

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**ON THE SOLUTIONS OF BIVARIATE NONLINEAR
ALGEBRAIC POLYNOMIAL SYSTEMS USING NEWTON'S
AND BROYDEN'S METHODS**

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

Over the past few decades, algebraic methods have been taught to solve equations. Several methods can be used, including substitution, factorization, quadratic formulas, and elimination. There are two types of equations in mathematics: linear and nonlinear. Basically, nonlinear algebraic equations occur when $f(x)$ is purely polynomial and its highest degree is greater than 1. Interesting methods to be studied in this project are Newton's method and Broyden's method in solving some bivariate nonlinear algebraic polynomial systems. Both numerical methods determine their solutions from the initial guess to the convergence points. However, the computations of the solutions at each iteration by both methods are unclear. Therefore, this project aims to reveal the calculation on the solutions of four bivariate algebraic polynomial systems by using the methods of Newton and Broyden. The solutions were demonstrated manually and the difference in iterations between the methods is analysed. Moreover, the investigation is carried out by analysing the effectiveness of the methods. In addition, the solutions from MATLAB software and error analysis are presented as a benchmark to the calculated solutions by the Newton's method where it terminates at the same values. Furthermore, the solutions of the systems as the intersection points on the graph by using Maple software are also depicted. The results of the study showed that Newton's method requires fewer iterations than Broyden's method even though the Broyden's method can reduce the cost of evaluating the Jacobian matrix at first iteration. Eventually, the Newton's method converges quadratically and Broyden's method converges linearly. Hence, Broyden's method delays the inclined roots for a longer period. Therefore, Newton's method has the fastest rate of convergence. This project can be extended to develop the Broyden's method in MATLAB for a new contribution besides applied both methods in a higher dimension and in mathematical model.