UNIVERSITI TEKNOLOGI MARA

SQUEEZED STATES IN QUANTUM COUPLER: THE ANALYTICAL PERTURBATIVE METHOD VERSUS POSITIVE-P REPRESENTATION

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ABSTRACT

Squeezed states of light promise great potential applications in many fields. It can surpass the standard quantum limit, hence making it crucial in precision measurements and quantum information processing. In this work, the generation of squeezed states of light is investigated theoretically using two different mathematical methods; the *phase space method* and the *analytical* perturbative *method*. Using these two methods, the generation of squeezed states of light in a two-guided waves coupler with Kerr nonlinearity and a three-guided waves coupler with second-order nonlinearity is studied. The overall behavior of both systems is defined by its dimensionless Hamiltonian operator. The phase space method is based on Schrödinger-picture in quantum mechanics, where the evolution of the system is specified by the density operator which represents the state vector (or wavefunction). In this method, the evolution of the Hamiltonian operator is described through the Von-Neumann equation. This equation is then converted into its corresponding classical Fokker-Planck equations using the positive-P representation. An equivalent set of Langevin stochastic equations are then obtained from the Fokker-Planck equation using the Ito calculus rules. Finally, the system is solved numerically and averaged over thousands of trajectories to obtain a reliable solution. On the other side, the analytical perturbative method is based on the Heisenberg picture in quantum mechanics where the operators are evolving in time, but the state vectors are time-independent. In this case, the Hamiltonian evolution is described by the Heisenberg equation of motion and a solution for the propagating modes is proposed in the form of the Baker-Hausdorff (BH) formula. The solution obtained using this method is validated by calculating the equal time commutation relation (ETCR). Single-mode squeezed states with frequency matching and mismatching generated from both methods at different combinations of input parameters are shown. The strength and weaknesses of each method are also discussed. From the obtained result, the two-channel system produces squeezed states with a steady oscillation pattern. Meanwhile, a more interesting pattern was shown by the three-channel system. For both systems, similar effects are observed when the coupling constants (linear and nonlinear) are manipulated. The increment of linear coupling results in the frequent squeezing signal, while the increment of nonlinear coupling yields the squeezing with high maximal amplitude. Under the existence of frequency mismatch, the two-channel system demonstrates collapses and revivals-like squeezed states, while the three-channel system produces squeezed states with erratic behavior. In the case of contradirectional propagation, the two-channel system shows a drastic improvement in the squeezing intensity. However, the contradirectional propagation does not significantly affect the squeezing intensity. As an overall conclusion, a good agreement between both methods was achieved, especially at early evolution stages and lower values of linear coupling coefficient. On one side, the analytical perturbative method seems insensitive to higher values of nonlinear coupling coefficients. Nevertheless, it demonstrated better numerical stability. On the other side, the solution of the stochastic equations resulting from the phase space method is numerically expensive as it requires averaging over thousands of trajectories. Besides, numerically unstable trajectories appear with positive-P representation at higher values of nonlinearity.

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