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Exploring the Computer Algebra System as A Tool in Learning the Fundamental of Column Buckling Behavior for Civil Engineering Students

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Abstract This paper presents the symbolic buckling analysis of an elastic column structure using Maple, a symbolic computer algebra system (CAS). The symbolic analysis was used as a learning tool in the third year of the undergraduate civil engineering programme at the School of Civil Engineering, Universiti Teknologi MARA Pulau Pinang Branch, Permatang Pauh Campus. The students were interviewed after using the symbolic buckling analysis and it was found that they were able to understand, justify their predictions and better visualise the solutions of the given problems. The tool serves as a new method for learning and understanding the basic behaviour of an elastic column structure made of any material and under a freely specified boundary condition at both ends under a concentric compressive load. Instructors should be trained in the use of the CAS software to train our future civil engineering graduates for the industrial markets.

Keywords: Buckling, CAS, Column, Elastic, Engineering Education

Introduction

Buckling of columns is taught at the end of the basic course on strength of materials because the topic requires extensive knowledge of integral calculus, differential equations, and statics. It is an important concept in the field of structural mechanics, as students need to understand the basic idea of column buckling (CB) behaviour before they can apply this knowledge in the higher semesters, e.g. in the structural design course.

Classically, the CB behaviour of straight-columns under a concentric compressive load can be modelled by a single second-order differential equation (DE) given by equation (1), using the beam-bending equation as given below;

$$EIy'' = M = -Py \tag{1}$$

where

E is the elastic modulus of the column
I is the moment of inertia of the column
y is the horizontal displacement of the column, which is the required solution of the equation (1)
M is the variation of the bending moment force along the column height
P is the compression force acting on the column

Engineering students must determine the solution to equation (1) using their basic knowledge DE from the previous mathematics course and then apply the solution to solve the CB problem. Mullis et al. [1] mentioned that the ability to model and solve this DE individually is very important to build skills essential for mathematics education. However, Camacho-Machín and Guerrero-Ortiz [2] reported that learners who solve DE face difficulties on selecting the most appropriate methods for solving DEs and find it difficult to evaluate integrals they encounter in solving them. In fact, Arslan [3] observed that the solutions of DEs can be found even without a deep understanding and conceptualization of DEs, making it unnecessary to understand DEs and related concepts, which is not the right attributes of civil engineering students. One of the students' comments taken from Maplesoft.com [4] highlights these issues as follows:

"When your answer doesn't match the answer in the back of the book, and you know the book is sometimes wrong, you don't know who to believe. Either the book is wrong, and I could relax, or I am wrong, and I'd know for sure that I had made a mistake, and that I wouldn't be wasting my time by going over the question again".

According to Raj [5], students will be more interested and even feel more enjoyable to understand the concept and engineering interpretation of the determined solution if they are able to visualise them. Visual stimulation allows students to make sense of solution forms and reduce their fear of mathematical language, leading to better understanding, development of interest, motivation to learn more, and even feel some enjoyment. Visualisation plays a very important role in understanding the dynamic aspects of a basic differential equation of a column buckling. It is used to understand the derivative as the slope of a column buckling shape; and it also helps to interpret graphs and read information from these graphs, such as the existence of an equilibrium state.

It appears that the DE solutions can be visualized by using special features of the digital technology such as the computer algebra systems (CAS). There are several advantages to using popular CAS software like Maxima, Mathematica and Maple that make learning mathematics DE more worthwhile. Boras [6] stated that the main advantage of Maxima software to the learner's community is the availability of the system to Android mobile device users and can be used offline. On the other hand, Mathematica [7] software claimed that using intelligent automation system on providing reliable and high-quality results. Despite of being similar with the Mathematica, Maple [8] offers ease of use and superior symbolic technology. A study feedback by Purnomo et al. [9] proved that Maple software significantly improves the understanding in multivariate calculus topic among their students. Lavicza [10] found that instructors who have proficiency in many CAS can successfully deliver the subject material well to their students. The selection of suitable CAS software is greatly dependent on the institution budget and the instructor's expertise However, to the best author's knowledge, the incorporation of this innovative teaching and learning approach is not yet adopted in any of Malaysian universities' civil engineering curriculum as students only study the CB via lectures, tutorials, and laboratories.

It is, therefore, an exploration of using CAS, i.e. Maple software, as a tool for the instructors in delivering the CB fundamental concept and its engineering interpretation to the civil engineering students at university level, was conducted in this paper. As studied by Uziak [11], there is an immediate need to introduce the engineering students with computer-assisted techniques in the very beginning of their engineering education to improve their learning experience especially the CB fundamental principles.

Method

The purpose of this study is to investigate students' engagement in learning differential equations with Maple and to improve the third-year students' basic understanding of column buckling problems in the subject of Computational Analysis for Engineers (CES513). This subject is related to the learning of numerical analysis approaches among civil engineering students to solve practical and real-world problems. Fourteen participants were selected based on the evaluation of civil engineering students' diagnostic tests in solving ordinary and second differential equations using the traditional approach. Seven students per group were selected for this study, representing the groups of very good students. The main reason for selecting the small number of participants was that the number of Maple licences in a computer laboratory is limited to fourteen users at one time and there are time issues.

It is worth noting that all students in previous semesters were taught differential equations at different levels of proficiency. In CES513, students had to learn various topics on numerical analysis, such as solving nonlinear equations, matrix manipulation, curve fitting, numerical integration, linear programming, finite element method, and solving differential equations in solving special problems in civil engineering. According to the curriculum, the solution of ordinary differential equations must be taught in a two-hour lecture, which is too short in time and does not motivate students enough to fully understand the special topic. The time allotment for this topic is two hours per week and another two hours for the computer lab. During the computer lab sessions, students were only introduced to standard numerical solutions of differential equations such as Euler's method, and second and fourth order Runge-Kutta methods using MATLAB software as required by the current curriculum. The procedural solution provided by this standard solution do not induce student's connections between the mathematical ideas and engineering knowledge that are able to enrich their life as engineering students.

To nurture an understanding of the real-world problem to CES513 students, column buckling was chosen as the specialised civil engineering problems on building structures. In this research, four different stages were employed in sequence as shown in Figure 1. The first stage is to collect the participants' ability on solving DE by using their mastery skills in their previous semester. As shown in equation (2) and equation (3). The established solution by the participants will be used later for comparison purposes in the later stage.

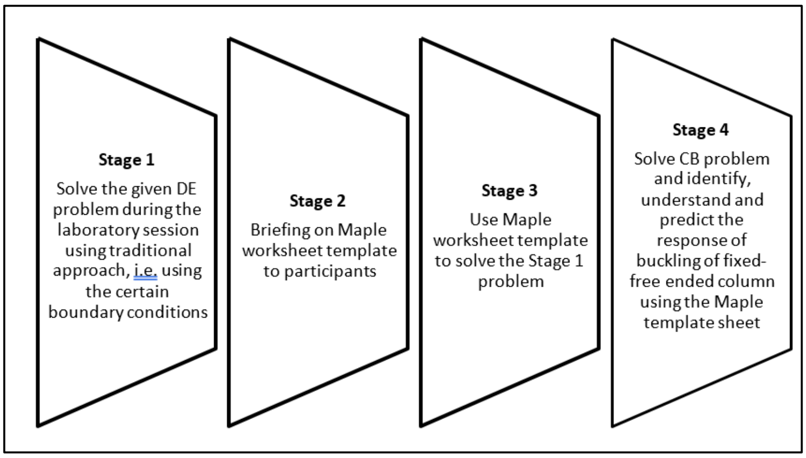
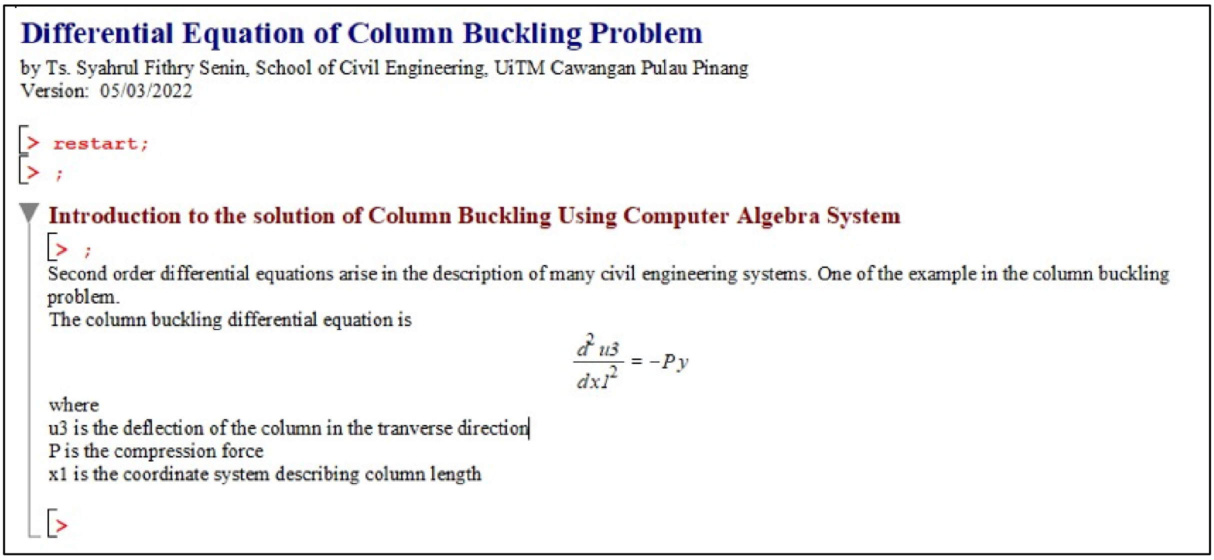


Figure 1: Four stages of the research activity

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=0 \tag{2}$$

$$y(0)=1\ ,\ y'(0)=0 \tag{3}$$

After finishing the first stage, there is a need to educate the participants on understanding the Maple worksheet template on solving the same previous DE. During this stage, participants were required to understand the given Maple template. The beginning topic was introduced to them by highlighting the main equation that needs to be solved with the involved important parameters as shown in Figure 2.



Differential Equation of Column Buckling Problem
by Ts. Syahrul Fithry Senin, School of Civil Engineering, UiTM Cawangan Pulau Pinang
Version: 05/03/2022

```
> restart;
> ;
```

Introduction to the solution of Column Buckling Using Computer Algebra System

```
> ;
```

Second order differential equations arise in the description of many civil engineering systems. One of the example in the column buckling problem.
The column buckling differential equation is

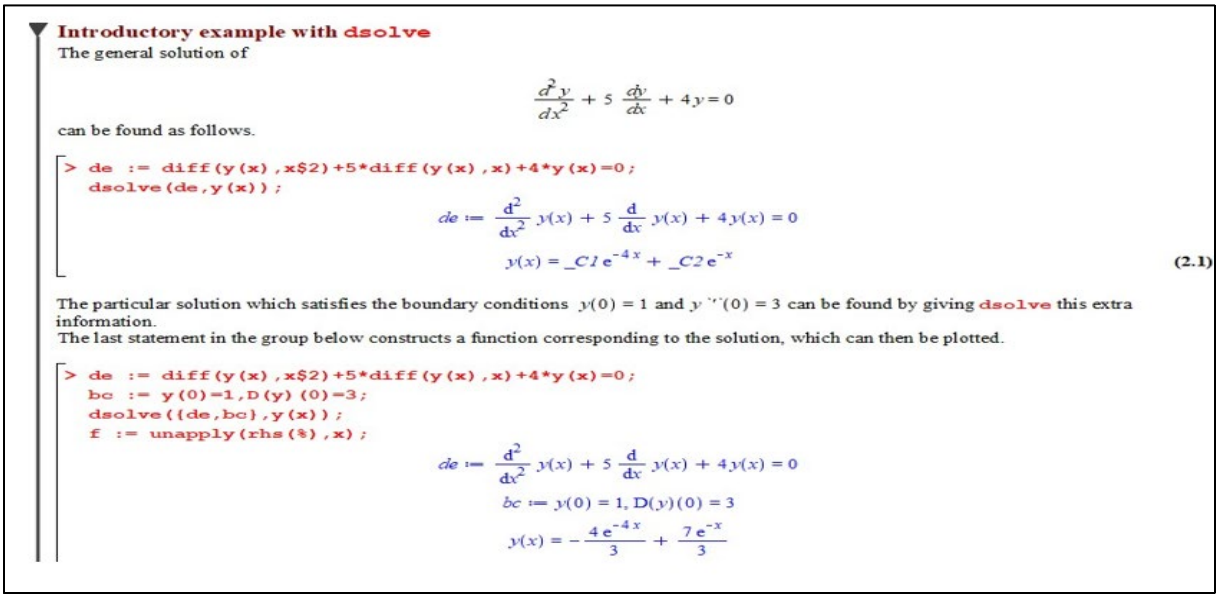
$$\frac{d^2 u3}{dx1^2} = -P y$$

where
u3 is the deflection of the column in the tranverse direction
P is the compression force
x1 is the coordinate system describing column length

```
>
```

Figure 2: The welcome screen of the created Maple worksheet

A specific simple step-by-step lesson on using the worksheet template to solve symbolically the second order DE were shown to the fifteen participants. The full set of instructions on how to create and display the second-order DE given by equation (1) was taught to the participants (line 2.1 in Figure 3). The incorporation of the boundary conditions, as mentioned in equation (2) to the created DE, was exposed to the participants. The dsolve command was employed in the Maple worksheet as the built-in function used to seek the symbolic DE solution with the supplied boundary conditions.



Introductory example with dsolve
The general solution of

$$\frac{d^2 y}{dx^2} + 5\,\frac{dy}{dx} + 4y = 0$$

can be found as follows.

```
> de := diff(y(x),x$2)+5*diff(y(x),x)+4*y(x)=0;
  dsolve(de,y(x));
```

$$de := \frac{d^2}{dx^2}\,y(x) + 5\,\frac{d}{dx}\,y(x) + 4\,y(x) = 0$$

$$y(x) = \_C1\,e^{-4x} + \_C2\,e^{-x} \qquad (2.1)$$

The particular solution which satisfies the boundary conditions $y(0) = 1$ and $y\,'(0) = 3$ can be found by giving dsolve this extra information.
The last statement in the group below constructs a function corresponding to the solution, which can then be plotted.

```
> de := diff(y(x),x$2)+5*diff(y(x),x)+4*y(x)=0;
  bc := y(0)=1,D(y)(0)=3;
  dsolve({de,bc},y(x));
  f := unapply(rhs(%),x);
```

$$de := \frac{d^2}{dx^2}\,y(x) + 5\,\frac{d}{dx}\,y(x) + 4\,y(x) = 0$$

$$bc := y(0) = 1,\ D(y)(0) = 3$$

$$y(x) = -\frac{4\,e^{-4x}}{3} + \frac{7\,e^{-x}}{3}$$

Figure 3: The step-by-step symbolic solution of second-order DE

In the third phase, all participants had to solve a linear, non-homogenous DE of CB given by equation (4) and equation (5), adapted from Jerath [12], based on a free-body diagram of the considered problem, as shown in Figure 4. Depending on the given boundary conditions at positions 1 and 2, which are listed in Table 1, each participant had to solve the equation (4). Each participant had to choose the appropriate parameter values for their CB problem to promote unique solutions among them. Boundary condition 1 and 2, represents either to be pinned, fixed or free condition at left and right ends of the column, are defined in Table 1.

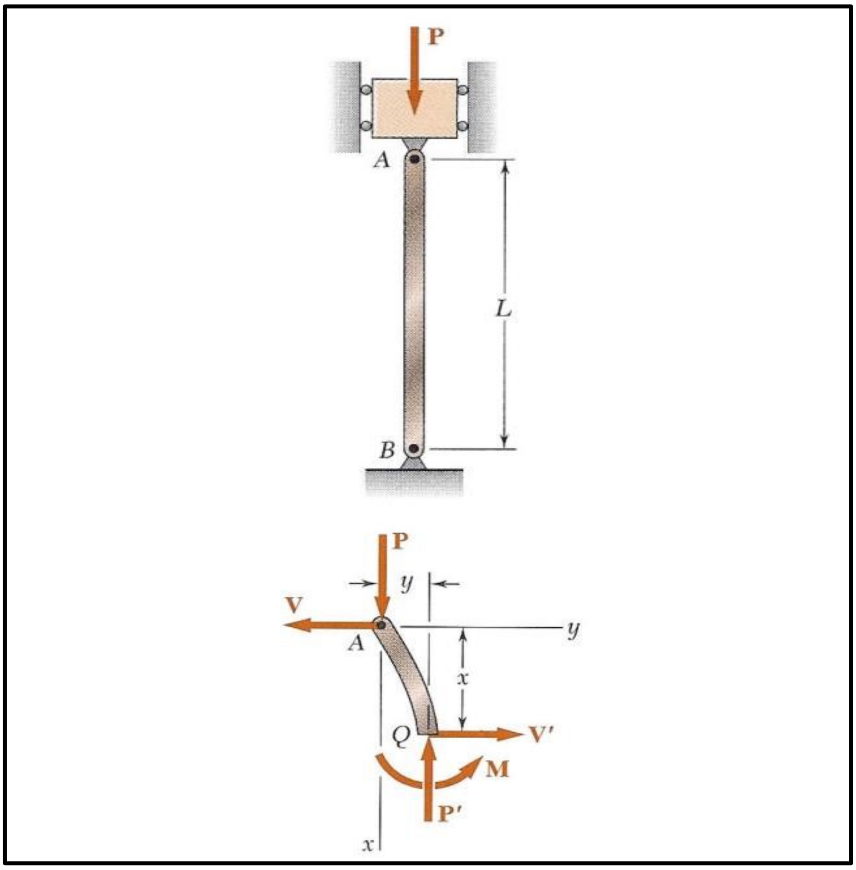


Figure 4: The undeformed column (top) and free-body diagram of deformed column (bottom)

$$\frac{d^2y}{dx^2}+k^2y=-\frac{v}{EI} \tag{4}$$

$$k^2=\frac{P}{EI} \tag{5}$$

where

P is the compression force acting through the column centreline
E is the column material and geometry properties
I is the moment of inertia of the column
y is the lateral displacement of column due to buckling
x is the coordinate system along the column's length in the horizontal axis
v is the induced horizontal force acting on the column

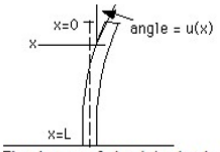
Table 1: Interpretation of the boundary conditions at 1 and 2 of CB problem

Boundary condition	u_3	$\frac{du_3}{dx_1}$	$\frac{d^2u_3}{dx_1^2}$	$\frac{d^3u_3}{dx_1^3}$
Pinned ended	0	-	0	-
Fixed ended	0	0	-	-
Free ended	-	-	0	0

Note: - = Not relevant

The final phase was intentionally designed to enable participants to understand, interpret, and predict the answer to the given CB problem, as shown in Figure 5. Both groups of participants were introduced to the idea of truncating the symbolic solution DE provided by Maple to a particular polynomial order to estimate the critical buckling length of the free-fixed ended support. The symbolic solution of a non-linear CB equation was also introduced as part of the activity in this stage. After obtaining the truncated solution, they were allowed to modify the appropriate parameters of the problem, i.e. change the length of the column, the type of materials, and the cross-sectional area of the column. Participants will test the problem and understand the limitations and validate their expectations or predictions of the column structure before constructing it in the real world. All these activities were guided based on a predefined Maple template to meet the needs of the research objectives as shown in Figure 6.

Consider a vertical column which consists of a circular steel rod of radius r and length *L*. The column is rigidly fixed at the base and is free at the top. As the length of the column is increased, a critical length is reached at which it buckles under its own weight. Our goal is to find the critical length of the column that first leads to buckling. Referring to the figure below, let *u(x)* be the angular deflection so that *u(x)=0* indicates no buckling. Buckling of the column will occur for the first value of *L* (as *L* increases from *0*) that leads to a nonzero solution for *u(x)*. The x -axis is positioned that *x=0* is the free end and *x=L* is the fixed bottom.



The theory of elasticity leads to the following equation (with boundary conditions) for the deflection:

- *u''(x) + a^2 x u(x) = 0*
- *u'(0)=0*
- *u(L)=0*

Figure 5 : Statement of the CB problem to be solved by the students in the final stage

```
Order := 8

E := 200e6;
b := 0.3;
h := 0.4;                         } User's input

Iner := b·h^3 / 12

g := 9.81;
rho := 7860·g / 1000;             } User's input
A := b·h;
a := g·rho·A / (E·Iner)
eqn := diff(u(x), x$2) + a^2*x*u(x) = 0      ➡ The CB equation

soln3 := dsolve({eqn, u(0) = 1, D(u)(0) = 0}, u(x), type = series);
                                             ➘ The truncated
assign(soln3);                                 symbolic solution
                                               of CB equation
poly3 := convert(u(x), polynom);

plot(poly3, x = 0 ..4)  ➡ Visualize the CB solution
```

Figure 6 : Maple template to guide both participants in the fourth stage of study

Results and Discussions

Participants, instructors, and the CAS software are the three types of items that will be analysed and discussed in this section. The way these components interact shows how important they are to the CB teaching and learning process. In the first stage, it was discovered that the weak participants lacked a fundamental comprehension of the calculus of integration and the solution of DE. This set of students struggled to recall what they had already learned about integration and differentiation. Another trait that may be highlighted for these seven weak participants is that, although trying to solve the provided DE step-by-step, they did so without having the necessary conceptual procedural comprehension. However, in the third stage, it was discovered that these weak participants' approach changed from a directed method to a non-directed one as a result of the learning process of utilising Maple to solve DE. According to their responses during the conducted interview, most of these weak participants believed that the Maple programme was a helpful tool for them because the DE solution entails several time-consuming processes before arriving at the final solution. They feel that utilising Maple considerably lessened the problem of providing lengthy computing processes and gave them a better knowledge of several key principles behind DE solutions. Legarde [13] and Yusof et al. [14] found that although it has helped the weak participants to shorten the time spent by them, the instructors must ensure that any misconception on DE learning concept be avoided. The provision of graphical representation of the DE solution as shown in Figure 7, also reinforced the weak participants on visualizing the solution and enjoying their learning process. This shows that special attention, efforts, and designed learning strategies are required to solve problems containing DE, particularly for weak civil engineering students, as observed by Bibi et al. [15].

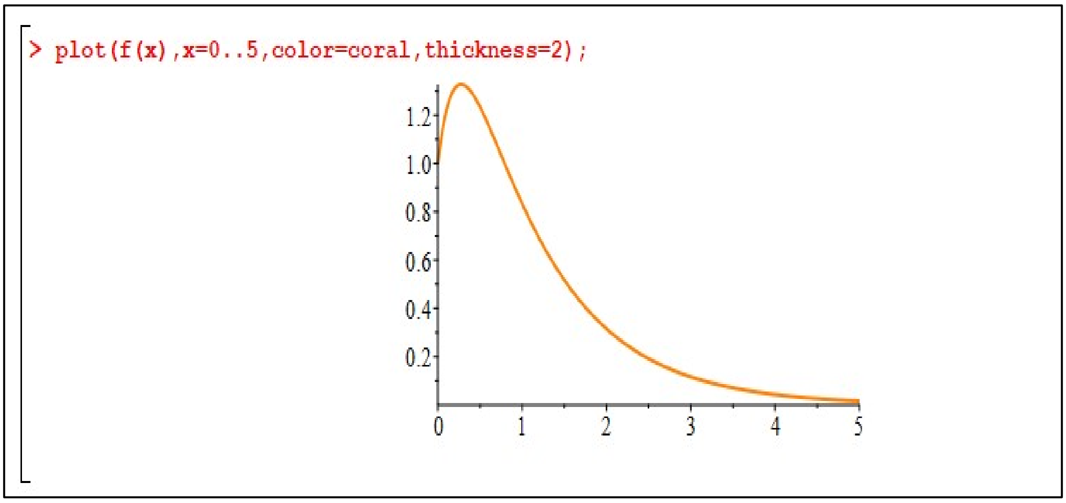


Figure 7: The graphical plot of the DE solution of the first stage research

Based on the feedback from the very good participants, it was expected that the other seven participants would show excellent performance in solving the given DE. They showed good mastery of correctly solving DE compared to the weak participants without using the Maple template. Solving DE correctly involves understanding the problems and planning the solutions using the correct calculus and algebra concept, which was clearly shown by the hand calculation solutions. Some of them were able to perform the verification steps to confirm that their solution is feasible for the given problem by back testing the solution into the given main task DE. Weak students were categorised as the group who manipulate Maple to access the correct solution without going through the traditional process of writing complete and long arithmetic sentences as the good students do. This opinion is sensitive when expressed directly to weak students, as it destroys their motivation and dignity in mastering CB. Therefore, while exploring the advantage of Maple in performance improvement between these two extreme groups, it is advisable to control these disagreements or misunderstandings about the use of Maple and preferably conduct the learning in this session separately between them.

In the final stage of the study, both groups of participants started the experiment by themselves in solving the CB problem of a rectangular steel column with a dimension of 300mm (breadth) x 400mm (width) x 4000mm (height). Before making use of the pre-defined Maple template sheet, each group managed to understand the briefing from the instructors given prior the problem solving. The graphical representation created in Figure 8 by executing the command shown in Figure 7 appears to improve the understanding of the topic CB for each of the participant group. The individual participants benefit from the DE solving skills through mastery of the Maple symbolic programming language as they acquire the skills sought by employers in the mathematics industry. One comment gathered from the participants during this stage is shown below:

"*One of my major disappointments with mechanics in this topic was when I would complete a problem, glance to the back of the book, and discover that I had given the incorrect answer. I couldn't figure out why. All I knew was that my response was incorrect, but I had no idea why or what I could do about it. I felt like I was hitting my head against a brick wall at times. I could have utilised Maple to doublecheck my steps and find out where I went wrong if I had it. Instead of becoming really upset, I could have used a tool to assist me in figuring things out*".

The tools provided at this stage, which allow participants to check their CB solutions and save time on difficult DE problems, appear to be well-liked by the participants. Plotting a graphical representation of the CB allows for both the validation and interpretation of answers as well as a comprehensive comprehension of mathematical concepts.

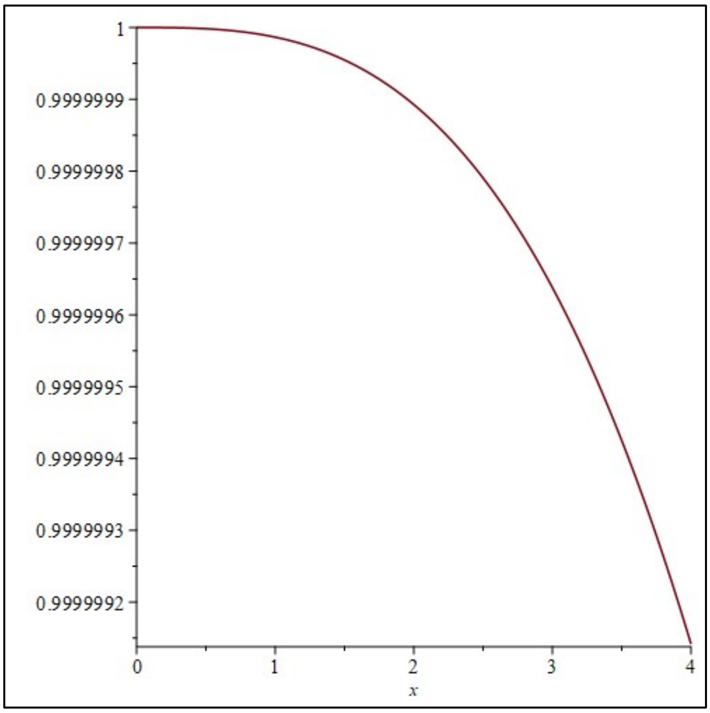


Figure 8: The graphical plot of the DE solution of the fourth stage

Conclusions

The solution of column buckling problems and their interpretation after solving DE is very important for civil engineering students. In this paper, Maple software was explored to promote structural mechanics concepts and understanding for civil engineering students specifically for the CB problem. Since the usage of Maple with a rather limited number of learners and with almost no programming skills among the learners, it reduced the efforts required to solve CB in terms of equations and graphical representation many times over. During the fourth phase of the study, the participants also showed great pleasure and high positive expectations of using this tool by trying the CB problem using different parameters. Further plans for this CAS software are also proposed for next year.

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