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NEW FUNDAMENTAL THEORY IN SOLVING THE ROYALTY PAYMENT PROBLEM

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ABSTRACT

This study aim to maximize the functional performance index of the non-classical Optimal Control problem. However, solving the problem, this project has to clarify a few constraints. Firstly, the final state is unknown which is resulting in the nonzero value of terminal costate. Besides, the royalty function is non-differentiable at certain process. Therefore, the indirect method was applied by involving new modified shooting method (Sufahani-Ahmad-Newton-Golden-Royalty Algorithm and Sufahani-Ahmad-Powell-Golden-Royalty Algorithm). The results comparison done with direct method (Euler, Runge-Kutta, Trapezoidal, Hermite-Simpson) as a validation process. Finally, it is expected that new modified shooting method yield highly accurate optimal solution compared to the direct method.

Keywords: non-classical optimal control, royalty payment, shooting method

1. INTRODUCTION

In 1981, Spence proposed an economic model formulation where the functional involved royalty function which is denoted as ρ. He did not specified or proposed a specific technique to solve the model. Therefore, this issue have been interested by Zinober and Kaivanto in 2008. Zinober and Kaivanto in 2008 considered a framework which consisted of a piecewise continuous cumulative royalty schedule. The main idea of the research was to maximize the performance index that consists of an unknown state variable value at final time T, and the integrand is a piecewise continuous. They proposed a methodology to solve Spence (1981) economic model where the problem is converted into matrix formulation. However, solving this interesting problem has presented two difficulties (Zinober & Kaivanto, 2008): (1) the discontinuous changes in the level of the regime royalty are not fully differentiable everywhere, (2) It is impossible to put the problem into separate time intervals because of the times at which the non-differentiable royalty level transition points are, depending on the level of overall optimization of $u(t)$ within the entire interval $t \in [0, T]$. Therefore, they encouraged another researcher to propose suitable technique to overcome those difficulties. After that, (Zinober & Sufahani, 2013) solved a nonstandard OC problem without implementation of Spence (1981) economic model. They wished to maximize the functional performance index, however, a certain function of the problem involved two-stage piecewise function. As a result, they have faced the same difficulties as Zinober and Kaivanto (2008). Despite that, they proposed an appropriate approach where the piecewise constant function then converted into continuous approximation of hyperbolic tangent (tanh) function in order to allow the differentiation in every process. This motivate us to extend the research for solving Spence (1981) economic model. Then, in 2015, Kaivanto and Zinober make another approach in solving nonstandard OC problem where they tried to solve a similar problem from the previous work, Zinober and Kaivanto in 2008. They applied the combination of Newton with Golden Section Search or Brent method in shooting technique. They proposed to solve the Spence (1981) economic model by using $n \times 2$ matrix formulation where n is

referring to the number of stages of royalty function. They wished to compare the results of optimal curve for the no royalty, three-stage royalty function and seven-stage royalty function. However, their solution faced the same difficulty as in Zinober and Kaivanto in 2008. In addition, their paper's lack of theoretical explanation regarding the problem in nonstandard OC problem where the final costate value, $\rho(T)$ is not equal to zero. This encourage us to provide a more detail theoretical explanation in solving Spence (1981) economic model.

2. MATERIAL AND METHOD

This research solved the problem through modified shooting method where it is a combination of Newton with Golden Section Search method. The problem also solved by using another combination which Powell with Golden Section Search method. Both results will be compared with the discretization method (Euler, Runge-Kutta, Trapezoidal and Hermite-Simpson method) as a validation process.

3. RESULT AND DISCUSSION

Based on Table 1, the shooting method give the result for similar only up to one decimal place when compared with the discretization technique. At the costate value give a response with comparative up to one decimal place for both shooting and discretization method. Referring to the Table 1, the values give a solution with similar up to one decimal place for all methods. Figure 1 shows the optimal curve for the performance index and state values are similar for the shooting and discretization methods. The plots for the costate and control values are slightly different for the discretization results when contrasted with the shooting results.

Table 1. Results for the shooting and discretization methods with royalty function

Method	Results				
	$y(T)$	$J(T)$	$p(0)$	$p(T)$	η_T
Shooting Method					
Newton & Golden Section Search	0.317978	0.606969	-1.254410	-0.356795	-0.356795
Powell & Golden Section Search	0.317988	0.606969	-1.254380	-0.356766	-0.356766
Discretization Method					
Euler	0.319074	0.611833	-1.26011	-	-
Runge-Kutta	0.323087	0.612860	-1.28002	-	-
Trapezoidal	0.322194	0.614032	-1.28354	-	-
Hermite-Simpson	0.328815	0.612950	-1.25833	-	-

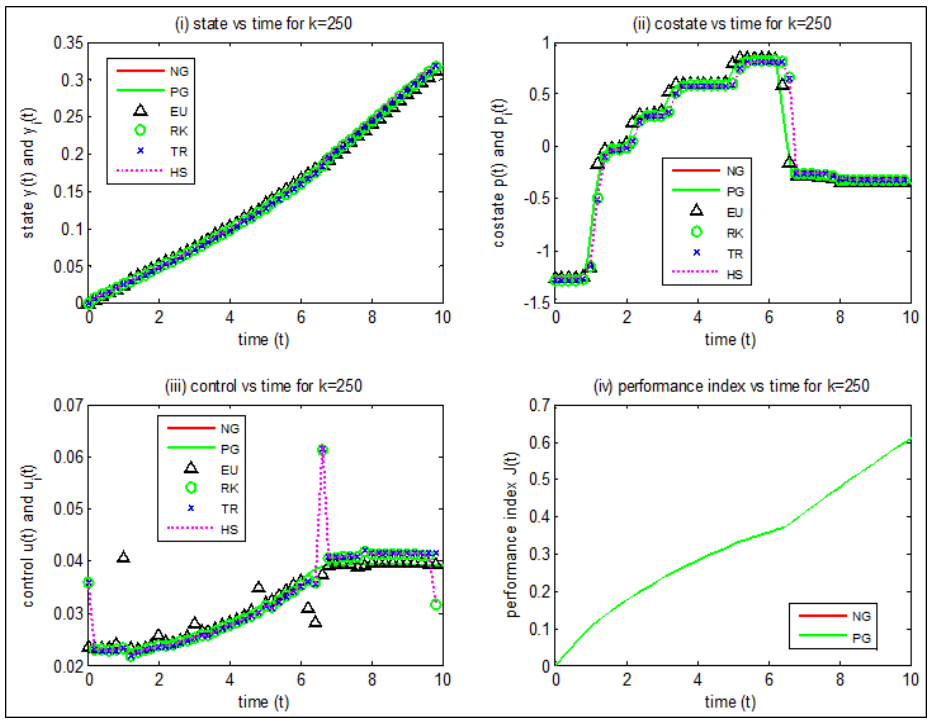

Figure 1. Optimal plot for seven-stage royalty function. (NG=Newton & Golden Section Search; PG=Powell & Golden Section Search; EU=Euler; RK=Runge-Kutta; TM=Trapezoidal; HS=Hermite-Simpson)

4. CONCLUSION

This research managed to solve the nonstandard OC problem with the involvement of royalty problem. The problem solved by using new modified shooting method. This will give a huge contribution to the research field and society. The research can be a stepping-stone to researcher to explore new method in solving real-world problem and lessen the time taken in solving the problem with discrete piecewise function.

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