DETERMINATION OF THE DETECTABLE MAXIMUM DIFFERENCE AMONG PARAMETERS IN HYPOTHESIS TESTING

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Abstract

Consider a one-way classification model with r treatments and n_i experimental units for the i^{th} treatment. Let $\mu_1, \mu_2, ..., \mu_r$ be the r treatment means and d the maximum difference between any two treatment means. This study focuses on determining the detectable maximum difference between any two means. We examine the scenario when the null hypothesis that the r treatment means are all equal ($H_0 : \mu_1 = \mu_2 = ... = \mu_r$) is accepted. When H_0 is accepted, the reliability of the result of the test is given by the power function which is a function of $\underset{\sim}{\mu} = (\mu_1, \mu_2, ..., \mu_r)^T$ and the error variance σ^2. It is not easy to use this power function to assess the reliability of the result of the test because it is a function of many variables. In this study, the information contained in the power function is summarized by an interval $[0, d^+]$ which represents the likely values of the detectable maximum difference between any two treatment means. Small value of d^+ will indicate that the acceptance result given by the test is reliable. The interval $[0, d^+]$ may also be viewed as a confidence interval for d when H_0 is accepted. This interval together with the confidence interval $(0, \infty)$ for d when H_0 is rejected form a mixed type confidence interval for d. We shall classify this mixed type confidence interval as one which is obtained by Method 1. Two other methods (Method 2 and Method 3) are also proposed for finding the mixed type confidence intervals for d. The coverage probabilities of the mixed type confidence intervals found by the three methods are then compared. It is found that in the case when r=3, $n_1 = n_2 = n_3 = 10$, the coverage probability of the confidence interval found by Method 3 is closest to the target value of 0.95.

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CHAPTER 1

Introduction

1.1 Overview of Study

Consider the one-way classification model

$$y_{ij} = \mu_i + \varepsilon_{ij} \qquad i = 1,2,\ldots,r; \quad j = 1,2,\ldots,n_i$$

in which y_{ij} is the observation obtained from the j-th unit receiving the i-th treatment, μ_i is the i-th treatment mean and the ε_{ij} are random errors which are independent and normally distributed with mean zero and unknown variance σ^2. We are often interested in testing the null hypothesis that all the treatment means are equal against the alternative hypothesis that not all the treatment means are equal. The above hypotheses may be stated as follows:

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_r$$
$$H_1 : \text{not all the } \mu_i \text{ are equal}$$

The usual test statistic used is the F statistic given by:

$$F = \frac{\sum_{i=1}^{r} n_i (\bar{y}_{i.} - \bar{y}_{..})^2 / (r-1)}{\sum_{i=1}^{r}\sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 / (\mathrm{N} - \mathrm{r})}$$

where $\bar{y}_{i.} = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}$, $\bar{y}_{..} = \frac{1}{N}\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}$ and $N = \sum_{i=1}^{r} n_i$.