ANALYTICAL FOURIER APPROXIMATION OF FIVE-PHASE SPACE VECTOR MODULATION VOLTAGE SOURCE INVERTER

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Abstract – **To utilize AC machine with phase number higher than three, electric drives in power electronic applied. For various applications, multiphase motor usage considered as typical solution. Multiphase drives are invariably supplied from multiphase voltage source inverters and adequate methods for VSI pulse width modulation (PWM) are therefore required. Proper modeling of voltage source inverters is important in devising appropriate control algorithm. This paper study of a five phase voltage source inverters based on space vector approach. The existing technique is elaborated utilizing only large space vectors. The Fourier analysis of output phase to neutral voltage, line to line voltage, and pole voltage are performed for ten step mode of operation (large vector). Simulation results are included throughout the paper to illustrate and verify the theoretical calculation.**

Keywords-five-phase, pulse width modulation, space vector, voltage source inverter, Fourier analysis.

I. INTRODUCTION

Due to additional x-y component in multi-phase drive system the control properties of multi-phase inverter can be more freedom in designing compared to three phase inverters. There is several research results over the year have been published and detailed review available in [1, 2]. Major advantages by using multiphase machine instead of a three-phase are review in [2-4] and are higher torque density, greater fault tolerance, greater efficiency, reduce torque pulsations, reduction in the required rating per inverter leg and therefore more reliable power conditioning equipment with noise characteristics of drive improve as well [5].

Development of appropriate control algorithm of multiphase inverter requires their details model. The variable voltage and variable frequency supply is obtained from a voltage source inverter by using proper pulse width modulation schemes. A five-phase VSI offers a total of 2^5= 32 space vectors, of which thirty are active state vectors, forming three concentric decagons, and two are zero state vectors.

The simplest method of realizing SVPWM is to utilize only ten large length vectors, belonging to the largest decagon in the d-q plane, in order to implement the symmetrical SVPWM [5-7]. Two active space vectors neighboring the reference space vector and two zero space vectors are utilized in one switching period to synthesize the input reference voltage. This method is a simple extension of space vector modulation of three-phase VSIs. While being the simplest possible, it leads to generation of low order output voltage harmonics of significant values, as shown in this paper.

A. Modeling of a Five-Phase VSI

Power circuit topology of a five-phase voltage source inverter is shown in Figure 1. The inverter input DC voltage is regarded further on as being constant. The load is taken as star-connected and the inverter output phase voltages are denoted in Figure 1 with lower case symbols (a,b,c,d,e), while the leg voltages have symbols in capital letters (A,B,C,D,E). Each switch in the circuit consists of two power semiconductor devices. One of these is a fully controllable semiconductor, such as a bipolar transistor of IGBT, while the second one is a diode. The model of the five-phase VSI is developed in space vector form in what follows, assuming an ideal commutation and zero forward voltage drop[9]. The relationship between the machine's phase-to-neutral voltages and inverter leg voltages is given with

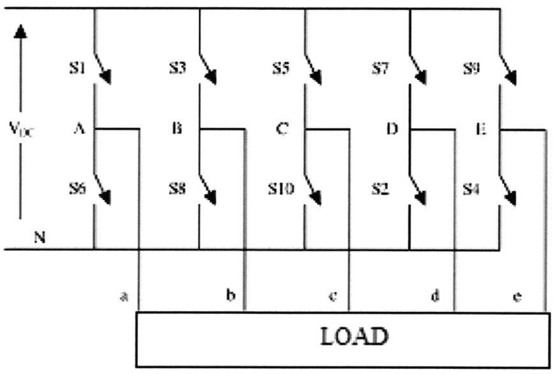


Figure 1: Five-phase voltage source inverter circuit

$V_a = (4/5)\,V_A - (1/5)\,(V_B + V_C + V_D + V_E)$
$V_b = (4/5)\,V_B - (1/5)\,(V_A + V_C + V_D + V_E)$
$V_c = (4/5)\,V_C - (1/5)\,(V_A + V_B + V_D + V_E)$
$V_d = (4/5)\,V_D - (1/5)\,(V_A + V_B + V_C + V_E)$
$V_e = (4/5)\,V_E - (1/5)\,(V_A + V_B + V_C + V_D)$ (1)

While the inverter leg voltages for large vector have magnitude of $\sqrt{\frac{2}{5}}$Vdc 2cos $(\pi/5)$.The phase voltages in different modes are obtained by substitute leg voltage into equation (1). The space vectors of phase to neutral voltage are identical to the leg voltage space vectors. The phases to neutral voltage for various modes are given in Figure 2.

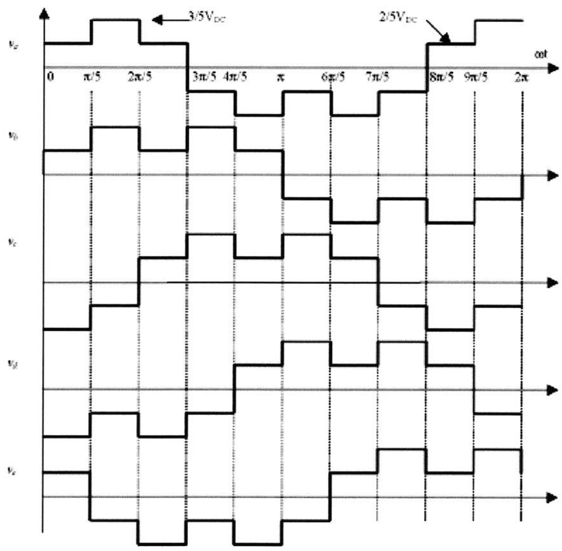


Figure 2: Phase-to-neutral voltages of the five-phase VSI in the ten step mode of operation (large vector).

B. Analytical Fourier approximation

Generally, the summation of a fundamental sinusoidal component is the representation of any continuous function repetitive in an interval T. the magnitudes and the phases of the fundamental and the harmonics content of a periodic waveform can be calculated by using the Fourier coefficient[8]. Using definition of the Fourier series for a periodic waveforms

$$V(t) = V_0 + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi\omega t}{L} + b_n \sin\frac{n\pi\omega t}{L}\right) \quad (2)$$

Where the coefficients of the Fourier series are given with

$$V_0 = \frac{1}{2\pi}\int_0^{2\pi} V(\theta)d(\theta)$$
$$a_n = \frac{1}{\pi}\int_0^{2\pi} V(\theta)\cos n\theta\, d(\theta) \quad (3)$$
$$b_n = \frac{1}{\pi}\int_0^{2\pi} V(\theta)\sin n\theta\, d(\theta)$$

And observing that the waveforms possess quarter-wave symmetry and can be conveniently taken as odd functions, one can represent phase to neutral voltages & line to line voltages with the following expressions:

$$V(t) = \sum_{n=0}^{\infty} (b_{2n+1}\sin(2n+1)\omega t)$$
$$= \sqrt{2}\sum_{n=0}^{\infty} V_{2n+1}(\sin(2n+1)\omega t) \quad (4)$$
$$b_{2n+1} = \sqrt{2}V_{2n+1}$$
$$= \frac{1}{\pi}4\int_0^{\pi/2} V(\theta)\sin(2n+1)\theta\, d(\theta) \quad (5)$$

The expression in bracket in the second equation of (5) equals zero for all the harmonics whose order is divisible by five. Hence one can write the Fourier series of the phase-to-neutral voltage as

$$V(t) = 2V_{DC}/\pi\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{7}\sin 7\omega t + \frac{1}{9}\sin 9\omega t + \frac{1}{11}\sin 11\omega t + \frac{1}{13}\sin 13\omega t + \cdots\right] \quad (6)$$

From (6) it follow that the fundamental component of the output phase-to-neutral voltage has an RMS value equal to

$$V_1 = \sqrt{\frac{2}{\pi}}V_{DC} = 0.45V_{DC} \quad (7)$$

It is important to note at this stage that the space vectors described by (1) provides mapping of inverter voltages into a two dimensional space. However, since five-phase inverter essentially requires description in a five dimensional space not all the harmonics contained in (6) will be encompassed by the space vector of (1). Harmonic of the order 5k, k=1, 2, 3....cannot appear due to the isolated neutral point.

Calculation on line voltage can be representing from equation (8 & 9)

Vab = Vao – Vbo

$$= \sum_{n=1,3,5,..}^{\infty} \frac{2Vdc}{\pi n}[\sin n\omega t - sinn(\omega t - \alpha)] \quad (8)$$

$$\sin A - \sin B = 2\sin\frac{1}{2}(A-B)\cos\frac{1}{2}(A+B) \quad (9)$$

By substitute (9) into (8) using trigonometric identity
The value in each line are shown which α is swithing pattern where is $2\frac{\pi}{5} = 72°$

$$Vab = \sum_{n=1,3,5,..}^{\infty} \frac{4Vdc}{\pi n} \sin\frac{n\alpha}{2} cosn(\omega t - \frac{\alpha}{2})$$
$$Vbc = \sum_{n=1,3,5,..}^{\infty} \frac{4Vdc}{\pi n} \sin\frac{n\alpha}{2} cosn(\omega t - \frac{3\alpha}{2}) \quad (10)$$
$$Vcd = \sum_{n=1,3,5,..}^{\infty} \frac{4Vdc}{\pi n} \sin\frac{n\alpha}{2} cosn(\omega t - \frac{5\alpha}{2})$$
$$Vde = \sum_{n=1,3,5,..}^{\infty} \frac{4Vdc}{\pi n} \sin\frac{n\alpha}{2} cosn(\omega t - \frac{7\alpha}{2})$$

II. METHODOLOGY

The flow chart in Figure 3 shows on designing five phase system waveform throughout the calculation and comparison with MATLAB.

A. Calculation part on Total Harmonic Distortion

Data from the Fourier function in each line can be representing in harmonic form by using this definition.

$$f(x) = Co + \sum_{n=1}^{\infty} (C_n \cos\,(n\omega t - \emptyset) \quad (11)$$

Where Cn = $\sqrt{an^2 + bn^2}$
$C_1 \cos\,(n\omega t - \emptyset)$: Fundamental frequency
$C_2 \cos\,(n\omega t - \emptyset)$: 2nd harmonic
$C_3 \cos\,(n\omega t - \emptyset)$: 3rd harmonic
$\emptyset$ = Phase angle

For the total harmonic distortion, the formula is given on equation (12)

$$THD = \frac{\sqrt{\sum_{1,3,5,..}^{\infty} Vn^2}}{V1} \quad (12)$$

B. Creating coding on Fourier graph

The plotting graph by x-axis is stop at 2π for one cycle. Harmonic n=1 until 10 and n=1 until 50 were used for differentiate between the graph. Fundamental frequency f=50 Hz is used.

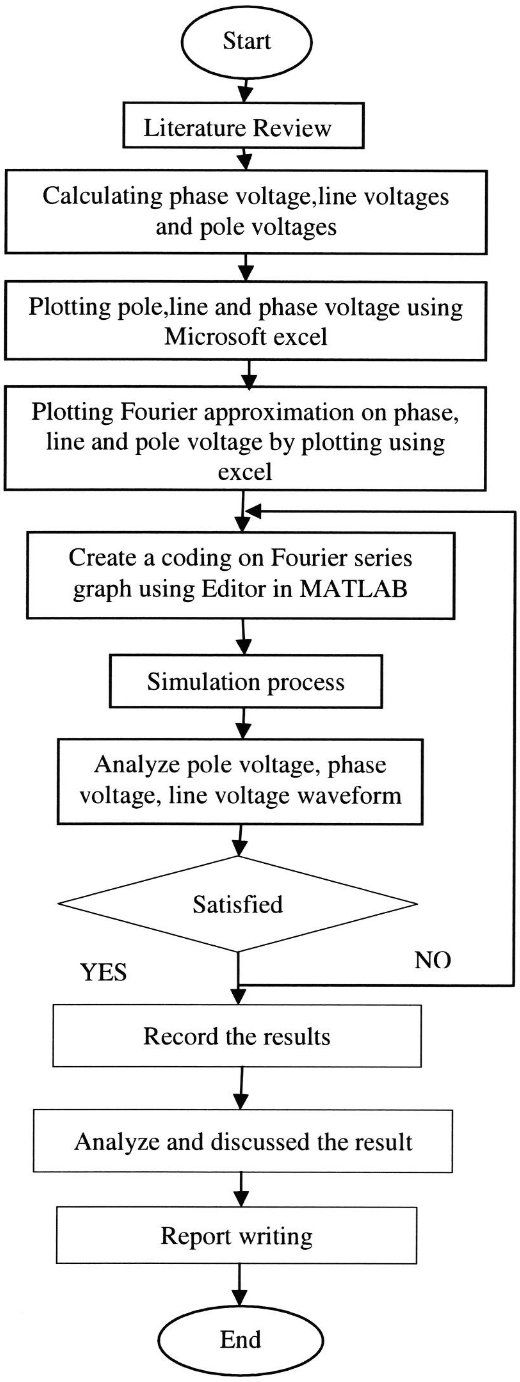
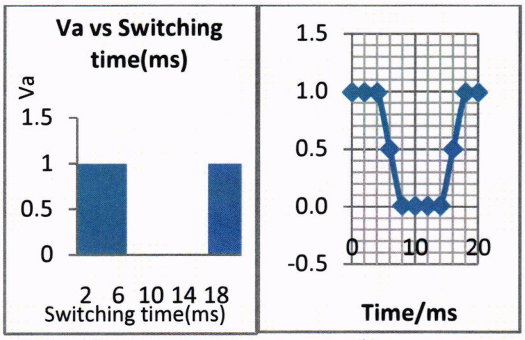


Figure 3: the flow chart on designing five phase waveform throughout the whole process

III. RESULT AND DISCUSSION

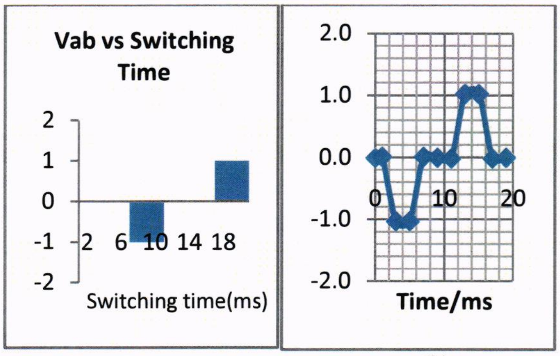
A. Comparison between Theoretical Waveform and Fourier approximation.

From theoretical, the waveforms of five phase space vector modulation from voltage 1 to voltage vector 10 have been plotted using Microsoft Excel. Calculation on Fourier series has been done referring to waveform of pole, line, and phase voltage in one cycle. Function is been plotted using Excel for comparison. Data from the function are shown in figure 3. Based on N=50 harmonic, total harmonic distortion been calculated.



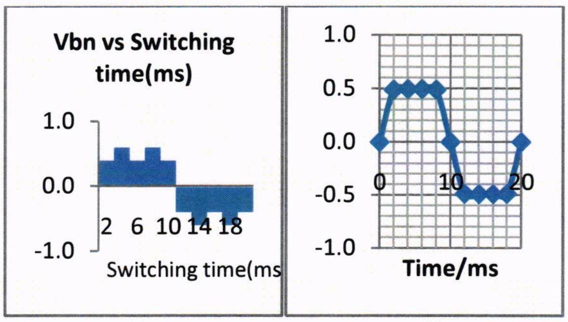
(a) (b)

Figure 4: Pole voltage waveform at phase A, (a) theoretical waveform and (b) Fourier approximation waveform.



(a) (b)

Figure 5: Line to line voltage waveform between phase A and Phase B, (a) theoretical waveform and (b) Fourier approximation waveform.



(a) (b)

Figure 6: Phase to neutral voltage waveform at phase B, (a) theoretical waveform and (b) Fourier approximation waveform.

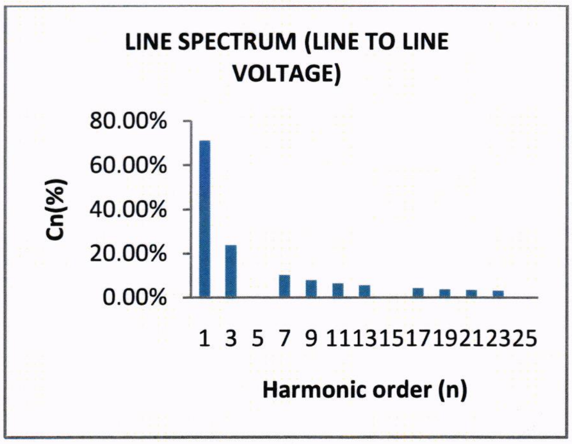
From Figure 4, Figure 5, and Figure 6 the Fourier approximation graph giving the similar waveform as predicted compare to theoretical part. The harmonic part in Fourier can be calculated.

B. Calculated on harmonic, THD .and line spectrum.

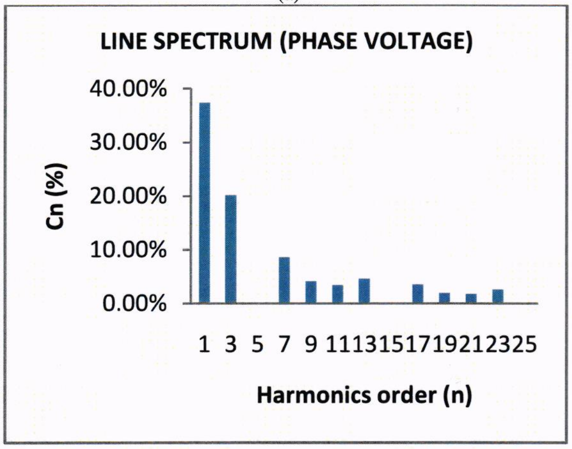
Table 1: Harmonic distortion of phase and line voltage

Harmonic order	phase	line
1	37.42%	71.18%
3	20.18%	23.73%
5	0.00%	0.00%
7	8.65%	10.17%
9	4.16%	7.91%
11	3.40%	6.47%
13	4.66%	5.48%
15	0.00%	0.00%
17	3.56%	4.19%
19	1.97%	3.75%
21	1.78%	3.39%
23	2.63%	3.09%
25	0.00%	0.00%
27	2.24%	2.64%
29	1.29%	2.45%
31	1.21%	2.30%
33	1.83%	2.16%
35	0.00%	0.00%
37	1.64%	1.92%
39	0.96%	1.83%
41	0.91%	1.74%
43	1.41%	1.66%
45	0.00%	0.00%
47	1.29%	1.51%
49	0.76%	1.45%
THD	118.91%	108.46%

It is worth noted that a five-phase voltage source inverter usually associated with odd harmonics and due to the half wave symmetry, the even harmonic are normally absent. The fifth and multiple of fifth harmonics are also eliminated.



(a)

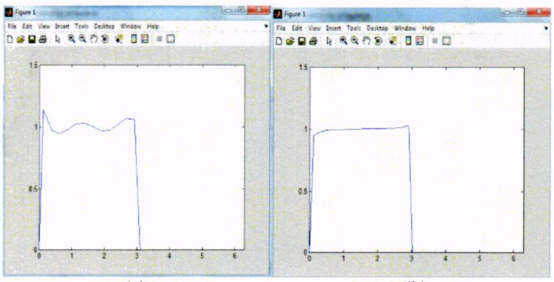


(b)

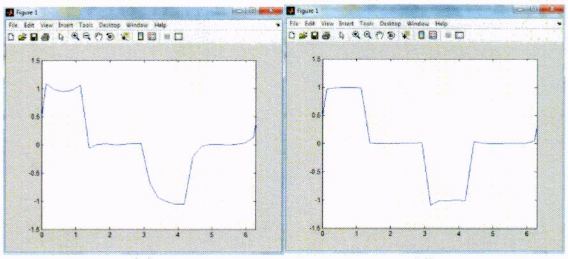
Figure 7: Line spectrum for each harmonic (a) line spectrum for line voltage, (b) line spectrum for phase voltage.

C. Simulation Result Using MATLAB

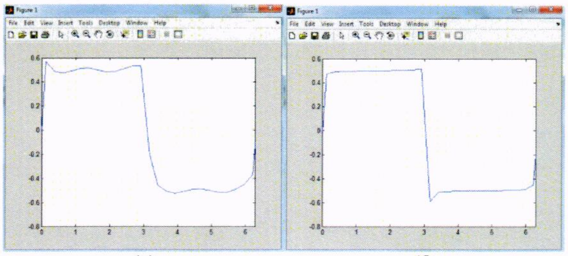
The Fourier function that has been calculated was coded to plot a graph. The f=50 Hz is used for one cycle with variable N (harmonic). MATLAB Editor was used to evaluate the graph with its harmonic.



(a) (b)



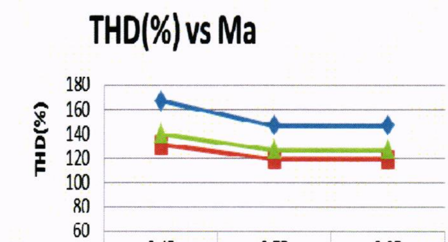
(c) (d)



(e) (f)

Figure 8: Simulation result with different harmonic which on (a), pole voltage with N=10, (b) pole voltage with N=50, (c) line voltage with N=10, (d) line voltage with N=50, (e) phase voltage with N=10, (f) phase voltage with N=50.

The Fourier approximation graph shown in Figure 8 were more similar compare to theoretical by using N=50. So the calculation using N=50 were used for the calculation on THD for more accurate result.



	0.45	0.75	0.95
line voltage between phase A and Phase B	167	147	147
line voltage between Phase A and Phase C	131	119	119
Phase Voltage at Phase A	140	127	127

Figure 9: The graph of THD versus modulation index (ma).

From the graph in Figure 9, it is obviously shows that the values of THD are decreasing through the rise of modulation index values for every phase voltage and line voltage.

IV. CONCLUSION

Low order harmonics can be substantially influenced by interaction between the ac and dc networks, and the converter control. This interaction can be an important factor in establishing the level of steady-state (or slowly-decaying) harmonic which will exist due to a given imbalance (or transient event) in the overall system.

This study deals with the details on five-phase voltage source inverter circuit. It has been developed using the outermost decagon which is ten step mode operation or large vector space vector approach. The model has produced the output of phase voltages, line voltages with substantial low order harmonic content.

From this study, learning and understanding on the characteristic of five phase system space vector modulation, multiphase drive and total harmonic distortion has been understood. In addition, knowing their specification and advantages gives more though on their expected results. The result of the simulation function Fourier waveforms gives the similar voltage waveforms as shown in theoretical.

V. FUTURE RECOMMENDATION

For the future works in the five phase space vector modulation, there are several suggestion on the improvements can be implement on this study. Firstly in order to reducing the THD values, adding filter in the circuit diagram of voltage source inverter is one of the methods that can be used. Furthermore, using multilevel concepts such as 3-level of five phase VSI will give the lowest THD values.

In addition, by using the combination of large and medium space vector will also lower the percentage of THD. This project also can be implemented into hardware using FGPA in order to get better performances for machines such as AC Induction Motor and Brushless DC Motor.

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