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COMPARISON ON QUEUING PERFORMANCE MEASURES ON CUSTOMER'S FLOW USING QUEUING THEORY MODEL AND DSW ALGORITHM

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1. Introduction

A queue is a line of people or things that are waiting to be handled. At any location, queues can form, including supermarkets, hospitals, manufacturing firms, and more (Priyangika & Cooray, 2016). The results of analyzing the queuing system are often used to make business decisions about waiting lines. Customer flow refers to the orderliness of a customer's trip which is important for any kind of business. The more efficient the customer flow, the higher the customer's satisfaction. Business owners can gain an understanding of customer flow by analysing flow patterns. Queuing theory is an area of mathematics concerned with the study and modelling the behaviour of waiting in lines (Whiting,2012). Queuing theory has been applied in many fields such as telecommunications, transportation, logistics, and more. The theory is used to calculate the length of time a customer must wait in a queuing situation (Yakubu & Najim, 2014). Fuzzy Queuing Model for the α-cut technique is based on the Dong, Shah, and Wong (DSW) algorithm. In a multi-server fuzzy queuing model, the approximate technique DSW algorithm is employed to describe a membership function regarding the performance measures (Thamotharan,2016). DSW approach simplifies the manipulation of the extension principle for continuous-valued fuzzy variables such as fuzzy integers defined on the real line. This research focuses on the queuing line system used by customers. The purpose of this research is to compare the queuing performance measures of a queuing system at service counters and to determine customer flow using the Queuing Theory Model and DSW Algorithm. There were five variables involved which were the number of customers, the arrival time of customers, the start and end of services, and average service time.

2. Methodology

The data obtained were average service time per day, total number of customer and average number of customer. These variables were calculated before proceed to include in the formulation:

λ = mean number of arrivals per time period

μ = mean number of customers or units served per time period

s = number of servers

2.1. *Queuing Theory Model : Multi-server Model*

Erlang initially proposed the Queuing Theory Model in 1909. For this study in Queuing Theory Model using multi-server model, L_q, L_s, W_q and W_s are computed. A mathematical model was built to analyze the situation, which is multiple queuing models with more than one server in the queuing system. The multiple server system illustrated here assumes that arrivals are Poisson distribution and that service times follow a Exponential probability distribution. Service is provided on a first-come, first-served basis, with all servers expected to perform at the same rate.

The average server utilization, ρ

$$\rho = \frac{\lambda}{s\mu} \tag{1}$$

The probability that there are zero customers in the system, P_0

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \left[\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{1}{1-\rho}\right)\right]} \tag{2}$$

The average number of customers in the queue, L_q

$$L_q = \frac{\rho\left(\frac{\lambda}{\mu}\right)^s P_0}{s!(1-\rho)^2} \tag{3}$$

The average number of customers in the system, L_s

$$L_s = L_q + \frac{\lambda}{\mu} \tag{4}$$

The average waiting time of a customer in the queue, W_q

$$W_q = \frac{L_q}{\lambda} \tag{5}$$

The average waiting time of a customer in the system, W_s

$$W_s = \frac{L_s}{\lambda} \tag{6}$$

2.2. *DSW Algorithm*

In Fuzzy Queuing Theory, by using the DSW Algorithm model, this model is an approximation approach for creating membership functions that use intervals at varying α-cut levels. The manipulation of the extension principle for continuous- valued fuzzy variables, such as fuzzy integers defined on the real line, is substantially facilitated by the DSW Algorithm approach. It can prevent the spreading of the consequent functional expression caused by conventional interval analysis techniques by preventing anomalies in the output membership function due to the employment of the discriminating reaching on the fuzzy variable domain.The α-cut depicts the fuzzy queuing performance measure's potential in the relevant range. Steps in the DSW Algorithm are as follows:

(1) An α-cut value between 0 and 1 is chosen.

(2) The intervals that correspond to the α-cut value in the arrival rate and service rate

membership functions are acquired.

(3) The intervals for the membership function for the chosen α-cut levels using conventional binary interval operations are calculated.

(4) Through making an α-cut representation of the result, steps 1 to 3 are repeated for different values of α.

2.3. *Standard Interval Analysis Arithmetic*

The operation for the general arithmetic property is described symbolically using $[+,-,\bullet,\div]$ as follows :

Addition :

$$[e,f]+[g,h]=[e+g,f+h] \tag{7}$$

Subtraction :

$$[e,f]-[g,h]=[e-g,f-h] \tag{8}$$

Multiplication :

$$[e,f]\bullet[g,h]=[\min(eg,eh,fg,fh),\max(eg,eh,fg,fh)] \tag{9}$$

Division :

$$[e,f]\div[g,h]=[e,f]\bullet\left[\frac{1}{h},\frac{1}{g}\right] \tag{10}$$

provided that $g,h \neq 0$

Range values for maximum and minimum:

$$\alpha\bullet[e,f]=[\alpha e,\alpha f] \tag{11}$$

with $\alpha > 0$

$$\alpha\bullet[e,f]=[\alpha f,\alpha e] \tag{12}$$

with $\alpha < 0$

By calculating the performance measures for the DSW algorithm, the different α-cut value is substituted into the formula. Below are the lists of formulas derived from (Shanmugasundaram & Venkatesh, 2015).

The utilization factor, ρ

$$\rho=\frac{x}{sy} \tag{13}$$

The probability that there are zero customers in the system, P_0

$$P_0=\frac{1}{\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{x}{y}\right)^n+\left[\frac{1}{s!}\left(\frac{x}{y}\right)^s\left(\frac{1}{1-\rho}\right)\right]} \tag{14}$$

The average number of customers in the queue, L_q

$$L_q = \frac{\rho\left(\frac{x}{y}\right)^s P_0}{s!(1-\rho)^2} \tag{15}$$

The average number of customers in the system, L_s

$$L_s = L_q + \frac{x}{y} \tag{16}$$

The average waiting time of a customer in the queue, W_q

$$W_q = \frac{L_q}{x} \tag{17}$$

The average waiting time in the system, W_s

$$W_s = \frac{L_s}{x} \tag{18}$$

3. Results and Discussions

Table 1: Performance Measures of Fuzzy Queuing Model Using DSW Algorithm

α	L_q (customer)	L_s (customer)	W_q (hour)	W_s (hour)
0	[0.00000085332, 0.0877]	[0.3333, 2.0877]	[0.000000085333, 0.0219]	[0.0333, 0.5219]
0.1	[0.0000014162, 0.0508]	[0.3590, 1.8810]	[0.000000146, 0.0121]	[0.0370, 0.4475]
0.2	[0.0000023238, 0.0298]	[0.386, 1.7084]	[0.00000024721, 0.0067727]	[0.0411, 0.3883]
0.3	[0.0000037704, 0.0176]	[0.4144, 1.56]	[0.00000041433, 0.0038261]	[0.0455, 0.3391]
0.4	[0.0000060529, 0.0105]	[0.4445, 1.4299]	[0.00000068783, 0.0021875]	[0.0505, 0.2979]
0.5	[0.0000096238, 0.0063223]	[0.4762, 1.314]	[0.0000011322, 0.0012645]	[0.0560, 0.2628]
0.6	[0.00001521, 0.0038174]	[0.5098, 1.2097]	[0.0000018549, 0.00073412]	[0.0622, 0.2326]
0.7	[0.000023854, 0.0023088]	[0.5455, 1.115]	[0.0000030195, 0.00042756]	[0.0691, 0.2065]
0.8	[0.000037275, 0.0013992]	[0.5834, 1.0284]	[0.0000049046, 0.00024986]	[0.0768, 0.1836]
0.9	[0.000058004, 0.0084622]	[0.6237, 0.9489]	[0.0000079458, 0.0001459]	[0.0854, 0.1636]
1	[0.000089816, 0.00051143]	[0.6668, 0.8755]	[0.0000012831, 0.000085238]	[0.0953, 0.1459]

There are eleven levels of α which are 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1. Table 1 lists the performance measures of the Fuzzy Queuing Model for Lq, Ls, Wq, and Ws by using the DSW Algorithm. If α = 0, there is a possibility that the performance measures will be presented. However, if α = 1, the performance measures will be presented. α = 1 is the fuzzy system's core for performance measures, while α = 0 is the fuzzy system's support for performance measures to show up. The calculations for levels α ranging from 0.1 to 0.9 show that the performance value is in the range [0,1]. The average mean of the study's performance measures ranges from α = 1 to α = 0, indicating that it never goes beyond the range α = 0.

Table 2: Performance Measures between Queuing Theory Model and Fuzzy Queuing Model Using DSW Algorithm

Performance Measures	Queueing Theory Model	Fuzzy Queuing Theory (DSW Algorithm) , **α**=1
The average number of customers in the queue, L_q	0.00023779	[0.000089816, 0.00051143]
The average number of customers in the system, L_s	0.7780	[0.6668, 0.8755]
The average waiting time of a customer in the queue, W_q	0.00003397 hours (0.002038 minutes)	[0.0000012831, 0.000085238]
The average waiting time of a customer in the system, W_s	0.1111 hours (6.6686 minutes)	[0.0953, 0.1459]

Table 2 shows that each performance measure for the Fuzzy Queuing Model and the Queuing Theory Model has its own values that can be deciphered. Queuing Theory Model is compatible with the Fuzzy Queuing Model as the value of L_q is approximately less than one customer queuing in the waiting line in an hour, L_s is approximately less than one customer queuing in the system in an hour. Next, W_q shows that the customer waits for less than one minute in the queue and W_s shows that the customer waits for less than 7 minutes in the system. Apart from that, all values including L_q, L_s, W_q and W_s for the Queuing Theory Model are inside of the Fuzzy Queuing Model's range for $\alpha = 1$. Consequently, the findings of performance measurements for the Queuing Theory Model and the Fuzzy Queuing Model indicate that the two models are equivalent. Since the obtained value of the Queuing Theory Model is within the range value of performance measures for the Fuzzy Queuing Model, this indicates that the obtained result is consistent.

4. Conclusion

As a conclusion, the objective of this research has been achieved where the performance measures of the Queuing Theory Model provide a single value, while the performance measures of the Fuzzy Queuing Model provide a range of values. When $\alpha = 1$, the performance measures never fall outside of the range when $\alpha = 0$. Hence, it has been demonstrated that in measuring multi-server performance in a queuing system, the Fuzzy Queuing Theory Model is considerably more effective and coherent. By using Fuzzy Queuing Theory, the customer flow is very smooth and long queues rarely occur as customers' waiting time is short and few people queue at one time. Besides that, the customer's flow and actual behaviour are easily conceived and comprehended.

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