UNIVERSITI TEKNOLOGI MARA

EXTENSION OF RMIL CONJUGATE GRADIENT METHOD FOR UNCONSTRAINED OPTIMIZATION

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ABSTRACT

The Conjugate Gradient (CG) methods have significantly contributed to solving Unconstrained Optimization (UO) problems. This research focused on the modification of existing CG method of Rivaie, Mustafa, Ismail and Leong (RMIL). RMIL is ubiquitous for its effectiveness as an optimization technique, yet their significance remains to be defined and their full potential is yet to be realized. Even the global convergence theoretical is available for RMIL method, it only applies for the positive RMIL parameter. Indeed, the numerical performance of RMIL method is impressive regardless of its parameter sign. Much efforts have been made previously to increase the efficiency of RMIL method. Hence, this research proposed a CG search direction named NEWRMIL by combining the scaled negative gradient as initial direction and a third-term parameter. Sufficient Descent Condition (SDC) and global convergence qualities for both the exact and the strong Wolfe line search were demonstrated to exist in NEWRMIL algorithm. The experiments were performed by a total of 44 multi-dimensional mathematical test functions with various levels of complexity. When compared with the existing CG methods, NEWRMIL performs similarly under precise line search, while under Strong Wolfe line search NEWRMIL is superior and relatively faster convergence speed. Additionally, the practicality of NEWRMIL was demonstrated in solving multiple linear regression problems. The findings show that the NEWRMIL algorithm is the most efficient and has the minimum NOI and CPU time when compared to the direct technique and existing CG methods.

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CHAPTER ONE
INTRODUCTION

1.1 Introduction to Research

This chapter will discuss on the research background, problem statement, research objectives, research questions, significance of study, and, scope and limitation of the study.

1.2 Research Background

One of the core components of computational mathematics is the optimization of an objective function involving unknowns that may be constrained in someway. Due to various mathematical problems that exist nowadays, various research are done to develop mathematical models in order to mimic real problems so those models could be manipulated by researchers to solve problems. Most mathematical models consist of finding maximum or minimum value or known as optimization.

Optimization is defined as the science of determining the best solution to certain mathematically defined problems, which are often called the models of physical reality. There are many studies that can be done on optimization including optimality criteria of problems, creation of model problems, determination of algorithmic methods of solution, establishment of convergence theory of the algorithms, and computational experimentation with methods both under trial conditions and real life problems (Snyman, 2005).

In this thesis, the focus is on unconstrained optimization (UO). Investigation on methods of UO is important for many reasons. If the model design does not have any active constraint, then the problem will need to involve unconstrained function minimization algorithm to determine the direction and distance traveled. Besides, many constrained optimization problems are frequently translated to UO problems by using the constraints to define relations among variables, resulting in a reduction of the number of effective variables (Luenberger & Ye, 2008). Last but not least, unconstrained minimization technique are widely used in linear and nonlinear problems (Nocedal & Wright, 2006b).