CALCULUS I : LIMITS

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LIMITS

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DISCLAIMER

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PREFACE

This e-book, Calculus 1: Limits aimed to help students in mathematic subject. Targeted users for this module is students who take foundation course. Mathematical tips and formulas will be placed in accordance to the subtopics whilst each questions will be displayed based on the syllabus carried out during the lesson. At the end of each topic, targeted students should meet up with the lecturer to discuss over the solution of mathematics problem. With the existence of this e-book, hopefully it will be beneficial and give positive impact towards teaching and learning for students and lecturers as a whole.

Basic Theorems on Limit

"*Let* $\lim\limits_{x \to a} f(x) = L$ *and* $\lim\limits_{x \to a} g(x) = M$ "

1) $$\lim_{x \to a} \left[f(x) + g(x) \right] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$
$$= L + M$$

2) $$\lim_{x \to a} \left[f(x) - g(x) \right] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$
$$= L - M$$

3) $$\lim_{x \to a} M f(x) = M \lim_{x \to a} f(x)$$
$$= M(L)$$

4) $$\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$
$$= L \cdot M$$

5) $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} f(x)}$$
$$= \frac{L}{M}$$

6) $$\lim_{x \to a} [f(x)]^c = \left[\lim_{x \to a} f(x) \right]^c$$
$$= L^c$$

7) $$\lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)}$$
$$= \sqrt{L}$$

Limit : Direct Substitution

Properties of Limits

$$\lim_{x \to c} f(x) = f(c)$$

Exercise 1:

Determine the limit for each of the following.

a) $\lim_{x \to 3} x^2 + 3x - 18$

b) $\lim_{x \to -2} 2x^3 - x^2 + 3x - 4$

c) $\lim_{x \to 5} \left(7 + \sqrt{x^2 + 11}\right)$

Exercise 2:

Evaluate the limit for the following functions.

a) $\lim_{x \to 1} \left(\frac{x^2 - 4}{x + 1}\right)$

b) $\lim_{x \to 2} \left(\frac{x^2 + 6x + 8}{x^2 + 4}\right)$

Solution:

Exercise 1

a) $\lim_{x\to 3} x^2+3x-18$

$=3^2+3(3)-18$

$=0$

b) $\lim_{x\to -2} 2x^3-x^2+3x-4$

$=2(-2)^3-(-2)^2+3(-2)-4$

$=-30$

c) $\lim_{x\to 5}\left(7+\sqrt{x^2+11}\right)$

$=7+\sqrt{5^2+11}$

$=13$

Exercise 2

a) $\lim_{x\to 1}\left(\frac{x^2-4}{x+1}\right)$

$=\frac{1^2-4}{1+1}$

$=-\frac{3}{2}$

b) $\lim_{x\to 2}\left(\frac{x^2+6x+8}{x^2+4}\right)$

$=\frac{2^2+6(2)+8}{2^2+4}$

$=\frac{24}{8}$

$=3$

Limit : Factorization

Find the limits by factoring

Don't leave your answer "Undefined".

Use other method to solve the problems.

Exercise 1 :

Determine the limit for the following function.

a) $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

b) $\lim_{t \to 0} \frac{2t^4 + 5t^3}{6t^6 + 4t^3}$

Exercise 2 :

Evaluate $\lim_{x \to 0} \frac{e^{2x} - e^{4x}}{1 - e^{2x}}$

Exercise 3 :

Evaluate the limit : $\lim_{x \to 3} \frac{3 - x}{x^2 - 3x}$

Exercise 4 :

Evaluate $\lim_{x \to -2} f(x)$ if $f(x) = \begin{cases} \frac{2x^2 - 5}{6 + x} & \text{if } x \leq -2 \\ \frac{x^2 - 2x - 8}{x^2 - 4x - 12} & \text{if } x \geq -2 \end{cases}$

Solution:

Exercise 1 :

a) 1st Method : Direct Substitution

$$\lim_{x\to 2}\frac{x^2+x-6}{x-2}$$

$$=\frac{2^2+2-6}{2-2}$$

$$=\frac{0}{0}\ (\textit{Undefined})$$

2nd Method : Factorization

$$\lim_{x\to 2}\frac{x^2+x-6}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+3)}{x-2}$$

$$=\lim_{x\to 2}\ x+3$$

$$=2+3$$

$$=5$$

b) 1st Method : Direct Substitution

$$\lim_{t\to 0}\frac{2t^4+5t^3}{6t^6-4t^3}$$

$$=\frac{2(0)^4+5(0)^3}{6(0)^6-4(0)^3}$$

$$=\frac{0}{0}\ (\textit{Undefined})$$

2nd Method : Factorization

$$\lim_{t \to 0} \frac{2t^4 + 5t^3}{6t^6 - 4t^3} = \lim_{t \to 0} \frac{t^3(2t+5)}{t^3(6t^3 - 4)}$$

$$= \lim_{t \to 0} \frac{2t+5}{6t^3 - 4}$$

$$= \frac{2(0) + 5}{6(0)^3 - 4}$$

$$= -\frac{5}{4}$$

Exercise 2 :

1st Method : Direct Substitution

$$\lim_{x \to 0} \frac{e^2 - e^{4x}}{1 - e^{2x}}$$

$$= \frac{e^{2(0)} - e^{4(0)}}{1 - e^{2(0)}}$$

$$= \frac{0}{0} \ (\textit{Undefined})$$

2nd Method : Factorization

$$\lim_{x \to 0} \frac{e^2 - e^{4x}}{1 - e^{2x}} = \lim_{x \to 0} \frac{e^{2x}(1 - e^{2x})}{1 - e^{2x}}$$

$$= \lim_{x \to 0} e^{2x}$$

$$= e^{2(0)}$$

$$= 1$$

Exercise 3 :

1st Method : Direct Substitution

$$\lim_{x \to 3} \frac{3 - x}{x^2 - 3x}$$

$$= \frac{3 - 3}{3^2 - 3(3)}$$

$$= \frac{0}{0} \; (\textit{Undefined})$$

2nd Method : Factorization

$$\lim_{x \to 3} \frac{3 - x}{x^2 - 3x} = \lim_{x \to 3} \frac{3 - x}{x(x - 3)}$$

$$= \lim_{x \to 3} \frac{-(-3 + x)}{x(x - 3)}$$

$$= -\lim_{x \to 3} \frac{1}{x}$$

$$= -\frac{1}{3}$$

Exercise 4 :

Left Hand Limit

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{2x^2 - 5}{x - 5}$$

$$= \frac{2(-2)^2 - 5}{(-2) - 5}$$

$$= \frac{3}{4}$$

Right Hand Limit

1st Method : Factorization

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{x^2 - 2x - 8}{x^2 - 4x - 12}$$

$$= \frac{(-2)^2 - 2(-2) - 8}{(-2)^2 - 4(-2) - 12}$$

$$= \frac{0}{0} \; (\textit{Undefined})$$

2nd Method : Factorization

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{x^2 - 2x - 8}{x^2 - 4x - 12}$$

$$= \lim_{x \to -2^+} \frac{(x-4)(x+2)}{(x-6)(x+2)}$$

$$= \lim_{x \to -2^+} \frac{x-4}{x-6}$$

$$= \frac{(-2) - 4}{(-2) - 6}$$

$$= \frac{-6}{-8}$$

$$= \frac{3}{4}$$

Limit : Conjugate

Find the limit using conjugate

Don't leave your answer "Undefined".

If you see a square root symbol you have to use conjugate.

Exercise 1 :

Evaluate each of the following limit :

a) $\lim\limits_{x \to 3} \dfrac{3x - 9}{\sqrt{x + 6} - 3}$

b) $\lim\limits_{x \to 2} \dfrac{\sqrt{x^2 + 5} - 3}{x - 2}$

Exercise 2 :

Find $\lim\limits_{x \to 0} \dfrac{(5x - 2) + \sqrt{x + 4}}{2x}$

Exercise 3 :

Determine :

a) $\lim\limits_{x \to 1} \dfrac{\sqrt{x + 6} - \sqrt{7}}{x - 1}$

b) $\lim\limits_{x \to 5} \dfrac{\sqrt{5} - \sqrt{10 - x}}{x^2 - 25}$

Solution:

Exercise 1 :

a) 1^{st} Method : Direct Substitution

$$\lim_{x\to3}\frac{3x-9}{\sqrt{x+6}-3}=\frac{3(3)-9}{\sqrt{(3)+6}-3}$$
$$=\frac{9-9}{3-3}$$
$$=\frac{0}{0}(\textit{Undef ined})$$

2^{nd} Method : Conjugate

$$\lim_{x\to3}\frac{3x-9}{\sqrt{x+6}-3}=\lim_{x\to3}\frac{3x-9}{\sqrt{x+6}-3}\cdot\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}$$
$$=\lim_{x\to3}\frac{(3x-9)\cdot\sqrt{x+6}+3}{x+6+3\sqrt{x+6}-3\sqrt{x+6}-9}$$
$$=\lim_{x\to3}\frac{(3x-9)\cdot\sqrt{x+6}+3}{x+6-9}$$
$$=\lim_{x\to3}\frac{3(x-3)\cdot\sqrt{x+6}+3}{(x-3)}$$
$$=\lim_{x\to3}3\cdot\sqrt{x+6}+3$$
$$=3\cdot\sqrt{(3)+6}+3$$
$$=3\cdot6$$
$$=18$$

b) 1^{st} Method : Direct Substitution

$$\lim_{x\to2}\frac{\sqrt{x^2+5}-3}{x-2}=\frac{\sqrt{(2)^2+5}-3}{(2)-2}$$
$$=\frac{3-3}{2-2}$$
$$=\frac{0}{0}(\textit{Undef ined})$$

2nd Method : Conjugate

$$\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}=\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}\cdot\frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$$
$$=\lim_{x\to 2}\frac{x^2+5-3\sqrt{x^2+5}+3\sqrt{x^2+5}-9}{(x-2)\cdot\sqrt{x^2+5}+3}$$
$$=\lim_{x\to 2}\frac{x^2-4}{(x-2)\cdot\sqrt{x^2+5}+3}$$
$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)\cdot\sqrt{x^2+5}+3}$$
$$=\lim_{x\to 2}\frac{(x+2)}{\sqrt{x^2+5}+3}$$
$$=\frac{[(2)+2]}{\sqrt{(2)^2+5}+3}$$
$$=\frac{2+2}{3+3}$$
$$=\frac{4}{6}=\frac{2}{3}$$

Exercise 2 :

1st Method : Direct Substitution

$$\lim_{x\to 0}\frac{(5x-2)+\sqrt{x+4}}{2x}=\frac{[5(0)-2]+\sqrt{(0)+4}}{2(0)}$$
$$=\frac{0}{0}(\text{ Undef ined})$$

2nd Method : Conjugate

$$\lim_{x\to 0}\frac{(5x-2)+\sqrt{x+4}}{2x}=\lim_{x\to 0}\frac{(5x-2)+\sqrt{x+4}}{2x}\cdot\frac{(5x-2)-\sqrt{x+4}}{(5x-2)-\sqrt{x+4}}$$
$$=\lim_{x\to 0}\frac{(5x-2)(5x-2)-(5x-2)\sqrt{x+4}+(5x-2)\sqrt{x+4}-(x+4)}{2x\cdot(5x-2)-\sqrt{x+4}}$$
$$=\lim_{x\to 0}\frac{25x^2-10x-10x+4-x-4}{2x\cdot(5x-2)-\sqrt{x+4}}$$
$$=\lim_{x\to 0}\frac{25x^2-21x}{2x\cdot(5x-2)-\sqrt{x+4}}$$
$$=\lim_{x\to 0}\frac{x(25x-21)}{2x\cdot(5x-2)-\sqrt{x+4}}$$
$$=\lim_{x\to 0}\frac{25x-21}{2\cdot(5x-2)-\sqrt{x+4}}$$
$$=\frac{25(0)-21}{2\cdot[5(0)-2]-\sqrt{(0)+4}}$$
$$=\frac{-21}{2\cdot(-4)}$$
$$=\frac{-21}{-8}=\frac{21}{8}$$

Exercise 3 :

a) 1^{st} Method : Direct Substitution

$$\lim_{x \to 1} \frac{\sqrt{x+6} - \sqrt{7}}{x-1} = \frac{\sqrt{(1)+6} - \sqrt{7}}{(1)-1}$$

$$= \frac{\sqrt{7} - \sqrt{7}}{1-1}$$

$$= \frac{0}{0} (\, Undef\,ined)$$

2^{nd} Method : Conjugate

$$\lim_{x \to 1} \frac{\sqrt{x+6} - \sqrt{7}}{x-1} = \lim_{x \to 1} \frac{\sqrt{x+6} - \sqrt{7}}{x-1} \cdot \frac{\sqrt{x+6} + \sqrt{7}}{\sqrt{x+6} + \sqrt{7}}$$

$$= \lim_{x \to 1} \frac{x+6+\sqrt{x+6}\sqrt{7} - \sqrt{x+6}\sqrt{7} - 7}{(x-1) \cdot \sqrt{x+6} + \sqrt{7}}$$

$$= \lim_{x \to 1} \frac{x+6-7}{(x-1) \cdot \sqrt{x+6} + \sqrt{7}}$$

$$= \lim_{x \to 1} \frac{(x-1)}{(x-1) \cdot \sqrt{x+6} + \sqrt{7}}$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x+6} + \sqrt{7}}$$

$$= \frac{1}{\sqrt{(1)+6} + \sqrt{7}}$$

$$= \frac{1}{\sqrt{7} + \sqrt{7}}$$

$$= \frac{1}{2\sqrt{7}}$$

b) 1^{st} Method : Direct Substitution

$$\lim_{x \to 5} \frac{\sqrt{5} - \sqrt{10-x}}{x^2-25} = \frac{\sqrt{5} - \sqrt{10-(5)}}{(5)^2-25}$$

$$= \frac{\sqrt{5} - \sqrt{5}}{25-25}$$

$$= \frac{0}{0} (\, Undef\,ined)$$

2nd Method : Conjugate

$$\lim_{x\to 5}\frac{\sqrt{5}-\sqrt{10-x}}{x^2-25}=\lim_{x\to 5}\frac{\sqrt{5}-\sqrt{10-x}}{x^2-25}\cdot\frac{\sqrt{5}+\sqrt{10-x}}{\sqrt{5}+\sqrt{10-x}}$$

$$=\lim_{x\to 5}\frac{5-\sqrt{5}\sqrt{10-x}+\sqrt{5}\sqrt{10-x}-(10-x)}{\left(x^2-25\right)\cdot\sqrt{5}+\sqrt{10-x}}$$

$$=\lim_{x\to 5}\frac{5-10+x}{\left(x^2-25\right)\cdot\sqrt{5}+\sqrt{10-x}}$$

$$=\lim_{x\to 5}\frac{(x-5)}{(x+5)(x-5)\cdot\sqrt{5}+\sqrt{10-x}}$$

$$=\lim_{x\to 5}\frac{1}{(x+5)\cdot\sqrt{5}+\sqrt{10-x}}$$

$$=\frac{1}{\left[(5)+5\right]\cdot\sqrt{5}+\sqrt{10-(5)}}$$

$$=\frac{1}{10\cdot\sqrt{5}+\sqrt{5}}$$

$$=\frac{1}{10\cdot 2\sqrt{5}}$$

$$=\frac{1}{20\sqrt{5}}$$

Limit : Infinity

Divide the numerator and denominator by the **highest power of x in the denominator**.

Exercise 1 :

Evaluate the limit for the following functions.

a) $\lim\limits_{x \to \infty} \dfrac{x^3 + 2x^2 - 3x + 5}{x^3 + 6x}$

b) $\lim\limits_{x \to -\infty} \dfrac{(x-5)^2}{4x^2 - 13x}$

Exercise 2 :

Evaluate $\lim\limits_{x \to \infty} \sqrt{\dfrac{x^4 - 7}{x^3 + 2x^2}}$

Exercise 3 :

Determine the limit for the following function.

a) $\lim\limits_{x \to \infty} \dfrac{\sqrt{6x^2 + 9}}{4x - 1}$

b) $\lim\limits_{x \to -\infty} \dfrac{9x^2 + 7x}{\sqrt{9x^4 + 6}}$

Solution:

Exercise 1 :

a) Answer :

$$\lim_{x\to\infty}\frac{x^3+2x^2-3x+5}{x^3+6x}=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}+\frac{2x^2}{x^3}-\frac{3x}{x^3}+\frac{5}{x^3}}{\frac{x^3}{x^3}+\frac{6x}{x^3}}$$

$$=\lim_{x\to\infty}\frac{1+\frac{2}{x}-\frac{3}{x^2}+\frac{5}{x^3}}{1+\frac{6}{x^2}}$$

$$=\frac{1+\frac{2}{(\infty)}-\frac{3}{(\infty)^2}+\frac{5}{(\infty)^3}}{1+\frac{6}{(\infty)^2}}$$

$$=\frac{1+0-0+0}{1+0}$$

$$=\frac{1}{1}$$

$$=1$$

b) Answer :

$$\lim_{x\to-\infty}\frac{(x-5)^2}{4x^2-13x}=\lim_{x\to-\infty}\frac{x^2-10x+25}{4x^2-13x}$$

$$=\lim_{x\to-\infty}\frac{\frac{x^2}{x^2}-\frac{10x}{x^2}+\frac{25}{x^2}}{\frac{4x^2}{x^2}-\frac{13x}{x^2}}$$

$$=\lim_{x\to-\infty}\frac{1-\frac{10}{x}+\frac{25}{x^2}}{4-\frac{13}{x}}$$

$$=\frac{1-\frac{10}{(-\infty)}+\frac{25}{(-\infty)^2}}{4-\frac{13}{(-\infty)}}$$

$$=\frac{1+0+0}{4+0}$$

$$=\frac{1}{4}$$

Exercise 2 :

$$\lim_{x\to\infty}\sqrt{\frac{x^4-7}{x^3+2x^2}}=\sqrt{\lim_{x\to\infty}\frac{\frac{x^4}{x^3}-\frac{7}{x^3}}{\frac{x^3}{x^3}+\frac{2x^2}{x^3}}}$$

$$=\sqrt{\lim_{x\to\infty}\frac{x-\frac{7}{x^3}}{1+\frac{2}{x}}}$$

$$=\sqrt{\frac{(\infty)-\frac{7}{(\infty)^3}}{1+\frac{2}{(\infty)}}}$$

$$=\sqrt{\frac{(\infty)-0}{1+0}}$$

$$=\sqrt{\frac{(\infty)}{1}}$$

$$=\infty$$

Exercise 3 :

a) Answer :

$$x=\sqrt{x^2}$$

$$-x=-\sqrt{x^2}$$

$$x^2=\sqrt{x^4}$$

$$-x^2=-\sqrt{x^4}$$

$$\lim_{x\to\infty}\frac{\sqrt{6x^2+9}}{4x-1}=\lim_{x\to\infty}\frac{\sqrt{\frac{6x^2}{x^2}}+\sqrt{\frac{9}{x^2}}}{\frac{4x}{x}-\frac{1}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{6}+\sqrt{\frac{9}{x^2}}}{4-\frac{1}{x}}$$

$$=\frac{\sqrt{6}+\sqrt{\frac{9}{(\infty)^2}}}{4-\frac{1}{(\infty)}}$$

$$=\frac{\sqrt{6}+0}{4-0}$$

$$=\frac{\sqrt{6}}{4}$$

b) Answer :

$$\lim_{x \to -\infty} \frac{9x^2 + 7x}{\sqrt{9x^4 + 6}} = \lim_{x \to -\infty} \frac{\frac{9x^2}{x^2} + \frac{7x}{x^2}}{\sqrt{\frac{9x^4}{x^4}} + \sqrt{\frac{6}{x^4}}}$$

$$= \lim_{x \to -\infty} \frac{9 + \frac{7}{x}}{\sqrt{9} + \sqrt{\frac{6}{x^4}}}$$

$$= \lim_{x \to -\infty} \frac{-\left(9 + \frac{7}{x}\right)}{\sqrt{9} + \sqrt{\frac{6}{x^4}}}$$

$$= \frac{-\left[9 + \frac{7}{(-\infty)}\right]}{\sqrt{9} + \sqrt{\frac{6}{(-\infty)^4}}}$$

$$= \frac{-9 + 0}{\sqrt{9} + 0}$$

$$= \frac{-9}{3}$$

$$= -3$$

There is some special case if $-\infty$

Either numerator or denominator have to put negative (-).

Limit : Trigonometric Function

Theorem a) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

b) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Exercise 1 :

Evaluate $\lim_{x \to 0} \frac{\sin 4x}{3x}$

Exercise 2 :

Evaluate the limit : $\lim_{x \to 0} \frac{\sin 7x}{3x(6 - 2\cos x)}$

Exercise 3 :

Evaluate $\lim_{x \to 0} \frac{\tan x}{2x}$

Exercise 4 :

Evaluate $\lim_{\theta \to 0} \frac{\sin 6\theta}{\sin 2\theta}$

Exercise 5 :

Evaluate the limit : $\lim_{x \to 0} \frac{x \cos 4x - \sin 2x}{3x}$

Solution:

Exercise 1 :

$$\lim_{x\to 0}\frac{\sin 4x}{3x} = \frac{1}{3}\lim_{x\to 0}\frac{\sin 4x}{x}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin 4x}{x}\cdot\left[\frac{4}{4}\right]$$

$$= \frac{4}{3}\lim_{x\to 0}\frac{\sin 4x}{4x}$$

$$= \frac{4}{3}(1)$$

$$= \frac{4}{3}$$

Exercise 2 :

$$\lim_{x\to 0}\frac{\sin 7x}{3x(6-2\cos x)} = \lim_{x\to 0}\frac{\sin 7x}{3x}\times\lim_{x\to 0}\frac{1}{(6-2\cos x)}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin 7x}{x}\times\lim_{x\to 0}\frac{1}{(6-2\cos x)}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin 7x}{x}\cdot\left[\frac{7}{7}\right]\times\lim_{x\to 0}\frac{1}{(6-2\cos x)}$$

$$= \frac{7}{3}\lim_{x\to 0}\frac{\sin 7x}{7x}\times\lim_{x\to 0}\frac{1}{(6-2\cos x)}$$

$$= \frac{7}{3}(1)\times\frac{1}{(6-2\cos 0)}$$

$$= \frac{7}{3}\times\frac{1}{4}$$

$$= \frac{7}{12}$$

Exercise 3 :

$$\lim_{x \to 0} \frac{\tan x}{2x} = \lim_{x \to 0} \frac{\left(\dfrac{\sin x}{\cos x}\right)}{2x}$$

$$= \lim_{x \to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{2x}$$

$$= \lim_{x \to 0} \frac{\sin x}{2x} \cdot \frac{1}{\cos x}$$

$$= \lim_{x \to 0} \frac{\sin x}{2x} \cdot \lim_{x \to 0} \frac{1}{\cos x}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x}$$

$$= \frac{1}{2}(1) \cdot \frac{1}{\cos 0}$$

$$= \frac{1}{2}(1)$$

$$= \frac{1}{2}$$

Exercise 4 :

$$\lim_{\theta\to 0}\frac{\sin 6\theta}{\sin 2\theta} = \lim_{\theta\to 0}\frac{\sin 6\theta \cdot \left[\frac{1}{\theta}\right]}{\sin 2\theta \cdot \left[\frac{1}{\theta}\right]}$$

$$= \lim_{\theta\to 0}\frac{\frac{\sin 6\theta}{\theta}}{\frac{\sin 2\theta}{\theta}}$$

$$= \frac{\lim_{\theta\to 0}\frac{\sin 6\theta}{\theta}}{\lim_{\theta\to 0}\frac{\sin 2\theta}{\theta}}$$

$$= \frac{\lim_{\theta\to 0}\frac{\sin 6\theta}{\theta}\cdot\left[\frac{6}{6}\right]}{\lim_{\theta\to 0}\frac{\sin 2\theta}{\theta}\cdot\left[\frac{2}{2}\right]}$$

$$= \frac{6\lim_{\theta\to 0}\frac{\sin 6\theta}{6\theta}}{2\lim_{\theta\to 0}\frac{\sin 2\theta}{2\theta}}$$

$$= \frac{6(1)}{2(1)}$$

$$= \frac{6}{2} \;/\; 3$$

Exercise 5 :

$$\lim_{x\to 0}\frac{x\cos 4x - \sin 2x}{3x} = \lim_{x\to 0}\left(\frac{x\cos 4x}{3x} - \frac{\sin 2x}{3x}\right)$$

$$= \lim_{x\to 0}\frac{\cos 4x}{3} - \lim_{x\to 0}\frac{\sin 2x}{3x}$$

$$= \lim_{x\to 0}\frac{\cos 4x}{3} - \frac{1}{3}\lim_{x\to 0}\frac{\sin 2x}{x}$$

$$= \lim_{x\to 0}\frac{\cos 4x}{3} - \frac{1}{3}\lim_{x\to 0}\frac{\sin 2x}{x}\left[\frac{2}{2}\right]$$

$$= \lim_{x\to 0}\frac{\cos 4x}{3} - \frac{2}{3}\lim_{x\to 0}\frac{\sin 2x}{2x}$$

$$= \frac{\cos 4(0)}{3} - \frac{2}{3}(1)$$

$$= \frac{1}{3} - \frac{2}{3}$$

$$= -\frac{1}{3}$$

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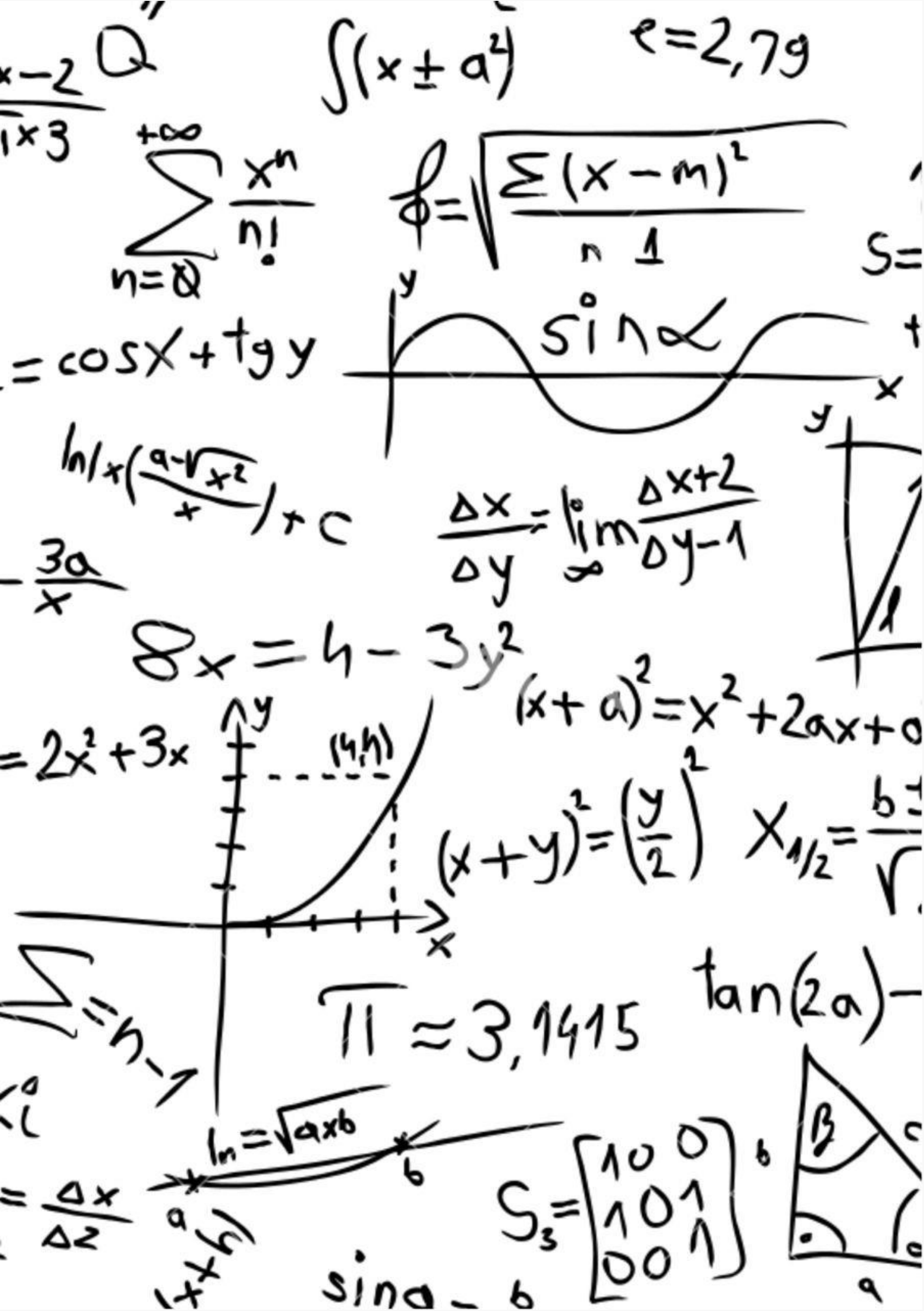
e=2,79
sin α
8x = h - 3y²
(x+a)²=x²+2ax+a
(x+y)²=(y/2)²
π ≈ 3,1415
tan(2a)
S₃ = [1 0 0; 1 0 1; 0 0 1]