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# MATHEMATICS

## IN APPLIED RESEARCH

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# MATHEMATICS
IN APPLIED RESEARCH

BULETIN RASMI
KOLEJ PENGAJIAN PENGKOMPUTERAN, INFORMATIK, DAN MEDIA, UITM CAWANGAN NEGERI SEMBILAN KAMPUS SEREMBAN
EDISI NOVEMBER 2022



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## ALUAN KETUA EDITOR

Al-Hamdulillah bersyukur kita ke hadrat Allah swt, dengan penerbitan edisi ketiga makalah Mathematics in Applied Research terbitan Kolej Pengajian Pengkomputeran, Informatik, dan Media UiTM kampus Seremban. Penerbitan ini merupakan satu usaha untuk menonjolkan hasil penyelidikan pelajar bersama pensyarah dalam Projek Tahun Akhir program ijazah sarjana muda di KPPIM Seremban.

Semenjak tahun 2014, mahasiswa tahun akhir KPPIM (sebelumnya dikenali FSKM) Seremban telah menghasilkan banyak penyelidikan yang berpotensi untuk diketengahkan dalam dunia akademik. Akan tetapi tidak banyak yang berjaya diterbitkan dalam jurnal atau pun prosiding konferensi akademik kerana halangan tertentu seperti kualiti penyelidikan dan penulisan ilmiah. Oleh itu, penerbitan makalah ini diharapkan dapat menambahkan lagi ruang bagi penerbitan hasil penyelidikan warga KPPIM Seremban

Disamping itu, pihak KPPIM Seremban mengharapkan makalah ini akan menjadi rujukan dan pemangkin kepada usaha menghasilkan penyelidikan Projek Tahun Akhir yang lebih bermutu tinggi. Makalah ini juga adalah batu asas kepada perkongsian penyelidikan terkini daripada pelajar dan pensyarah KPPIM Seremban.

Bagi pihak editorial, saya mengucakan Syabas dan tahniah kepada pasukan penerbitan yang berusaha keras untuk menghasilkan makalah Mathematics in Applied Research. Saya juga ingin mengambil kesempatan ini untuk menyampaikan ucapan terima kasih khas buat dua editor yang akan meninggalkan kita untuk bersara iaitu;

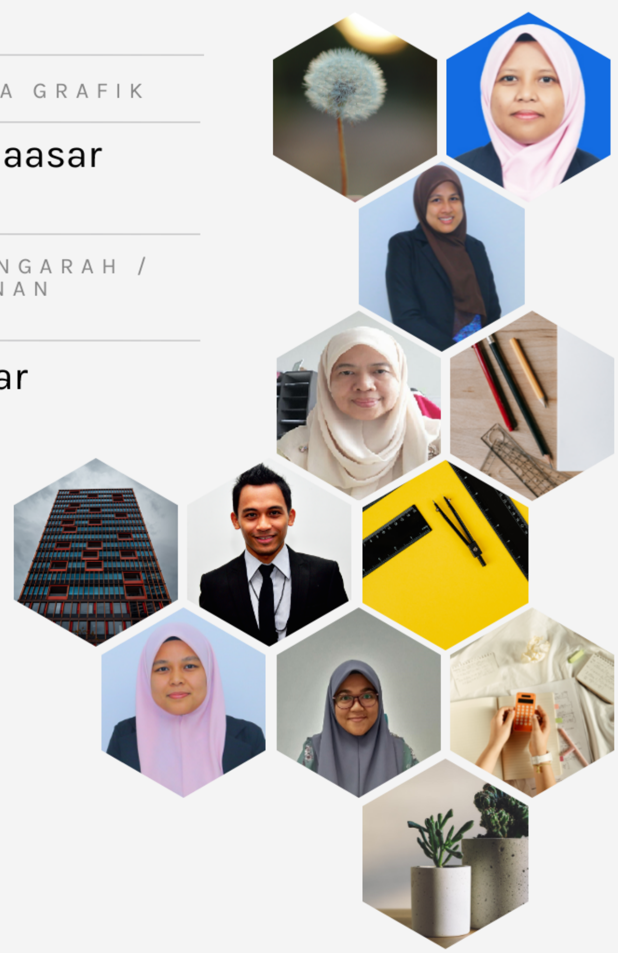
- Profesor Madya Dr. Nur Azlina Abd Aziz
- Dr. Nor Azni Shahari

semoga kecemerlangan perkhidmatan yang ditunjukkan oleh kedua editor-editor ini akan menjadi pendorong kepada editorial board yang seterusnya.

Sekian. Terima kasih.

## SIDANG REDAKSI





# Thank You!

for being with us,

**Dr. Nor Azni Shahari**
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MiAR 2021, 2022

**Dr. Nur Azlina**
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MiAR 2021, 2022

**for all the dedications and**

# Happy Retirement

**From:**
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Universiti Teknologi MARA Cawangan Negeri Sembilan

## Contents

# ROOT FINDING FOR NON LINEAR EQUATION BASED ON IMPROVEMENT NEWTON'S METHOD

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## 1. Introduction

A root-finding process is a process for finding zeroes of continuous functions and provide approximations to the roots, which are represented as small isolating intervals or as floating-point integers. One of the most prevalent problems encountered in the root finding process is the rapidity of convergence rate to the actual root and the accuracy of the root approximation. However, the procedure is dependent on the initial estimate, neither stability nor convergence are guaranteed.

Mathematicians and scientists have used Newton's Method, an iterative methodology, for centuries to find the solution to nonlinear equations. While there is an alternative numerical method for determining the roots of nonlinear function, such as Secant methods and Bisection methods, the Newton's Method is by far the most popular among academia and industry owing to its quick convergence rate. Based on many research publications, Newton's Method converges quickly in comparison to the other methods (for example see: Akram and Ann (2015), Azure et al. (2019), Mehtre and Singh (2019)). This was clearly seen in the number of iterations taken by each of the methods to converge to the exact solution.

According to Mehtre and Singh (2019) and Hasan and Ahmad (2015), Newton's method diverge if the starting value given is not near enough to the root. Newton's Method may fail to converge or may converge to the incorrect root or may converge too slowly. Although it is broadly acknowledged that Newton's Method is better than other methods like Secant and Bisection method in finding the root, as functions get more complex, Newton's Method might be difficult to perform the optimal number of iterations and may not provide an accurate root. If too many iterations required, it may cause the speed of the convergence rate to decrease. As a result, several enhancements to Newton's Method have been developed and implemented to tackle challenging science and engineering issues (Alleme and Azad, 2012). Hence, an improvisation Newton's Method is needed to decrease the rounding error which will improve the accuracy of the result obtained. Therefore, the study will be focused on a comparison of the efficiency and accuracy between Newton's and Improvised Newton's Methods in some transcendental function.The methods that have been studied include the Midpoint Newton's Method (MNM), the Improvised Midpoint Newton's Method (IMNM), and the Chebyshev's Method (CM). All these methods are assessed using various transcendental functions that have been chosen, and the number of iterations and convergence rate as well as average elapsed time for each method are compared to determine the optimal method. The pace of convergence and uniqueness of the solution are two ongoing concerns with the current numerical methods.

## 2. Mathematical Formulations of Newton and Improvised Newton's Method

The mathematical formulations of the two methods employed in this research which are the Newton's Method, and the Improvised Newton's Method that consists of three components: the Midpoint Newton's Method, the Improvised Midpoint Newton's Method, and the Chebyshev's Method.

### 2.1. *Newton's Method (NM)*

The Newton's method is:

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \quad \text{for} \quad n = 0, 1, 2, ... \tag{1}$$

### 2.2. *Midpoint Newton's Method (MNM)*

The Midpoint Newton's method is an improvised method based on midpoint rule for solving nonlinear equations. The formula is:

$$x_{n+1} = x_n - \frac{f(x)}{f'\left(x_n - \frac{f(x_n)}{2f'(x_n)}\right)} \tag{2}$$

### 2.3. *Improvise Midpoint Newton's Method (IMNM)*

Then, an improvised Midpoint Newton's method by using Midpoint Newton's method (Alleme and Azad, 2012) is shown below, for any one initial approximation as $x_0$ of the root. The improvised Midpoint Newton's method formula is:

$$x_{n+1} = x_n - \left(\frac{f(x)(y_n - x_n)}{f(y_n) - f(x_n)}\right) \tag{3}$$

### 2.4. *Chebychev Method (CM)*

The new iterative formula named Chebyshev Method is a formula that have a cubically convergence. The procedure was based on Newton's method and the expansion of the inverse of the function as a power series. The Chebyshev method is defined by:

$$x_{n+1} = x_n - \frac{f(x)}{f'(x_n)} - \frac{f^2(x_n)f''(x_n)}{2(f'(x_n))^3} \tag{4}$$

## 3. Graphical User Interface (GUI)

The Graphical User Interface (GUI) that was developed by using a MATLAB Graphical User Interface Development Environment (GUIDE) has been used in this study to compute the roots of nonlinear equations. A sample run of this simulation was used to find the first positive root of $f(x) = \sin(x) - 0.5x$ as shown in Figure 1. The interface is divided into three parts, which are input, output, and graph. To get the results, the user must follow the provided instructions. Firstly, the user must manually input the information needed, including the functions, initial value, and maximum number of iterations in the input section. The tolerance of the computation has been fixed up to $10^{-6}$. Then, to solve the general functions, the user must select the Pop-up Menu button to choose the method from either Newton's Method, Midpoint Newton's Method, Improvised Midpoint Newton's Method, or the Chebyshev's Method. After entering all of the required information, the user may track the progress in the output area. The results for each iteration are presented in the table provided the precise root is specified. The iteration value, approximation root, and error for each root will be shown in tabular form on the table displayed in the output section to make the generated results more representable. As seen in Figure 1, the function has stopped at the fifth iteration with an approximate root of 1.8955 for Newton's Method. The user may also plot the function graph by clicking the plot button on the interface. Furthermore, the user can attempt another function, but, before doing so, the user must clear all data in the interface by using the clear button.

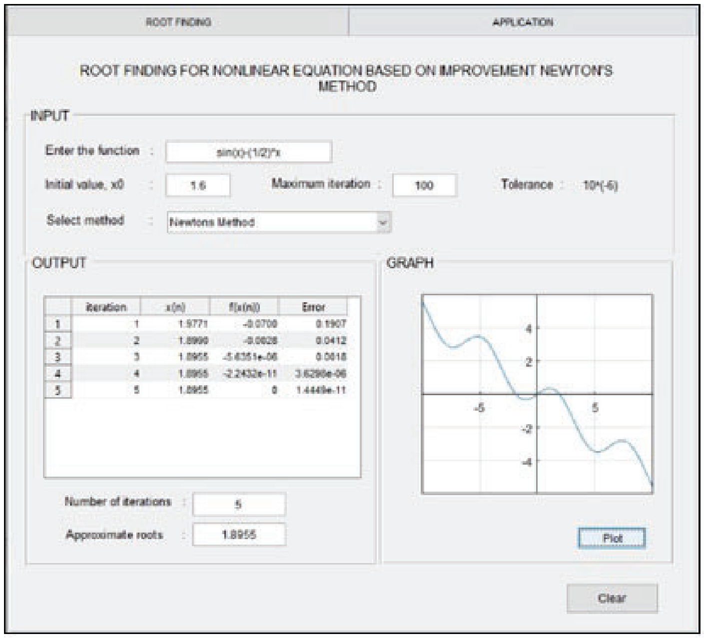


Figure 1: Graphical User Interface (GUI) to generate the results by MATLAB software

## 4. Results and Discussions

The study obtained two essential findings:

1. Analysis of the number of iterations and average elapsed time Table 1 and Table 2 below shows the comparison between four methods where NM, MNM, IMNM and CM are Newton's Method, Midpoint Newton's Method, Improvised Newton's Method and Chebyshev's Method, respectively. For this paper, the number of iterations and average elapsed time for several functions was calculated by using random initial values in a MATLAB programming software. When the relative error is less than or equal to a given tolerance, the number of iterations of the function $f(x)$ will terminate. This indicates that the $x_n$ value from the last iteration served as the exact root for the functions.

Based on the Table 1, it can be concluded that MNM can be utilized successfully and efficiently since it always showed that MNM requires the fewest iterations when compares to

Table 1: Comparison among method according to number of iterations

| Function | Initial value | No of Iteration | | | |
|---|---|---|---|---|---|
| | | NM | MNM | IMNM | CM |
| $f_1(x) = 10e^{-x^2} - 1$ | 0.75 | 5 | 3 | 4 | 4 |
| | 1.6 | 3 | 2 | 3 | 3 |
| $f_2(x) = \sin(x) - \frac{1}{2}x$ | 1.6 | 4 | 2 | 3 | 3 |
| | 2 | 4 | 2 | 3 | 2 |
| $f_3(x) = \tan(x) - \tanh(x)$ | 4.5 | 6 | 4 | 5 | 4 |
| | 3.5 | 5 | 3 | 3 | 3 |
| $f_4(x) = \ln(x^2 - 3) - 2$ | 1.75 | 6 | 4 | 5 | 4 |
| | 2.1 | 4 | 2 | 3 | 3 |

Table 2: Comparison among method according to average elapsed time

| Function | Initial value | Average elapsed time (seconds) | | | |
|---|---|---|---|---|---|
| | | NM | MNM | IMNM | CM |
| $f_1(x) = 10e^{-x^2} - 1$ | 0.75 | 0.006780 | 0.003364 | 0.003190 | 0.007599 |
| | 1.6 | 0.001969 | 0.001705 | 0.002833 | 0.002646 |
| $f_2(x) = \sin(x) - \frac{1}{2}x$ | 1.6 | 0.005127 | 0.002348 | 0.005382 | 0.005912 |
| | 2 | 0.03779 | 0.003561 | 0.002643 | 0.006480 |
| $f_3(x) = \tan(x) - \tanh(x)$ | 4.5 | 0.0066286 | 0.003228 | 0.003952 | 0.004319 |
| | 3.5 | 0.002546 | 0.002380 | 0.002643 | 0.002725 |
| $f_4(x) = \ln(x^2 - 3) - 2$ | 1.75 | 0.006628 | 0.002596 | 0.003889 | 0.007304 |
| | 2.1 | 0.002504 | 0.001621 | 0.003566 | 0.002507 |

the other methods. Additionally, when the initial value is selected closer to the root, the fewer number of iterations may be obtained.

On the other hand the average elapsed time is also considered. According to Badr et al. (2021), the elapsed time is also critical for evaluating an algorithm, while the majority of studies have overlooked the subtleties of calculating the elapsed time. Each algorithm will be performed five times to achieve an exact elapsed time, and the average of those elapsed times will be determined as given in Table 2. Table 2 shows that MNM has the shortest average elapsed time most of the time, while CM indicates the longer time to converge to the exact roots. According to the number of iterations in Table 1, CM outperforms NM. However, in Table 2, when elapsed time is included, NM performs better. This occurs because the CM formula is quite complex, including second derivatives and cubic roots of the first derivative, which makes CM take more time to converge to the exact roots. In conclusion, MNM gives the fastest average elapsed time, followed by IMNM, NM, and CM. Thus, in terms of overall performance, MNM is the best method because it takes the least number of iterations and also has the shortest average time spent on each iteration

2. Analysis of the absolute relative errors

The Figure 2 compare the absolute relative errors associated with determining the roots of logarithmic functions with initial value $x_0 = 1.8$ using four distinct techniques with accuracy to six significant digits.

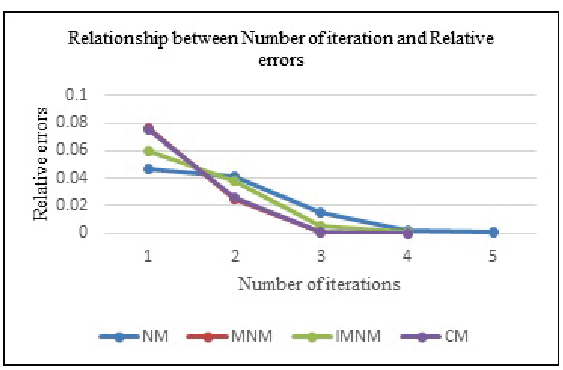


Figure 2: Graph absolute relative error for $f_4(x) = \ln(x^2 - 3) - 2$

As seen in Figure 2, the absolute relative errors for MNM and CM are almost identical. Both approaches demonstrate a significant drop, with MNM falling from 0.07653 to 0.02514 and CM falling from 0.07554865 to 0.02607997. However, MNM outperforms CM in terms of performance, since MNM requires only three iterations to get the exact root, while CM requires four. It demonstrates that MNM is superior to other methods, despite the fact that it begins with a huge absolute relative error and requires fewer iterations than the other methods.

## 5. Conclusion

This study concludes the following three main contributions: the number of iterations, the accuracy of the desired root, and the average elapsed time all have a substantial influence on an iterative method's performance. In term of the number of iterations, Improvised Newton method can outperform Newton's method with the rank of MNM become the superior method continued by IMNM and CM. When all approaches were evaluated based on average elapsed time, MNM remained the best. Due to the complexity of the IMNM formula, this method cannot perform very well in the comparison of number iterations as compared to other method.

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