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DAFTARDAR-GEJJI AND JAFARI METHOD FOR SOLVING SIR EPIDEMIC MODEL

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1. Introduction

In this letter, the classical susceptible-infectious-removed (SIR) epidemic models has been solved using Daftardar-Gejji and Jafari method (DJM)(Daftardar-Gejji and Jafari, 2006). The SIR model aims to predict the number of individuals who are susceptible to infection, are actively infected, or have recovered from infection at any given time. The SIR model is given by the following system:

$$\begin{aligned} \frac{dx}{dt} &= -\beta x(t)y(t) \\ \frac{dy}{dt} &= \beta x(t)y(t) - \gamma y(t) \\ \frac{dt}{dt} &= \gamma y(t) \end{aligned} \tag{1}$$

where x, y and z represent the number of susceptible, infective and recovered at time t, respectively. The parameter β represent transmission rate and γ represent recovery rate. This model does not consider birth rate or death rate. Therefore, SIR model can be described as the simplest model on an epidemic of a non-fatal disease.

2. Basic Idea of Daftardar-Gejji and Jafari Method

Consider the general functional equation (Hemeda and Eladdad, 2018):

$$u(x) = f(x) + N(u(x)) \tag{2}$$

where N is a nonlinear operator from a Banach space $B \to B$ and f is a known function of the Banach space B. The solution $u(x)$ can be given in the form:

$$u(x) = \sum_{i=0}^{\infty} u_i(x) \tag{3}$$

The nonlinear operator N can be decomposed as:

$$N\left(\sum_{i=0}^{\infty} u_i(x)\right) = N(u_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j(x)\right) - N\left(\sum_{j=0}^{i-1} u_j(x)\right)\right\} \tag{4}$$

Therefore, (2) is equivalent to:

$$\sum_{i=0}^{\infty} u_i(x) = f + N(u_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j(x)\right) - N\left(\sum_{j=0}^{i-1} u_j(x)\right)\right\} \tag{5}$$

Then, the solution can be obtained from recurrence relation:

$$\begin{aligned} u_0 &= f, \\ u_1 &= N(u_0), \\ u_{r+1} &= N(u_0 + u_1 + ... + u_r) - N(u_0 + u_1 + ... + u_{r-1}), \quad r = 1, 2, ... \end{aligned} \tag{6}$$

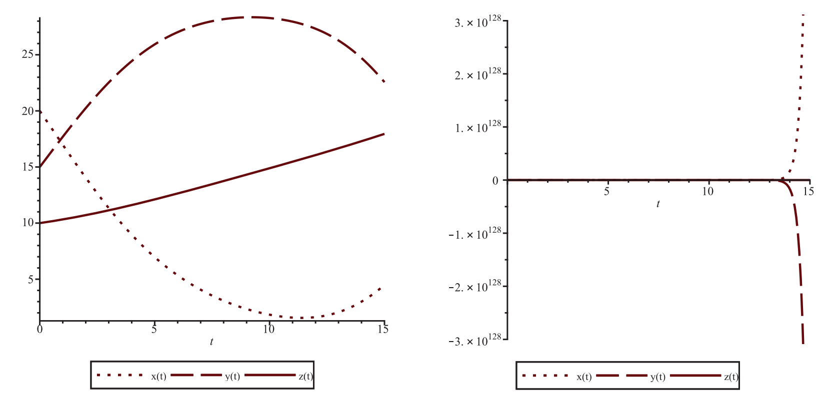
and

$$u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right). \tag{7}$$

The $r-$term approximate solution of (2) and (3) is given by $u(x) = \sum_{i=0}^{r-1} u_i$.

3. Results

We consider the epidemic model (1) with $x(0) = 20$, $y(0) = 15$ and $z(0) = 10$. Figure 1 show the results for sixth terms approximation with a) $\beta = 0.01$ and $\gamma = 0.02$ and b) $\beta = 1$ and $\gamma = 1$. It shows that when the β and γ parameters are relatively small, the solution given is almost the same as the solution in Biazar (2006) and Rafei et al. (2007). But the increase to the β and γ parameters causes the results to be unreasonable. This is in line with expectations by Fernández (2009) for iterative methods such as DJM. The calculation algorithm has been coded through MAPLE 2020 software.



a) $\beta = 0.01$ and $\gamma = 0.02$ b) $\beta = 1$ and $\gamma = 1$

Figure 1: Sixth terms approximation

4. Conclusion

In this study, DJM was applied to solve the epidemic model. The results show that the reliability of the results is limited to certain parameters only i.e $\beta = 0.01$ and $\gamma = 0.02$. Therefore DJM is not a method that can be used in all situations.

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