= 3\cos 3x \end{aligned}$$

c)
$$\begin{aligned} f'(x) &= -\sin 2x(2) \\ &= -2\sin 2x \end{aligned}$$

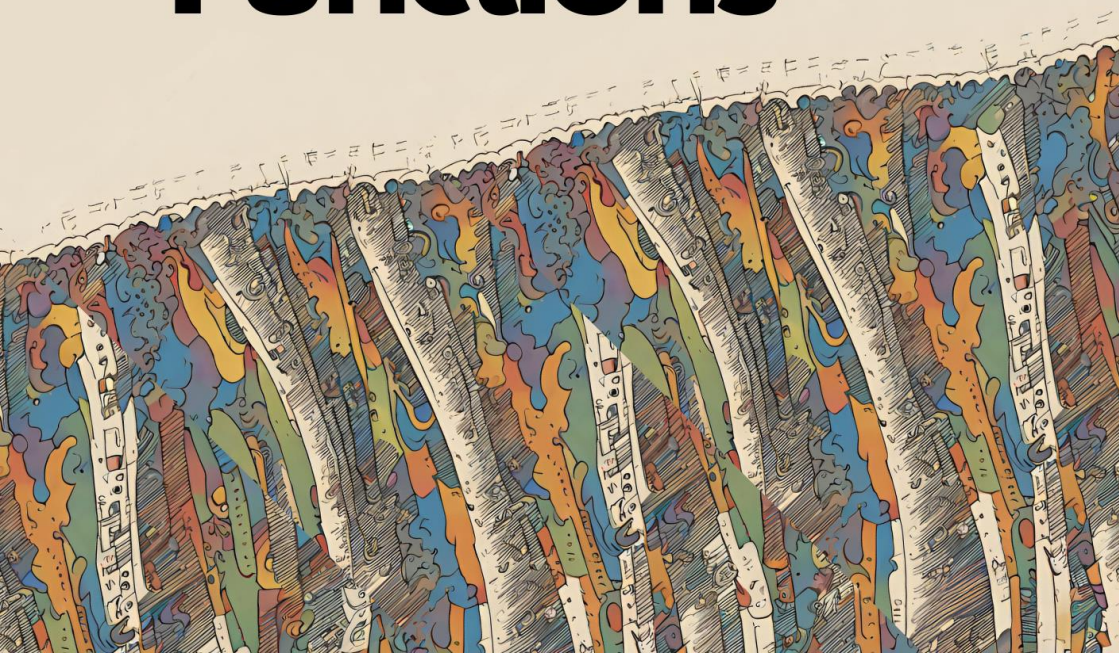
d)
$$f(x) = \tan x \cot x$$

$$\begin{aligned} u &= \tan x \quad & v &= \cot x \\ u' &= \sec^2 x & v' &= -\csc^2 x \end{aligned}$$

$$\begin{aligned} f'(x) &= uv' + vu' \\ &= (\tan x)\left(\csc^2 x\right) + (\cot x)\left(\sec^2 x\right) \\ &= \tan x \csc^2 x + \cot x \sec^2 x \end{aligned}$$

DERIVATIVES OF

Exponential Functions



QUESTIONS AND SOLUTIONS

Example:

$f(x) = e^x$

$$f'(x) = e^x\left(\frac{d}{dx}x\right)$$
$$= e^x$$

1.Find $f'(x)$ of the following functions.

a) $f(x) = e^{3x}$

b) $f(x) = e^{-2x}$

c) $f(x) = e^{4x}$

d) $f(x) = 2e^{-3x}$

Solution

$$f'(x) = e^{3x}(3) \text{ a)}$$
$$= 3e^{3x}$$

$$f'(x) = e^{-2x}(-2) \text{ b)}$$
$$= -2e^{-2x}$$

$$f'(x) = e^{4x}(4) \text{ c)}$$
$$= 4e^{4x}$$

$$f'(x) = 2e^{-3x}(-3) \text{ d)}$$
$$= -6e^{-3x}$$

2.Differentiate each of the following with respect to x.

a) $f(x) = e^{2x} + 3e^{-x}$

b) $f(x) = e^{2x} - e^{-x}$

c) $f(x) = e^{5x} + 4e^{2x}$

d) $f(x) = e^{-4x} - 3e^{3x}$

e) $f(x) = e^{4x}e^{-3x}$

Solution

a) $$\begin{aligned} f'(x) &= e^{2x}(2) - 3e^{-x} \\ &= 2e^{2x} - 3e^{-x} \end{aligned}$$

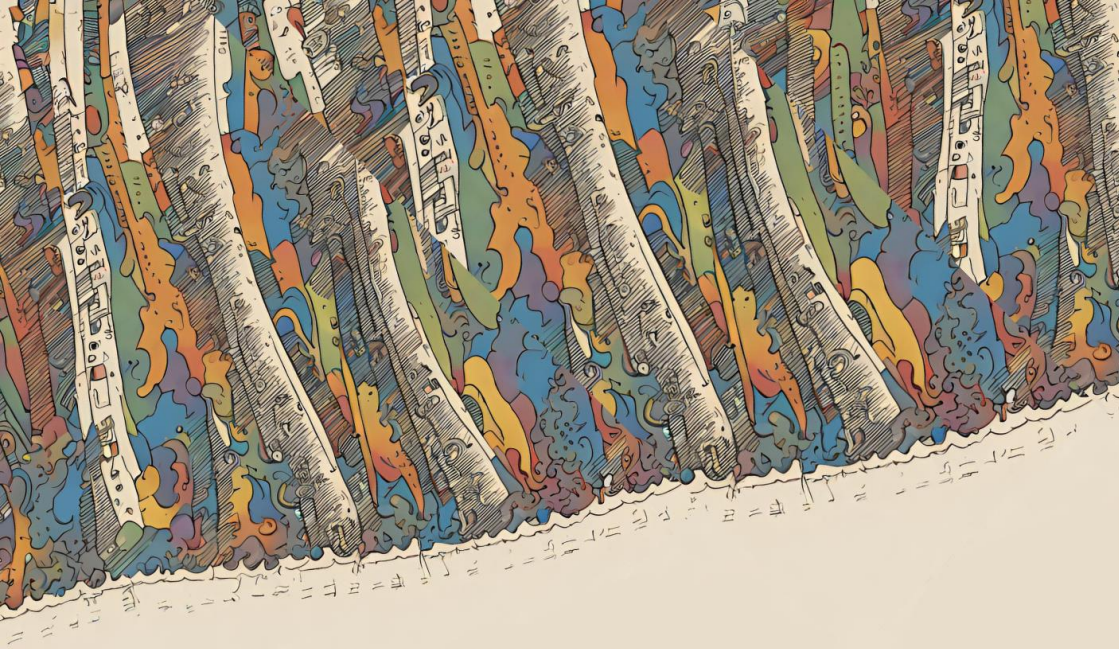
b) $$\begin{aligned} f'(x) &= e^{2x}(2) + e^{-x} \\ &= 2e^{2x} + e^{-x} \end{aligned}$$

c) $$\begin{aligned} f'(x) &= e^{5x}(5) + 4e^{2x}(2) \\ &= 5e^{5x} + 8e^{2x} \end{aligned}$$

d) $$\begin{aligned} f'(x) &= e^{-4x}(-4) - 3e^{3x}(3) \\ &= -4e^{-4x} - 9e^{3x} \end{aligned}$$

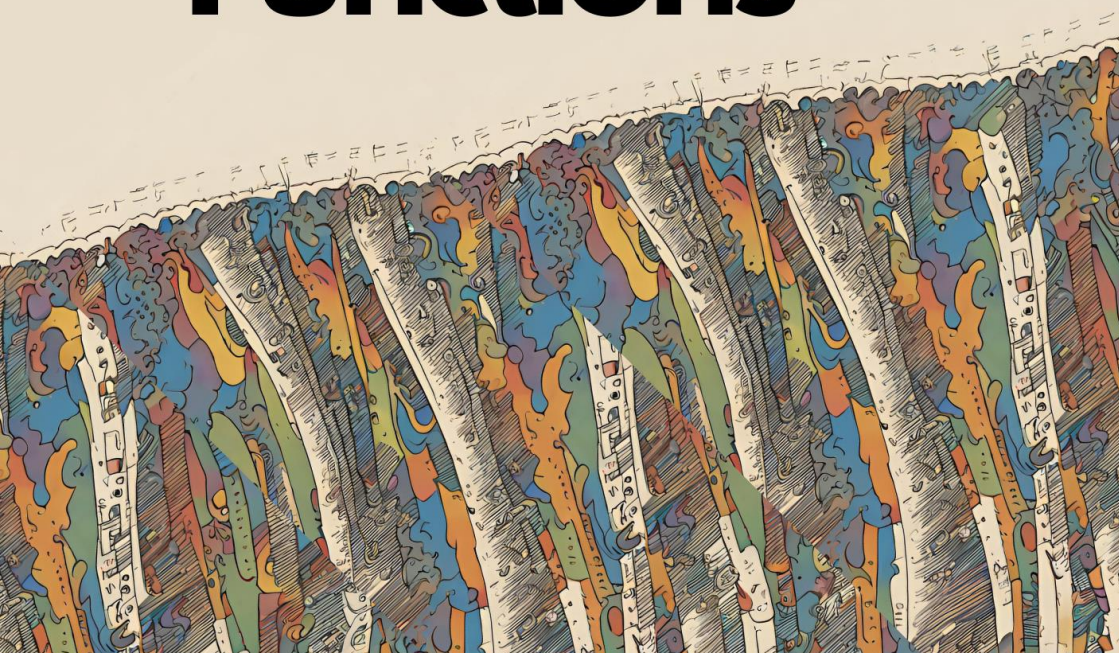
e) $$\begin{aligned} f(x) &= e^{4x-3x} \\ &= e^{x} \end{aligned}$$

$$f'(x) = e^{x}$$



DERIVATIVES OF

Logarithmic Functions



QUESTIONS AND SOLUTIONS

Simple Logarithmic Functions

1.Find the derivatives of the following functions

a) $f(x) = \ln\left(x^{2)}\right)$

b) $f(x) = \ln(\sin x)$

c) $f(x) = \ln(\cos x)$

d) $f(x) = \ln\left(e^{2x}\right)$

Solution

a)
$$f'(x) = \frac{1}{x^2}(2x)$$
$$= \frac{2}{x}$$

b)
$$f'(x) = \frac{1}{\sin x}(\cos x)$$
$$= \cot x$$

c)
$$f'(x) = \frac{1}{\cos x}(-\sin x)$$
$$= -\tan x$$

d)
$$f(x) = 2x$$
$$f'(x) = 2$$

Sums and Differences in Logarithmic Functions

1.Differentiate the following functions with respect to x.

a) $f(x) = \ln\left(x^4 + 2x^2\right)$

b) $f(x) = \ln(2x + 1)$

c) $f(x) = \ln(3x + 4)$

d) $f(x) = \ln\left(5x^2 + 2x\right)$

Solution

a) $f'(x) = \dfrac{1}{x^4 + 2x^2}\left(4x^3 + 4x\right)$

$= \dfrac{4x^3 + 4x}{x^4 + 2x^2}$

b) $f'(x) = \dfrac{1}{2x + 1}(2)$

$= \dfrac{2}{2x + 1}$

c) $f'(x) = \dfrac{1}{3x + 4}(3)$

$= \dfrac{3}{3x + 4}$

d) $f'(x) = \dfrac{1}{5x^2 + 2x}(10x + 2)$

$= \dfrac{10x + 2}{5x^2 + 2x}$

2.Find the derivatives of the following functions.

a) $f(x) = \ln\left(x^3 + 2x\right)$

b) $f(x) = \ln\left(4x^2 + 3x + 1\right)$

c) $f(x) = \ln\left(2x^3 + x\right)$

d) $f(x) = \ln\left(5x^4 + 2x^2 + 3\right)$

Solution

a) $$f'(x) = \frac{1}{x^3 + 2x}\left(3x^2 + 2\right)$$
$$= \frac{3x^2 + 2}{x^3 + 2x}$$

b) $$f'(x) = \frac{1}{4x^2 + 3x + 1}(8x + 3)$$
$$= \frac{8x + 3}{4x^2 + 3x + 1}$$

c) $$f'(x) = \frac{1}{2x^3 + x}\left(6x^2 + 1\right)$$
$$= \frac{6x^2 + 1}{2x^3 + x}$$

d) $$f'(x) = \frac{1}{5x^4 + 2x^2 + 3}\left(20x^3 + 4x\right)$$
$$= \frac{20x^3 + 4x}{5x^4 + 2x^2 + 3}$$

Trigonometry in Logarithmic Functions

1.Differentiate the following functions with respect to x.

a) $f(x) = \ln(\tan x)$

b) $f(x) = \ln(\sin 2x)$

c) $f(x) = \ln(\cos 3x)$

d) $f(x) = \ln(\tan 4x)$

Solution

a) $$f'(x) = \frac{1}{\tan x}\left(\sec^2 x\right)$$
$$= \frac{\sec^2 x}{\tan x}$$

b) $$f'(x) = \frac{1}{\sin 2x}(2\cos 2x)$$
$$= 2\cot 2x$$

c) $$f'(x) = \frac{1}{\cos 3x}(-3\sin 3x)$$
$$= -3\tan 3x$$

d) $$f'(x) = \frac{1}{\tan 4x}\left(4\sec^2 4x\right)$$
$$= \frac{4\sec^2 4x}{\tan 4x}$$

Exponential in Logarithmic Functions

1.Find the derivatives of the following functions.

a) $f(x) = \ln\left(e^{-3x}\right)$

b) $f(x) = \ln\left(e^{4x}\right)$

c) $f(x) = \ln\left(e^{2x}\right) - \ln\left(\cos 2x\right)$

d) $f(x) = \ln\left(e^{2x} * \sin x\right)$

Solution

a) $f(x) = -3x$

$f'(x) = -3$

b) $f(x) = 4x$

$f'(x) = 4$

c) $f'(x) = 2 - \dfrac{1}{\cos 2x}(-2\sin 2x)$

$= 2 + 2\tan 2x$

d) $f(x) = \ln\left(e^{2x}\right) + \ln\left(\sin x\right)$

$f'(x) = 2 + \dfrac{1}{\sin x}(\cos x)$

$= 2 + \cot x$

2.Differentiate the following functions.

a) $f(x) = \ln\left(e^{-3x+4}\right) - \ln(\tan 3x)$

b) $f(x) = \ln(\csc 4x)$

c) $f(x) = \ln(\sec 5x)$

d) $f(x) = \ln(\cot 6x)$

Solution

a) $f(x) = (-3x + 4) - \ln(\tan 3x)$

$$f'(x) = -3 - \frac{1}{\tan 3x}\left(3\sec^2 3x\right)$$
$$= -3 - \frac{3\sec^2 3x}{\tan 3x}$$

b) $$f'(x) = \frac{1}{\csc 4x}(-4\csc 4x \cot 4x)$$
$$= -4\cot 4x$$

c) $$f'(x) = \frac{1}{\sec 5x}(5\sec 5x \tan 5x)$$
$$= 5\tan 5x$$

d) $$f'(x) = \frac{1}{\cot 6x}\left(-6\csc^2 6x\right)$$
$$= \frac{-6\csc^2 6x}{\cot 6x}$$

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