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SEMI ANALYTICAL ITERATIVE METHOD FOR SOLVING PAINLEVÉ EQUATION I

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1. Introduction

The Painlevé equation I is one of the six Painlevé equations which are second order nonlinear ordinary differential equations. The generic solutions of the Painlevé equations are transcendental in the sense that they cannot be expressed in terms of known functions that is difficult to solve analytically (Clarkson, 2003). Therefore, some numerical methods are recommended to solve this problems. In this study, the Painlevé equation I will be numerically simulated by a semi analytical iterative method (SAIM) (Temimi and Ansari, 2011) and the results will be compared with the fourth-order classical Runge-Kutta method (RK4) and other references. The calculation algorithm has been coded through MAPLE 2020 software. The Painlevé equation is given by (Behzadi, 2010):

$$\frac{d^2u}{dx^2}(x) = 6u^2 + x \tag{1}$$

with initial condition: $u(0) = 0, \quad u'(0) = 1.$

2. Basic Idea of Semi Analytical Iterative Method

We begin by noting any differential equation

$$L(u(x)) + N(u(x)) + g(x) = 0 \tag{2}$$

with boundary conditions

$$B\left(u, \frac{du}{dx}\right) = 0 \tag{3}$$

where x denotes the independent variable, $u(x)$ is an unknown function, $g(x)$ is known function, L is a linear operator, N is a nonlinear operator and B is a boundary operator.

By assuming that $u_0(x)$ is an initial guess to solve the problem $u(x)$ and the solution begin by solving the initial value problem:

$$L(u_0(x)) + g(x) = 0, \quad \text{and} \quad B\left(u_0, \frac{du_0}{dx}\right) = 0. \tag{4}$$

The next iteration is:

$$L(u_1(x)) + N(u_0(x)) + g(x) = 0 \quad \text{and} \quad B\left(u_1, \frac{du_1}{dx}\right) = 0. \tag{5}$$

Thus, we have the general iterative procedure which is the solution of a set of problems i.e.,

$$L(u_{n+1}(x)) + N(u_n(x)) + g(x) = 0 \quad \text{and} \quad B\left(u_{n+1}, \frac{du_{n+1}}{dx}\right) = 0. \tag{6}$$

Then, the solution for problem (2) with boundary condition (3) is given by

$$u(x) = \lim_{n\to\infty} u_n(x). \tag{7}$$

3. Results

Figure 1 shows a comparison of Painlevé I equation solutions using RK4 and fourth iteration of SAIM. It is clear that the results obtained for both methods are almost the same. This result is

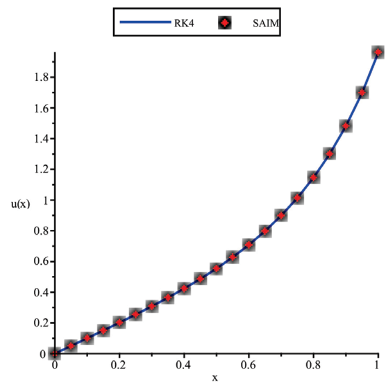

Figure 1: Comparison of solution by RK4 and SAIM

further strengthened by error analysis as in Table 1. Since the Painlevé I equation does not have an exact solution, the error is calculated by comparing the results obtained by SAIM compared to the results obtained by RK4. Table 1 also shows the errors by variational iterative method (VIM) (Hesameddini and Peyrovi, 2009) and Daftardar-Gejji and Jafari method (DJM) (Selamat et al., 2017). It was found that the error obtained by SAIM is almost the same as the error by VIM and DJM.

Table 1: Error measurements between RK4 and various methods

x	RK4–SAIM ($u_3(x)$)	RK4–DJM	RK4–VIM
0.1	5E-08	-5.5E-05	5E-08
0.2	1.28E-07	-1E-06	1.28E-07
0.3	2.336E-07	-1E-06	2.336E-07
0.4	3.088E-07	1E-06	3.088E-07
0.5	6.122E-07	5E-06	6.082E-07
0.7	2.00209E-05	3.3E-05	2.07429E-05
0.9	0.000576177	0.000746	0.000746301
1.0	0.002374358	0.003708	0.003708178

4. Conclusion

In this study, the reliability of SAIM is proven by the accuracy of results equivalent to other methods such as RK4, DJM and VIM.

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