Differentiation and Integration: Students’ Mistake and The Correction

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Introduction

Calculus 1 and Calculus 2 are compulsory subjects for engineering students in UiTM Penang. The mathematics course codes involving these two topics are MAT183 for semesters 1 or 2 and MAT235 for semesters 2 or 3. Each semester the student's weaknesses can be identified based on the topic differentiation which is learned in the subject of calculus 1 or MAT183. In this chapter, I will share a little bit of knowledge to correct what is wrong with examples of student error in the step of calculation for differentiation and integration.

Basic differentiation

Differentiation is important to understanding the concept of integration. A common weakness identified is that students cannot remember the basic concepts of differentiation for functions such as algebra, exponential, logarithmic and trigonometric. Students can also easily forget the techniques available such as product rule and quotient rule. There are some examples of students’ mistake that involving basic of differentiation. Table 1 shows the question, student mistake and the correction in basic differentiation.

Table 1 : Student's Mistake and The Corrections.

Example	The mistake	The correction
1. Find $\frac{d}{dx}\frac{1}{6x^2}$	$\frac{d}{dx}\frac{1}{6x^2}=\frac{d}{dx}6x^{-2}$ $=-12x^{-3}$ • student change both positions 6 and x	$\frac{d}{dx}\frac{1}{6x^2}=\frac{d}{dx}\frac{x^{-2}}{6}$ $=-\frac{2}{6}x^{-3}=-\frac{1}{3x^3}$ • Bring the variable x only • Differentiate
2. Differentiate $f(x) = 2xe^{2x}$	$f'(x)=(2)(2e^{2x})=4e^{2x}$ • differentiate the function separately. • there are product of function $2x$ and e^{2x} , so must use the appropriate technique	$f'(x)=2x(2e^{2x})+e^{2x}(2)$ $=4xe^{2x}+2e^{2x}$ • suitable method is by using product rule $\frac{d}{dx}(uv)=uv'+vu'$
3. Differentiate $f(x) = \tan(x^3)$	$f'(x)=\sec^2(3x^2)$ • differentiated x^3 and replace with origanal angel • original angle was eliminate	$f'(x)=[\sec^2(x^3)](3x^2)$ $=(3x^2)\sec^2(x^3)$ • Differentiate the inside function • Multiply with $\sec^2(x^3)$
4. Differentiate $f(x) = (2x^2 + 1)^4$	$f'(x)=(4)(4x)^3=256x^3$ • differentiate the inside function and eliminate the real function. • need to use the appropriate technique	$f'(x)=4(2x^2+1)^3\bullet(4x)$ $=16x(2x^2+1)^3$ • By using power rule: $f'(x)=n[f(x)]^{n-1}f'(x)$

Method of Integration by Part

The two important thing that student have to remembers are

1) How to choose u and dv by using acronym LIATE as follows

L	I	A	T	E
Logarithmic function	Inverse trigonometric	Algebraic	Trigonometric Function	Exponential Function

2) Formula of integration by part ⟶ $\int u\,dv = uv - \int v du$

Figure 1 and 2 are two examples of student's mistake in the topic of integration by parts. The question is taken from the past year examination question on Jun 2019(Question 1a).

Question 1: Solve $\int \tan^{-1} x\, dx$ by using integration by parts.

Let's see what's the student have done for this topic?

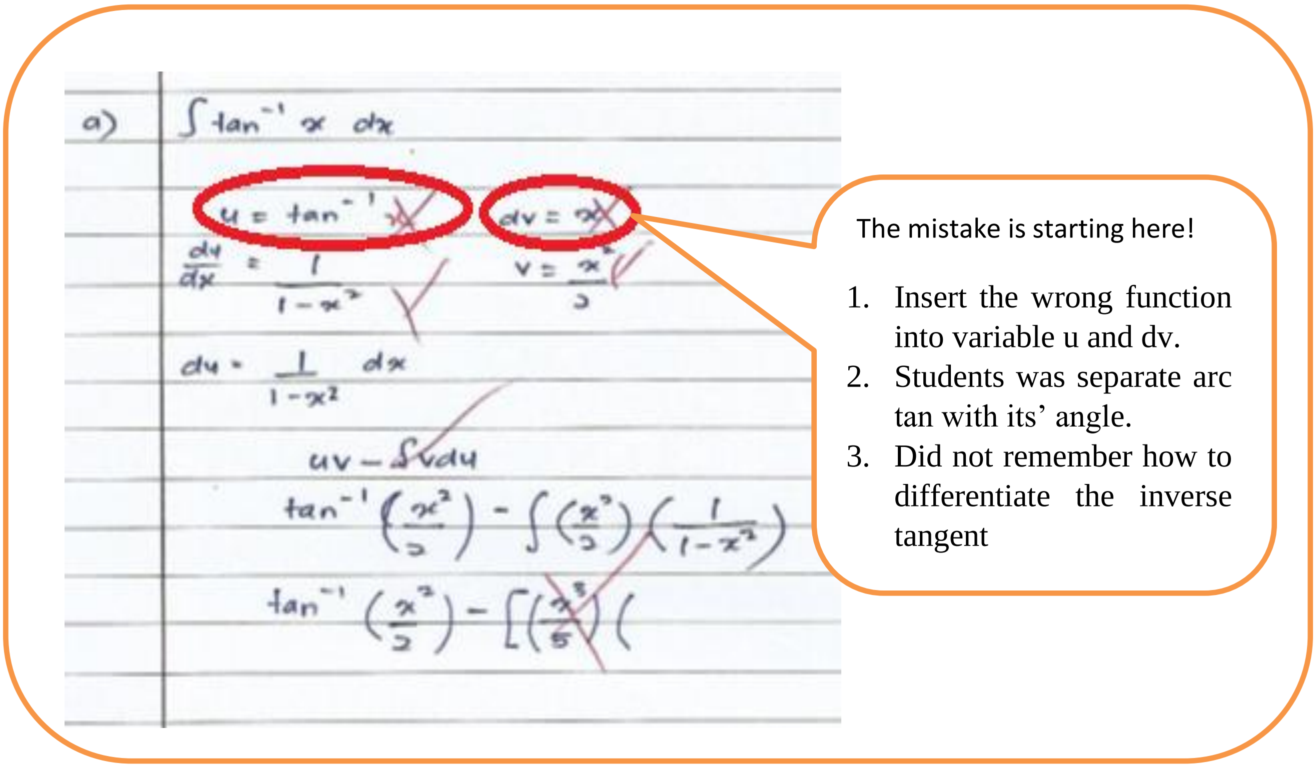


Figure 1: Student's mistake (1)

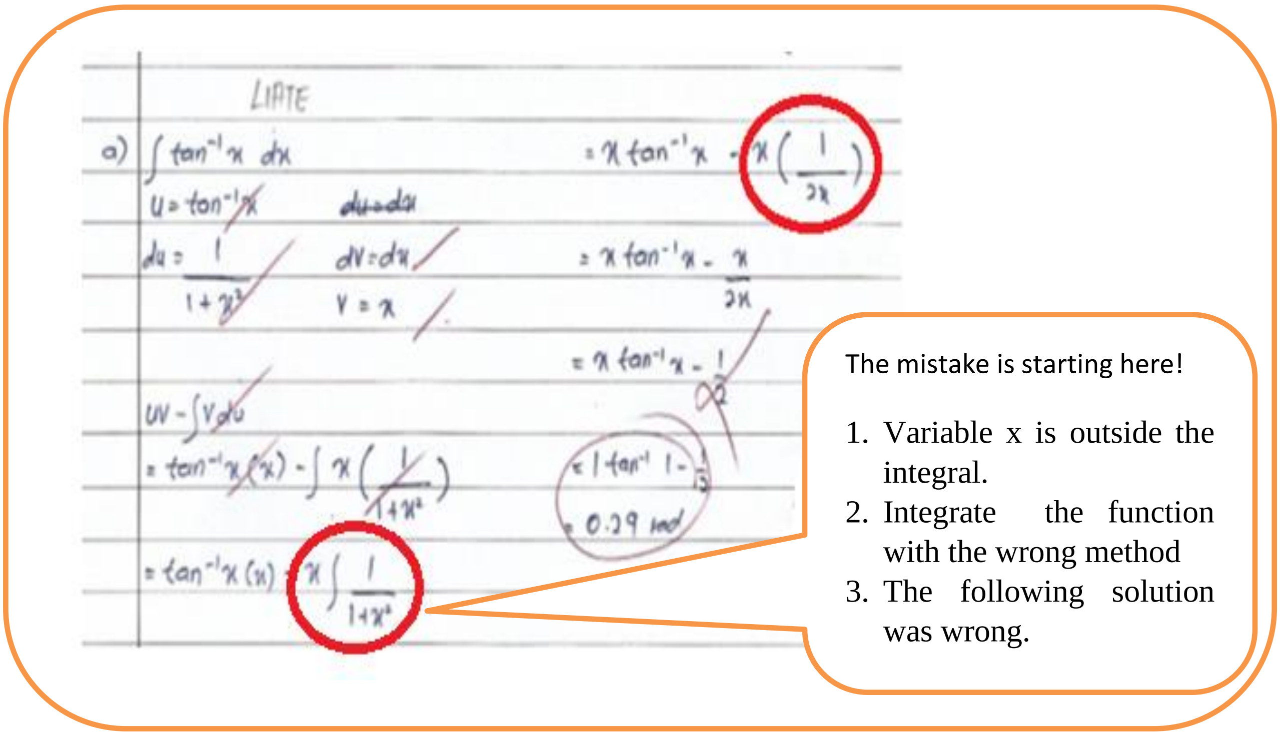


Figure 2: Student's mistake (2)

Table 2 show that the correction of the students' mistake. Make it perfect!

Table 2 : The Correction of Integration by Part

Correction:	**Method**
$\int \tan^{-1} x \ dx$ $u = \tan^{-1} x \quad , \quad dv = \int dx$	**Step 1: Choose u and dv.** *Remember!!!Single function that involving inverse trigonometric or logarithmic can be choose as u.* *Using LIATE when the integral involving product of 2 different function.*
$du = \frac{1}{1+x^2} dx \qquad v = x$	**Step 2: Differentiate u and integrate dv** *Remember the formula for integration of inverse trigonometric function!* *Basic formula:* $\frac{d}{dx} \tan^{-1} f(x) = \frac{f'(x)}{1+f(x)^2} dx$
$\int \tan^{-1} x \ dx = uv - \int vdu$ $= x \tan^{-1} x - \int x\left(\frac{1}{1+x^2}\right) dx$	**Step 3: Substitute into formula** *Substitute what do you find in step 2 into the formula of integration by part* $uv - \int vdu$
$\int x\left(\frac{1}{1+x^2}\right) dx = \int \frac{x}{1+x^2} dx$ let $u = 1+x^2$ $du = 2x \ dx$ $\int \frac{x}{1+x^2} dx = \int \frac{1}{u}\left(\frac{du}{2}\right)$ $= \frac{1}{2} \int \frac{1}{u} du$ $= \frac{1}{2} \ln u + c$ $= \frac{1}{2} \ln (1+x^2) + c$	**Step 4: Solve the integral** *Integrate by using method of substitution* *Remember!!* *Differentiate:* $\frac{d}{dx} \ln x = \frac{1}{x}$, *then* *integrate* $\int \frac{1}{x} dx = \ln x + c$ *Don't forget the last step for u substitution, that is replace* u *with* $\ln(1+x^2)$.
$\int \tan^{-1} x \ dx$ $= x \tan^{-1} x - \frac{1}{2} \ln (1+x^2) + c$	**Step 5: Write the final answer** *Don't forget to combine the answers.*

Method of Trigonometric Substitution

They are many things that student have to remember:

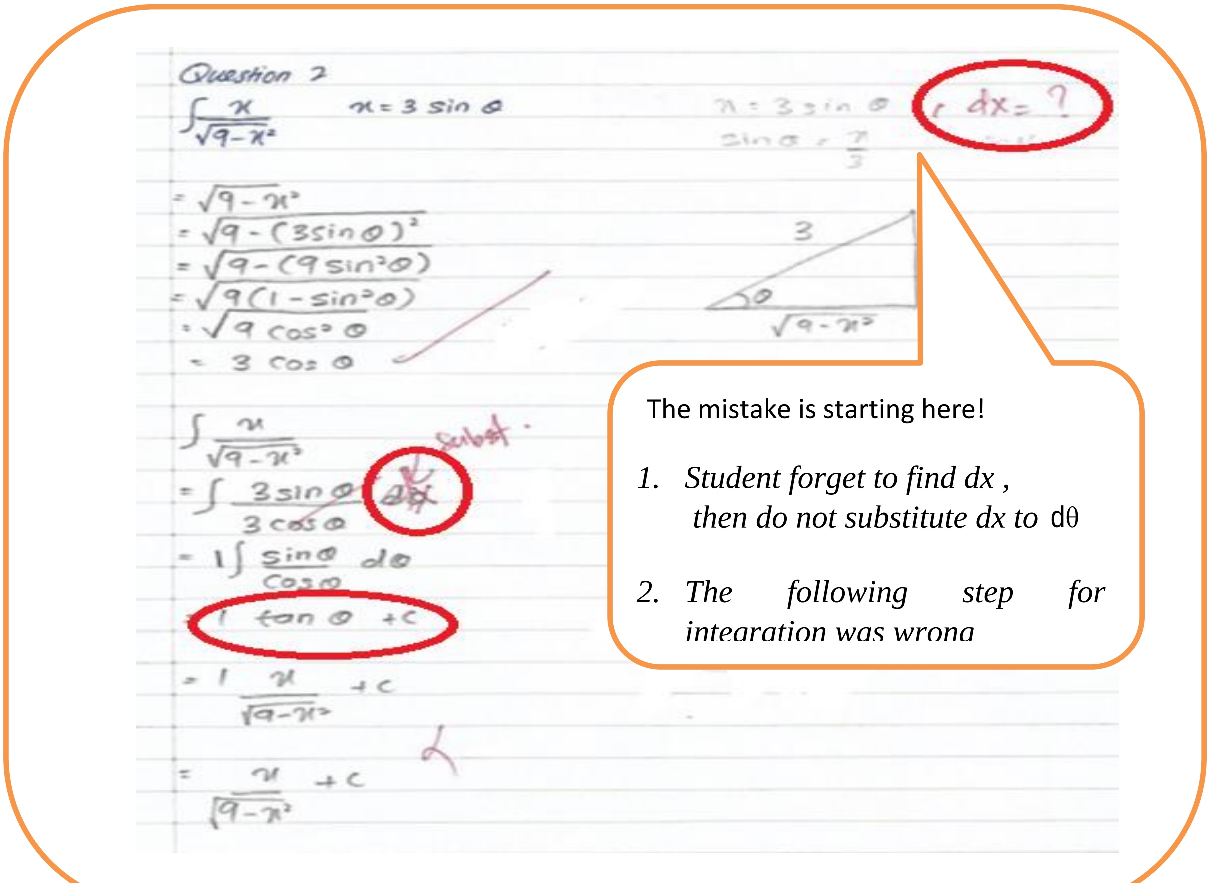
1. Three pattern of trigonometric substitution as follows:

   $\sqrt{x^2 + a^2}$, $\sqrt{x^2 - a^2}$, $\sqrt{a^2 - x^2}$

2. Each pattern have different steps to solve with different trigonometric substitution
3. Remember the basic differentiation and integration of trigonometric function
4. Using the right step for differentiation expecially the method of u substitution.

Let's see what's the student have done for this topic as shown on figure 3?

Question 2: Find $\int \frac{x}{\sqrt{9 - x^2}} dx$ by using trigonometric substitution $x = 3\sin\theta$.



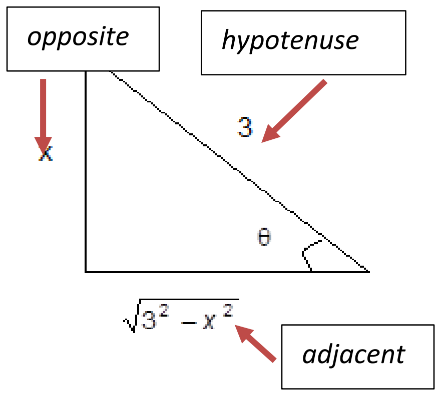
(sources: Test 1(April 2019))

Figure 3: Example of Students' Mistake on Integration of Trigonometric Substitution

The following table 3 show the correction of the above question 2.

Table 3: The Correction of Integration Trigonometric Substitution

Correction:	**Method**
$\int \frac{x}{\sqrt{9-x^2}}dx$, given $x=3\sin\theta$	**Step 1: Simplify the radical function using trigonometric substitution given.**
$\sqrt{9-x^2} = \sqrt{9-(3\sin\theta)^2}$ $= \sqrt{9-9\sin^2\theta}$ $= \sqrt{9(1-\sin^2\theta)}$ $= \sqrt{9\cos^2\theta}$ $= 3\cos\theta$	*Substitute* $x=3\sin\theta$ *and then expand it.* *Factorize the value 9 and change the* $1-\sin^2\theta$ *by using identity* $1-\sin^2\theta=\cos^2\theta$. *The purpose is to change the subtracting to the product and the square roots can be done.*
$x=3\sin\theta$ $dx=3\cos\theta\ d\theta$	**Step 2: Differentiate the given trigonometric substitution** *Differentiate x with respect to* θ
$\int \frac{x}{\sqrt{9-x^2}}dx$ $= \int \frac{3\sin\theta}{3\cos\theta}(3\cos\theta\ d\theta)$ $= \int \sin\theta\, d\theta = -\cos\theta$	**Step 3 Substitute into The Real Question** *Substitute the answer in step 1 and step 2* • $x=3\sin\theta$ • $dx=3\cos\theta\ d\theta$ • $\sqrt{9-x^2}=3\cos\theta$ *into the real question.*
	Step 4 : Simplify and Integrate *The answer is in term of* θ, *so this is not the final answer. You should change to variable x.*



Step 5 : Using the triangle of θ

Using the triangle of θ to make a change from θ to variable x.

From the trigonometric substitution, $x = 3\sin\theta$,

the $\sin\theta = \frac{x}{3}$. *Fill the value into triangle, that is opposite , adjacent and hypotenuse of* θ

Remember!! $\sin\theta = \frac{opposite}{hypothenus\ e} = \frac{x}{3}$

So, the adjacent is $\sqrt{3^2 - x^2}$

$$= \int \sin\theta \, d\theta = -\cos\theta$$

$$= \frac{\sqrt{3^2 - x^2}}{3} + c$$

Step 6: Write down the final answer

From the triangle,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypothenus e}} = \frac{\sqrt{3^2 - x^2}}{3}$$

Write down the final answer in terms of x.

References:

Faridah Hussin, Fadzilawani Astifar Alias, et al. 2019. "Module: Common Mathematics Errors (Algebra & Calculus)." 1-22.

Hasfazila Ahmat, Peridah Bahari et al. 2015. "Calculus 11 for Engineers." eISBN:978-967-841-07-6. Chapter 1.

Maisurah Shamsuddin, Siti Balqis Mahlan, et al. 2014 "Mathematical Errors in Advanced Calculus: A Survey among Engineering Students" In the ESTEEM Journal Special Issue 11(2).