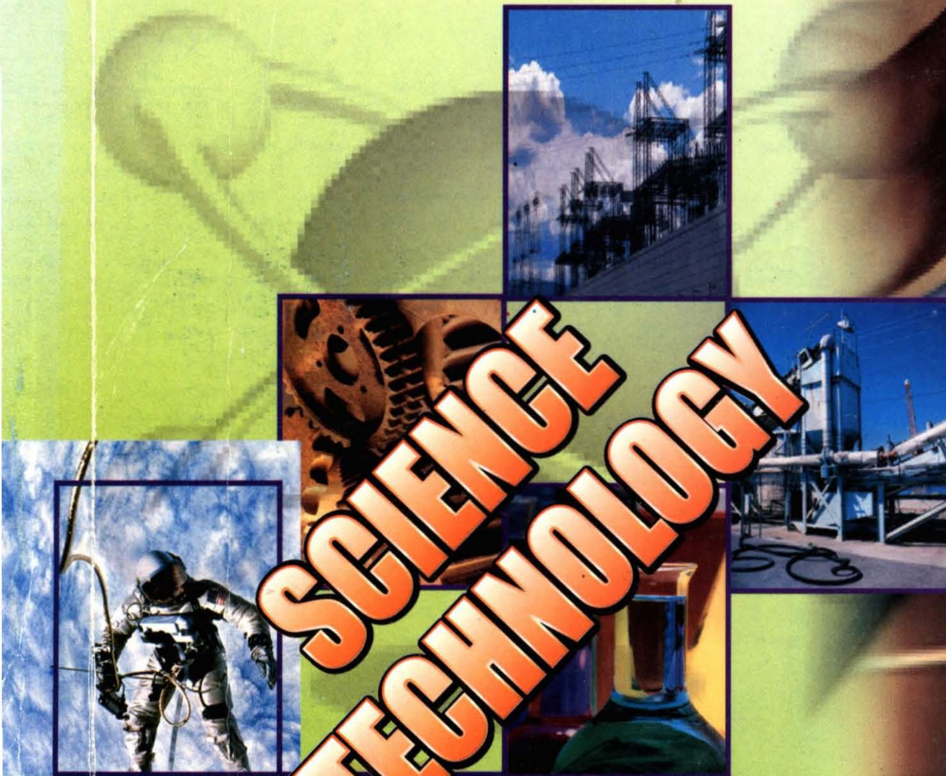
UNIVERSITI TEKNOLOGI MARA

ISO 9001:2000 No Sijil :0500132

# Globalising Knowledge and Information

# NATIONAL SEMINAR ON SCIENCE TECHNOLOGY & SOCIAL SCIENCES 2006




30-31 May 2006

Swiss Garden Resort & Spa

Kuantan, Pahang

# Application of Stochastic Model in Field Of Engineering: Monte Carlo Method in Modeling Fatigue Crack Propagation

F.R. M. Romlay

## ABSTRACT

*This paper deals with the modeling of fatigue crack propagation on a gear tooth using a dual boundary element method. The effects of life cycle to the multiple site fatigue crack propagation were studied. Analysis of stress intensity factor was performed by the deterministic approach using a dual boundary element method. The dual boundary element method was used to simplify the crack model through the numerical approach. The complex problems have been solved using the information from a boundary condition only. Next, the initial crack and life cycle of the structure have been predicted using stochastic method which is Monte Carlo. The crack size and fatigue life were computed until failure of the structure. The failure analysis was performed by a linear elastic fracture mechanics. The scenarios of the fatigue crack propagation were given by an integration of both dual boundary element and Monte Carlo method. Therefore, fatigue life of multiple site crack structure can be predicted.*

**Keywords**: *Crack propagation; fatigue; Monte Carlo; stochastic; boundary element method*

## Introduction

Monte Carlo method is a stochastic characteristic which is nondeterministic behaviour of various physical and mathematical applications. In the area of fatigue reliability, an estimation of failure is required. This is caused by uncertainties in initial crack, surface roughness, material property, applied load, flaw, defects such as scratches or weld defects from manufacturing process (Yang et. al 1988). In other words, as the crack grows, the crack size has a variation according to those uncertainties and the residual life of the structure is not deterministic but stochastic. Fatigue crack propagation is inherently a random process because of the inhomogeneous of material, connected with its crystal structure and with variations of convective film coefficient at the structures surface due to it non-smoothness and other similar reasons (Cherniavsky 1996). The experimental results for the fatigue crack growth under constant amplitude loading show that the material resistance against crack propagation has the inter-specimen as well as the intra-specimen variability. A stochastic model considering both types of variability is thus needed for the rational assessment of fatigue crack propagation. Therefore, the analysis of fatigue crack propagation should be based on the probabilistic approach and the inspection interval or the repair method must be determined according to the possibility of structural failure considering the uncertainties mentioned above.

This paper presents the development of an inspection programme for the fatigue crack propagation which is an enhancement of an earlier programme (Kebir et. al. 2001), and the major differences between these two programmes are summarized below:

The crack propagation has been modelled using BEM principal of Beasy software with the combination of random function of Matlab program.

The life cycle of a centre member bar with more than one notch has been analyzed using dual boundary element and Monte Carlo.

## Law of Fatigue Crack Propagation

In 1963, Paris and Erdogan created a Paris law equation for calculating fatigue crack propagation rate, *da/dN* as given in Equation (1).

$$\frac{da}{dN} = C\left(\Delta K\right)^m \quad (1)$$

$\Delta K = K_{max} - K_{min}$ is the range of stress concentration factor and $C$ and $m$ are the material properties.

The stress concentration factor is one of the parameters that are considered in linear elastic fracture mechanic.

The theory is only acceptable for the situation when there is no yield occurs at the crack tip. Therefore, Equation (1) can be used for high cyclic fatigue cases. Forman et al. (1967) tried to modify the Equation (1) as it does not include the stress concentration ratio, $R = K_{min} / K_{max}$ and the fracture strength, $K_C$.

From the definition of $\Delta K = K_{max}(1 - R)$ and $K_{max} = K_c$, the boundary condition for the crack propagation rate is:

$$\lim_{\Delta K \to (1-R)K_c} it \frac{da}{dN} = \infty \tag{2}$$

Substituting Equation (2) in Equation (1) gives:

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \tag{3}$$

From Equation (3), Forman (1967) found that the $m$ value for aluminium alloy 7075-T6 and 2024-T3 was 3. Equation (3) is known as Forman equation. Starting from the Forman equation and considering the crack will not propagate if the $\Delta K$ value below $\Delta K_{th}$ (Figure. 1), a growth law was introduced as in Equation (4) to calculate the fatigue crack propagation rate.

$$\frac{da}{dN} = C\left(\frac{\Delta K - \Delta K_{th}}{K_c - K_{max}}\right)^2 + C' \tag{4}$$

This model is valid for a soft metal that under both the fix and random loading amplitude, which $C'$ is approaching to $2.4 \times 10^{-7}$ mm/cyclic.

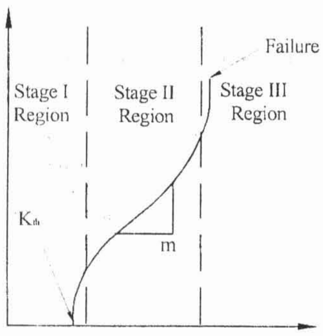



Fig. 1: Scheme diagram of short and long fatigue crack propagation (Dharani 2001).

Crack behaviour is determined by the values of the stress intensity factor, which are the function of an applied load and the geometry of the crack structure. The crack growth process is performed by the analysis of the crack extension. The stress intensity factor is evaluated and the crack path is defined in terms of the stress intensity factor. Damage tolerance analysis is developed based on linear elastic fracture mechanics. The stress intensity factor is described the behaviour of cracks.

## Crack Modeling Strategy

The domain region has been treated as dual boundary element by Boundary Element System (BEASY) software. It is necessary to calculate the related stiffness matrix and effective stress intensity factor, $K_{eff}$ by means of Dual Boundary Elements Method (DBEM). The crack modelling strategy shown by algorithm below:

Carry out a dual boundary element method for stress analysis of the structure.
Compute the effective stress intensity factors, $K_{eff}$ with the $J$-integral technique.
Compute the direction of the crack-extension increment.

Extent the crack one increment along the direction computed in the previous step.

Repeat all the above steps sequentially until a specified number of crack-extension increments have been achieved.

The boundary stiffness matrix and $K_{eff}$, after condensation, have been inserted into crack initial and propagation routine using Monte Carlo method by MATLAB source code. Using the S-N curve at 50% includes the deterministic approach.

By running a Monte Carlo method by MATLAB program, it possible to calculate the cycle number for each of the propagation and the crack length. The initial point also indicated by random process as shown in Figure 2. The modified data files in BEASY has been run to update the crack parameter.

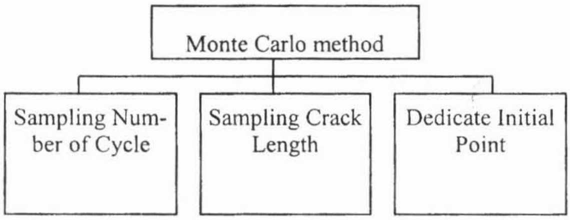



Fig. 2: Random parameter for fatigue crack propagation

## Numerical Results Of Plate 14 Holes

In order to validate the global probabilistic approach, the results have been compared with the fatigue test on a plane plate with 14 free holes that was conducted by Kebir et. al. (2001) at Aerospatiale-Matra laboratory in Suresnes, France as shown in Figure 3.

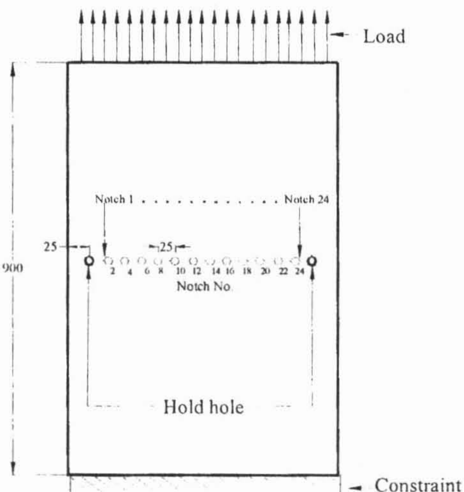



Fig. 3: Schematic diagram of plate 14 holes.

The sample material was aluminum alloy 2024-T3 sheets with a thickness of 1.6 mm. The load has been applied on transversal direction. The Young Modulus of the sample was 72.7 GPa. The initial structure has been discreet with 262 elements, in one zone with 1202 degrees of freedom. It has 897 internal points patch in the model. The numerical results have a good compromise between the test results. The total numbers of cycles with the probabilistic approach are closely similar to the test expressed in Figure 4.

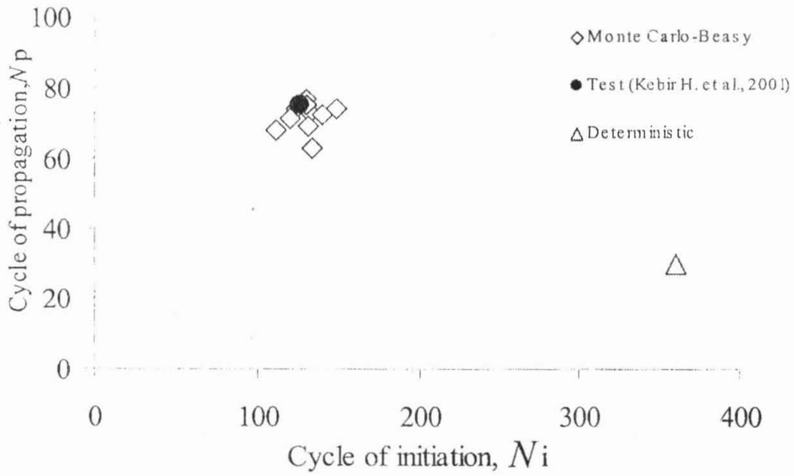



Fig. 4: Fatigue prediction life

The synthesis of the probabilistic results is expressed in Figure 5. In the deterministic approach, the propagation phase was short with is $30\times10^3$ cycle. It was because all the cracks assumed begin at the same time, since all the sites were undergoing the same stress level. So, the probabilistic approach has an advantage of giving the view of initial crack propagation. A large crack size has been dominated the failure probability at the beginning of the failure process. In a long duration time, the small cracks size may have the most dominant effect on the failure probability.

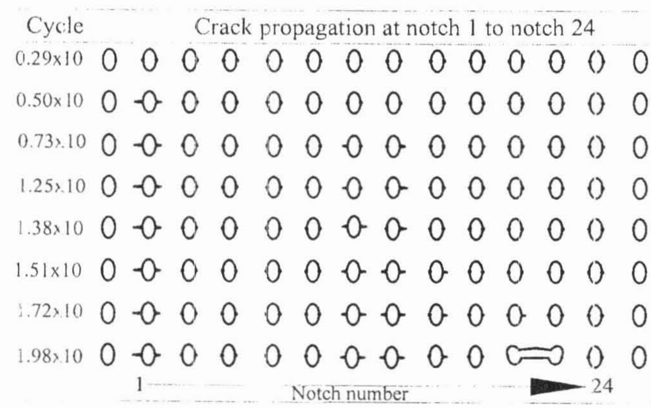



Fig. 5: Life cycle of fatigue crack propagation by iterations

Table 1 shows a maximum crack length before failure is 3.5293 mm. At this moment, the life cycle only $0.7354\times10^5$ cycles as presents at 7th iteration. This happened because a crack notch 14 had enough energy to propagate initial crack. The crack propagated very fast and verifies the high propagation rate. However, the failure of the sample did not happen until the life cycle reached $1.9825\times10^5$ cycles.

Table 1: Results of fatigue crack propagation

| Iteration | Crack Length | Cycle, $N_{Total}(10^5)$ | Point No. |
|---|---|---|---|
| 1 | 0.0056 | 0.2957 | 7 |
| 2 | 0.1702 | 0.3058 | 2 |
| 3 | 0.1034 | 0.3985 | 1 |
| 4 | 0.3706 | 0.4319 | 1 |
| 5 | 0.2498 | 0.5012 | 1 |
| 6 | 0.2077 | 0.6541 | 11 |
| **7** | **3.5293** | **0.7354** | **14** |
| 8 | 0.2219 | 1.2595 | 14 |
| 9 | 0.2557 | 1.3871 | 12 |
| 10 | 0.6363 | 1.4735 | 13 |
| 11 | 0.1043 | 1.5108 | 16 |
| 12 | 0.1557 | 1.7244 | 21 |
| 13 | Failure | 1.9825 | 21 |

Any notch that has been chosen randomly to propagate initial crack was increasing its stress intensity factor. The stress intensity factor values have been constantly increased in a few iterations, which the value slowly trended to achieve one fix value that was being the maximum stress intensity factor value at that moment. For the fourteen holes plate, notches 1, 2, 11, 12 and 14 have been chosen for having an initial crack as shown in Figure 6. The increasing was continuing for certain iterations. After that, the crack has been randomly propagated at other notch, which had lower stress intensity factor. In this scenario, the notch 6, 15, 16, 17, 18 have been randomly propagated the crack. The crack has been continuing propagate for a certain iterations to get close with the maximum stress intensity factor at that time. The increased of the life cycle was continuing the crack propagation at any ready crack randomly. At this moment, the crack propagation can make the sample fail. Notch 21 was having a catastrophic failure when its effective stress intensity factor reached the value of 276,659.75 $MPam^{1/2}$. The high potential energy assembled at a low stress intensity factor notch and catastrophic failure occurred because of the high grow crack propagation rate.

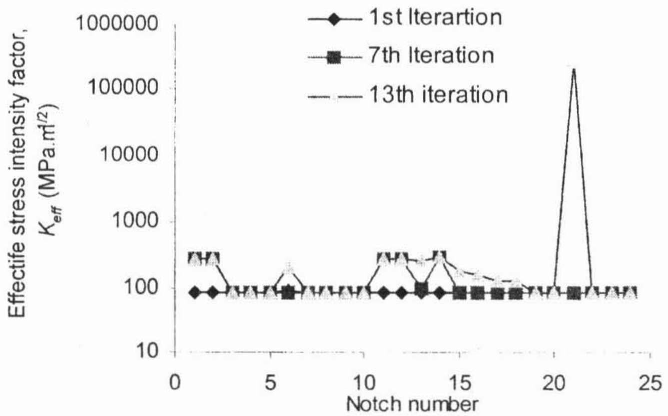
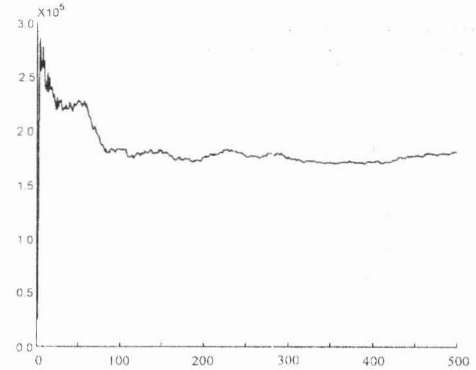
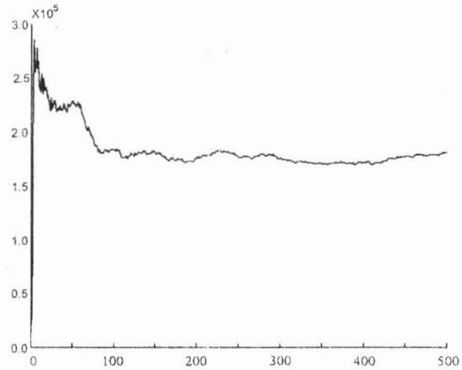



Fig. 6: Graph of effective stress intensity factor versus notches number for 1st, 7th and 13th iteration.

Figures 7 and 8 show a mean life and a standard deviation prediction for the tenth iterations. It is seen that the number of samples influences the fatigue life. The results are constant when the number of samples is over 300. So, the Monte Carlo-BEM result here gives a statistical value. The mean life and a standard deviation prediction for other iterations have given the same result like the tenth iteration.




## Numerical Results of Gear Tooth

The same method was continued for modeling of fatigue crack propagation on a gear tooth as shown in Figure. 9. Two notches were chosen as example on the left and right side and name as notch 1 and notch 2. The results of applying Monte Carlo method for gear tooth are shown in Table 2. Figure 10 illustrates the major crack at notch 1. This is due to the stress applied at the left side by considering the real load when gear tooth is operating. However, the random processes still happen because of the small different value of stress intensity factor. So, a very small crack happened at notch 2 in the third iteration.

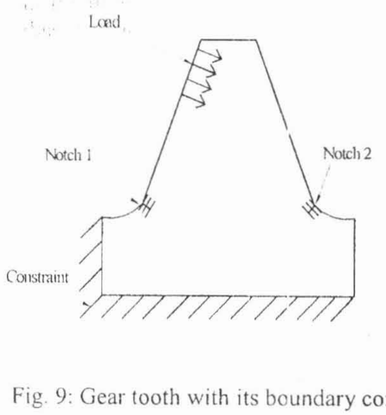



Fig. 9: Gear tooth with its boundary condition

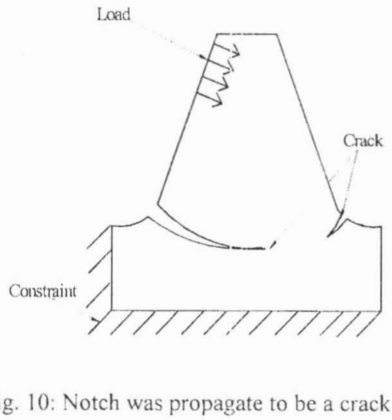



Fig. 10: Notch was propagate to be a crack

Table 2: Life cycle and crack size of gear tooth by Monte Carlo analysis.

| Iteration no. | Crack size $(10^{-3})$ (mm) | Notch no. | $N_{Initiation}$ $(10^5)$ | $N_{Propagation}$ $(10^3)$ | $N_{Total}$ $(10^5)$ |
|---|---|---|---|---|---|
| 1 | 0.3862 | 1 | 0.2840 | 35.7596 | 0.6416 |
| 2 | 7.9654 | 1 | 0.7782 | 8.6242 | 0.8645 |
| 3 | 0.4273 | 2 | 0.9752 | 0.3350 | 0.9786 |
| 4 | 4.7054 | 1 | 0.9967 | 4.2570 | 1.0393 |
| 5 | (Failure) | 1 | 1.0594 | 6.8270 | 1.1277 |

The modeling of fatigue crack propagation was repeated to a gear tooth but by fully deterministic approach and without using Monte Carlo method. The result in Table 3 shows that the crack directly propagated without any crack inter-phase happened and failed at 6.5984□$10^4$ cycles. The geometry of the crack is viewed in Figure. 11. The crack propagated at the notch with maximum principle stress, $\sigma_{max}$. That was why the crack propagated very fast and the calculation of the crack length, *da* finished directly by a single iteration. The modeling work by fully deterministic method could not provide the characteristic of a random process.

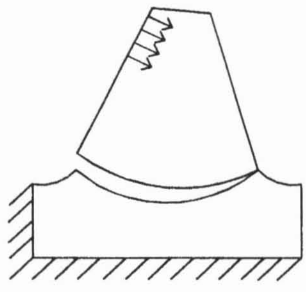


Fig. 11: Failure of the gear tooth

Table 2: Life cycle and crack size of gear tooth by Monte Carlo analysis.

| Iteration No. | Crack Length ($10^{-3}$)(mm) | Notch No. | $N_{Initiation}$ ($10^4$) | $N_{Propagation}$ ($10^4$) | $N_{Total}$ ($10^4$) |
|---|---|---|---|---|---|
| 1 | Failure | 1 | 4.5411 | 2.0573 | 6.5984 |

## Conclusion

An overall assessment method proposed in this paper was developed in order to validate the fatigue crack propagation with the probabilistic method through the Monte-Carlo. The modeling process used the dual boundary element method. Using this method makes the work simpler than finite element method, which is common method that using today. The results from the boundary element method and Monte Carlo analysis show that the life cycle of structure can be predicted and obtained in good agreement with the experiment results. The results obtained performed that such an application seem to be possible, taking into account the fact that the computer simulation can be used to predict fatigue crack propagation. The proposed algorithm can be used for a guideline to have a risk and reliability analysis and life expectancy of the structure.

## References

Brebbia, C.A. & Dominguez, J. (1989). *Boundary Elements – an Introductory Course*, Computational Mechanics Publications, Southampton, UK,

Cherniavsky A.O. (1996). *Probabilistic Approach to Calculation of Kinetics of Crack Meshes. Dynamics, Strength & Wear-Resistance of Machines.* Vol. 3.

Kebir, H., Roelandt, J.M. & Gaudin, J. (2001). *Monte-Carlo simulations of life expectancy using the dual boundary element method.* Engineering Fracture Mechanics, 68: pp. 1371-1384.

Yang J. N. Salivar G. C. & Schwartz B. J. (1988). *Statistical model for prediction of fatigue crack growth under a block type spectrum loading.* Engineering Fracture Mechanics, 31: pp. 371-388.

F.R.M. ROMLAY, Faculty of Mechanical Engineering, Kolej Universiti Kejuruteraan & Teknologi Malaysia (KUKTEM), Gambang 25000 Kuantan, Pahang, Malaysia. fadhlur@kuktem.edu.my