Islands of Superficial Knowledge without a Canoe to get from One End to the Other: The Nature of College Mathematics

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ABSTRACT

Successful mathematics students do indeed construct a fairly large number and variety of algorithms in order to continue to achieve good results in college mathematics courses. However, what is the quality of this mathematical knowledge 'accumulated' from the courses taken? How well do the college examination grades in mathematics courses reflect the student's mathematical thinking? Utilizing both quantitative and qualitative approaches, the findings indicate that the sixty-four college students involved in this study have learnt how to do numerical computation at the expense of learning how to think mathematically. The findings poignantly revealed that the accumulation of mathematics courses taken and the grades obtained in their end of semester examination do not correlate with their mathematical knowledge. The clinical interviews findings indicate that these students have an instrumental understanding rather than a relational understanding due to their emphasis on procedure rather than the process of learning. They ignore things like context, structure and situation as being crucial for constructing mathematical knowledge. Unfortunately, the development of mathematical thinking in their learning is overshadowed by an instructional focus on decontextualized content and the imparting of facts necessary to pass end semester examinations. They end up with islands of superficial knowledge without a canoe to get from one end to the other.

Keywords: algorithms, college mathematics, mathematical thinking, teaching and learning

INTRODUCTION

What is the nature of college mathematics? Before answering this question as the foundation of the paper, we will briefly discuss the evolution of mathematics teaching and learning in colleges over the last few decades. The evolution of mathematics learning over the past few decades has been dramatic, where in the 90s, the focus had been on doing computation and applying procedures in solving a problem. Then in the later stages of the 19^{th} century and early 20^{th} century, the conception of mathematics learning tilted from computation emphasis to understanding abstract concepts and relationship. This was a shift in teaching with emphasis from *doing* to *understanding* (Devlin, 2014). This era led to the emergence of mathematics as the science of patterns. According to Resnik (1999), the nature of mathematics is the espionage of a system of ideas using factual subject matter towards the existing of reality. Devlin (2003) elucidated that the Science of Patterns explores the many ways mathematics helps us in understanding the perception of reality--both the physical, biological, and social worlds without, and the realm of ideas and thoughts within. According to him, this development yields the rapid growth of computing and applications have helped to cross-fertilize the mathematical sciences, yielding an unprecedented abundance of new methods, theories, and models. No longer just the study of number and space, mathematical science has become the science of patterns, with theory built on relations among patterns and on applications derived from the fit between pattern and observation.

This shift was from heavily relying on formulas to solve problems towards the teaching on *what* and *why* on the conceptualization of the problem given. Then in the early part of the current century, mathematical thinking has been the focus of attention. According to Ridgway (2001), "*thinking mathematically is about developing habits of mind that are always there when you need them - not in a book you can look up later (p. 1).* It is a pre-built thinking of mathematical thinking in the mind of an individual when solving problems. Mason and Johnston-Wilder (2004) provide a detailed list of words they believe denote processes and actions that mathematicians employ when they pose and tackle mathematical problems: exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing, refuting" (p.109).

They propose that questions function to enable students to draw on these words in allowing them to experience aspects of mathematical thinking. While Mason and Johnston detail the words used, Stacey (2007) on the other hand emphasizes the processes applied in developing mathematical thinking. The two pairs of processes through which mathematical thinking very often proceeds: *Specialising and Generalising , Conjecturing and Convincing.* We can surmise to a certain extent that mathematical thinking is something that is cumbersome to definition but researchers in general agree that the important characteristics must include: conjecturing, reasoning and proving, abstraction, generalization and specialization (Breen & O'Shea, 2010). This is a way of thinking that permits the building up of concepts, processes and applications, exploring and unravelling problems, making and examining inferences and communicating complex ideas to the world precisely and concisely. Therefore, the ability to be able to think mathematically has substituted the process of memorising a set of procedures and the solution of routine-based problems as the focus of mathematical learning.

Now, we revert to the context of this paper on the nature of college mathematics, especially in the Malaysian setting. Has this evolution of mathematics learning and teaching affected the settings in the Malaysian paradigm? Much of the issues on the low standard of college student's mathematics performance have been barged on the foundation laid by school mathematics. This is more prevalent in the participation of Malaysian schools in the international studies of Trends in Mathematics and Science Studies (TIMSS) and PISA. Both TIMSS (TIMSS 2003, 2007, 2011) and PISA results have revealed dismal performance by Malaysian School students in mathematics over the last decade. This scenario has been debated by both researchers and mass media (Asadullah, 2014; Ghagar, Othman & Mohammadpour , 2011; Noor Azina & Halimah Awang, 2009; Mohammadpour, Moradi, & Najib ; 2009; Fensham , 2007; Malay Mail Online, 2013; Bernama, 2013) on the decry standards of mathematics in Malaysian schools over the years. While it can be argued that TIMSS and PISA rankings are never going to be "accurate" in any study, it's fair to say that it does provide some indications to the quality of our education system. For example, there may not really be any difference between a country ranked 11th and 15th, but there's likely to be a significant gap between the countries placed 15^{th} vs 35th vs 56^{th} . These ranking results have sent a ripple of anxiety among researchers and educators which provoked perturbation. Inadvertently, it has to a large extent, brought education reform movements in school mathematics by the education ministry, especially with the Malaysian Education Blue Print (2013). However, the question to ponder upon is " what about the nature of college students learning of mathematics?" How is student performance being measured?

With the paradigm shifts of the nature of mathematics that have taken place as stated earlier, did the philosophical stance at the college level move in tandem with this shiftt? The fundamental Mathematics courses taught in colleges today for students (major and non-major requirement) include Calculus, Algebra (modern and linear), Number Theory, Topology, Logic, Geometry, probability etc. These courses have been taught and re-taught over the years by instructors and students are getting good grades in their transcripts based on the number of students graduating with honours (Rojstaczer & Healy, 2012; Rampell, 2011; Kristina , 2011;Parmjit, 2009; Parmjit & White, 2006; Ridhancock, 2003). However, these grades in their transcripts are not being translated into the development of their mathematical thinking (Devlin, 2003; Parmjit, 2009; Liu & Niess, 2006). The alarming pattern where faculties aspire in the development of students' thinking skills were documented by Gardiner (1998). However, research proves otherwise as when it comes to practice we focus more on facts and concepts within the disciplines and also at the lowest cognitive levels in comparison to developing intellect and values. How has this debilitating perspective of mathematics as a collection of arcane procedures and rules become so prevalent among our students at college?

These students' low level of mathematical thinking seem to indicate that the notion of college mathematics is based almost exclusively on formal mathematical procedures and concepts that, of their nature, are very remote from the conceptual world of the students who are to learn them. It seems to indicate that instructors are diligently teaching mathematics but students are not learning! These findings have been documented two decades ago and are still prevalent in today's college classroom. Successful mathematics students do indeed construct a fairly large number and variety of algorithms in order to continue to achieve good results in college mathematics courses. However, what is the quality of this mathematical knowledge 'accumulated' from the courses taken?

OBJECTIVES OF THE STUDY

This study was undertaken to investigate college students, who have taken mathematics courses (e.g. calculus courses, mathematics foundation and logic courses), repertoire of cognitive strategies in solving problems that were within the zone of potential constructions. It aims to develop a comprehensive description of college freshman's thinking and reasoning capabilities in solving these problems. Students who are considered successful are able to construct a variety of algorithms in line with achieving excellent results in college mathematical courses. Nevertheless, what the quality of the mathematical knowledge that is accumulated from the mathematical courses? Apart, from that, it is also aims to investigate how examination grades obtained from mathematical courses reflect the mathematical thinking of students.

Specifically, the questions addressed are:

1. What are the levels of college students thinking in solving non-routine problems?

2. What difficulties do college students encounter in solving non-routine problems?

METHODOLOGY

The methodology used in this study was both qualitative and quantitative in nature. Qualitative data from clinical interviews with selected students gave the researchers an in-depth understanding of these students' heuristic actions, exploration of the mathematical processes, and tacit mathematical understanding that constitute thinking mathematically in problem solving. The written assessment provided both quantitative and qualitative data about these students' relational understanding of their application of mathematical concepts in problem solving. A total of 64 students from from three different classes (semester 3 and semester 4) majoring in Physics, Chemistry and Biology were involved in the study. All these students have taken Logic and Foundation, Calculus 1 and Calculus 2 courses in Year 1 and Year 2 of their college settings. From these students performance in the written assessment, nine were selected (three within the high achievers, three within average achievers and three within the low achievers) for the interviews to elicit information on their cognitive processes in solving problems. The academic demographic data regarding their mathematics acumen is shown in Table 1. (To be noted that the grades for the Mathematical Logic and Proving Techniques course was not available for this paper).

Demographics of Students Acumen in Mathematics

Table 1 depicts the mathematics grade obtained by students in their SPM examination and current college settings. It depicts that 96.9% (n =62) and 23.5% (n=15) of the students obtained an A grade (A+, A and A-) in the subject Modern Mathematics and Additional Mathematics respectively during their SPM examination. Whereas for the current college settings, 29.7% (n = 19) and 4.7% (n=3) of the students obtained A grade in their Calculus 1 and Calculus 2 respectively. More than two thirds of the students obtained at least a grade B in all four subjects except for Calculus 2. This demographic academic acumen results indicates that these students have an accepted level of mathematics expectancy based on the grades achieved. From this group, 71.9% (n=46) and 28.1% (n=18) entered this program via matriculation and dipoloma qualifications.

There were all together 5 items in the test and the responses were grouped based on categories in accordance to the criterion behavior exhibited in which a numerical value was assigned. Students' responses were categorized based on the reasoning employed on a 4 point scale and was than computed as correct (0 to 2) and incorrect responses (3 to 4).The 4 point scale used was :

4. all correct, good reasoning
3. good reasoning, small error(s)
2. some promising work but it is not clear a solution would be reached
1. some work but unlikely to lead to a solution
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Table 1: Distribution of Grades Obtained in SPM (Modern Math and Add Math) and College (Calculus 1 and Calculus 2)

Grade	SPM Math Grade	%	SPM Add Math Grade	%	Calculus 1	%	Calculus 2	%
A+	13	20.3	1	1.6	1	1.6	0	0
A	43	67.2	4	6.3	9	14.1	1	1.6
A-	6	9.4	10	15.6	9	14.1	2	3.1
B+	2	3.1	18	28.1	12	18.8	4	6.3
B	-	-	6	9.4	9	14.1	9	14.1
B-	-	-	6	9.4	3	4.7	10	15.6
C+	-	-	7	10.9	8	12.5	14	21.9
C	-	-	11	17.2	10	15.6	17	26.6
C-	-	-	1	1.6	3	4.7	5	7.8
D	-	-	-	-	-	-	2	3.1
Total	64	100.0	64	100	64	100	64	100
Minimum Grade B		**100%**		**70.3%**		**67.2%**		**40.6%**

FINDINGS OF THE STUDY

Table 2 shows the correct and incorrect responses for each of the five written test items attempted in the written assessment. The data can be an important indication of college student's fundamental relational thinking of mathematical concepts in solving problems.

Table 2: Item Analysis of Written Assessment Test

Item	Frequency	Percentage Correct	Incorrect
1	9	14.1	85.9
2	5	7.8	92.2
3	1	1.6	98.4
4	7	10.9	89.1
5	10	15.6	84.4

From Table 2, it can be deduced that students faced great difficulties in all the problems as the incorrect responses for all the problems are more than 85%. The highest level of difficulty is for item 3 (98.4%), followed by item 2 (92.2%), item 4 (89.1%), item1 (85.9%) and item 5 (84.4%).

Some of the verbatim during the interviews were translated to English as the students faced difficulty in corresponding in English language.

Question 1

Eva and Alex want to paint the door of their garage. They first mix 2 cans of white paint and 3 cans of black paint to get a particular shade of grey. They add one more can of each. Will the new shade of gray be lighter, darker or they are the same?

Approximately 85.9 % of the students got this item wrong with approximately 59.4% (n = 38) reasoning it as the same. The data from the interview depicted their thinking in solving the given problem:

This group of students used primitive additive reasoning. The reasoning employed is that if an equal number of cans for each type of paint are added to the mixture, the shade will remain the same. They were unable to see the proportion of white paint to the black paint before and after the addition of two cans of paint.

S_{F3} : *In my opinion, it is the same if you add one can of white paint and one can of black paint as the differences are the same. If we intend the outcome to be lighter, we should put in more white paint and if we want to have a darker effect, we put in more black paint.*
(Pada pendapat saya, sama, jika ditambah satu tin cat putih dan satu tin cat hitam,kerana bezanya sama. Jika ingin mendapatkan yang lebih terang, kita akan menambahkan lebih banyak cat putih dari cat hitam dan jika ingin mendapatkan yang lebih gelap, kita akan tambah lebih banyak cat hitam dari cat putih.)

R : *Therefore, you believe that if you add another can of white paint and another can of black paint, the color will be...?*

(Jadi anda berpendapat kalau ditambah satu tin cat putih dan satu tin cat hitam, warna kelabuhnya adalah)

S_{F3} : *Same*
(Sama)

The reasoning employed is that if an equal number of cans for each type of paint are added to the mixture, the shade will remain the same. They were unable to see the proportion of white paint to the black paint before and after the addition of two cans of paint. Approximately 45.3 % of the students gave this additive reasoning. In short, these students failed to construct a coordination of two ratios simultaneously as: 2 white to 3 black and 3 white to 4 black. Their thinking was based on the primitive additive reasoning and not the expected multiplicative thinking.

Question 2

If it takes 9 workers 5 hours to mow a certain lawn, how long would it take 6 workers to mow the same lawn? (Assume that the workers are all performing at the same rate and all working for the entire time.)

In this item, 92.2 % of the students obtain an incorrect answer with the majority failing to see an inverse proportion relationship. They algorithmically solved the question by utilizing a cross multiplicative structure:

9 workers = 5 hours
6 workers = x
x/5 = 6/9; 9x = 30; x = 30/9 = 3 1/3 hours.

Here, they did not reason the representation of each number and what they were actually computing. Logically, they should have realized that the answer they produced (3 1/3 hours) implied that more people take a longer time to finish up the job!

An interview with a student who used a similar method revealed that he was aware that less workers means longer working hours but was unable to answer why his cross multiplication strategy did not give a logical solution.

R : *What's your answer?*
(Apa jawapan kamu?)

S_{M6} : *3 hours and 33 minutes*
(3 jam 33 minit)

R : *Is your answer logical?*
(Adakah jawapan kamu logic)?

S_{M6} : *(Silence)*

R : *What is it that you are thinking?*
(Apakah yang kamu sedang berfikir?)

S_{M6} : *It should've been that 9 people with 5 hours. So, if 6, it must be more.*
(Sepatutnya 9 orang 5 jam. Kalau 6 orang mesti lebih lagi.)

R : *What must be more?*
(Apa yang lebih?)

S_{M6} : *Time*
(Masa)

R : *In your opinion, where is it that you went wrong?*
(Pada pendapat kamu, mana kesilapannya?)

S_{M6} : *Well, my working steps were correct (showing his steps, utilizing the cross multiplication method, in his worksheet)...I'm uncertain.*
(Jalan kerja saya betul) ... Saya tak pasti.)

An interview with another student yielded a similar response.

R : *You wrote 5 hours multiply by 60 minutes, then divide by 9 (From his worksheet). Why?*
(Awak tulis 5 jam darab 60 minit, bahagi 9 (From his worksheet). Kenapa?)

S_{F5} : *In order to find out how many minutes a person works. After that, we multiply by 6.*
(Untuk dapat setiap orang berapa minit dia kerja. Lepas itu darab 6.)

R : *So, how long does it take for 6 people to finish the job?.*
(Jadi, 6 orang buat kerja berapa lama?)

S_{F5} : *3.33 hours*
(3.33 jam)

R : *Is your answer logical?*
(logik tak jawapan kamu?)

S_{F5} : *Yes*
(Ya)

However, this student who used proportional reasoning by cross multiplication, unitization multiplication by 6 to get the total numbers of hours work by 6 workers, did not realize that his answer was not logical. Majority of the students failed to see an inverse proportion relationship embedded in this question.

Question 3

Rahman takes 20 hours to paint a house, whereas Ah Beng takes 30 hours. How long will it take for them to paint a house if they work together?

Surprisingly, 98.4% of students obtained an incorrect answer for this question. Only one student obtained a correct response for this question.

From all the answer scripts, majority of them were just not able to get started to solve the problem.

During one of the interviews:

R : What is your answer?

S_{F1} : This question is tricky.

R : Why do you say that?

S_{F1} : (silence)

R : What are you thinking?

S_{F1}: Well most of the time, in this type of question….. the question will be like if Rahman takes 20 hours to paint a house, how long will it take to paint three or four house... something like that.

R : What about this question?

S_{F1} : Here they ask if we combine both of them
Silence for about 4-5 minutes

S_{F1}: I am not sure

Problems which involve proportions are traditionally categorized into two types: missing-value problems and comparison problems. In missing value problems, one needs to determine the unknown x in a/b = c/x where a, b and c are given. A comparison problem, on the other hand, compares a/b and c/d where a, b, c and d are given. These students faced great difficulty in this type of problem which relates to comparison of two ratios. They were just not to apply their thinking from missing value problems to comparison problems.

Question 4

An old antique bicycle has wheels of unequal size. The front wheel has a circumference of 8 feet. The back wheel has a circumference of 10 feet. How far has the bicycle gone when the front wheel has turned 20 more revolutions than the back wheel?

For this problem, 89.1% of the students obtained an incorrect response. They faced great difficulties in expressing the problem into a mathematical expression. From the interviews it revealed that the majority did not realize that both tyres travelled the same distance. Some of the heuristics by students who got it correct was:

To travel a certain distance, the front wheel has made 5 revolutions, while the back has made 4. Therefore, the ratio is 5 : 4, and the difference is 1 revolution. So, to get a difference of 20, multiply ten on each side to get 100 : 80. This shows that the front has made 100 revolutions. Hence, the wheel has travelled 100 x 8 = 800 feet.

Question 5

A dog chasing a rabbit, which has a start of 45m, jumps 3m every time the rabbit jumps 2m. In how many leaps does the dog overtake the rabbit?

In this item, merely 15.6% of the students were able to give correct response. This is an algebraic task and some students were observed using interesting heuristic action to solve the problem such as:

> *Difference in the distance of every leap is 1 m. To cover the difference of 45 m, it requires 45 leaps. Therefore, the dog will overtake the rabbit in the 46th leap.*

Majority of them saw the problem as a difference of 1 meter between each jump of the rabbit and dog. Then they classified the problem as sequences:

> *2, 4, 6, 8, ... as the sequence for the distance travelled by the rabbit and*
> *3, 6, 9, 12, ... as the sequence for the distance traveled by the dog.*

These heuristics were correct but unfortunately they got the wrong answer because they did not take into consideration that the rabbit was already 45m ahead of the dog. Another variation of these heuristics is exhibited by a student that was being interviewed.

R : *You said that it's 45 divided by 3, you get 15. Why 45 divided by 3?*
(Awak kata 45 bahagi 3, awak dapat 15. Kenapa 45 bahagi 3?)

S_{M1} : *The dog can jump for 3 meters, the rabbit can jump for 2 meters. For the dog to outjump the rabbit, it will be 45 meters divide by 3 meters. So, the dog gets 15 jumps and in those 15 jumps, the dog can outjump the rabbit.*
(Anjing boleh lompat 3 meter, arnab boleh lompat 2 meter. Untuk anjing memintas arnab, jadi 45 meter bahagi 3 meter. Anjing dapat 15 lompatan. Jadi dalam 15 lompatan tu, anjing boleh memintas arnab).

He saw that the rabbit is 45 metres in front of the dog, therefore, in 15 leaps the dog will overcome the rabbit. This student was not aware of the extra information in the problem, that is, when the dog leaps, the rabbit also leaps.

Surprisingly, the students faced difficulty in expressing the problem into a mathematical algebraic equation of $45 + 2(x) = 3x$ where x is the number of jumps. None of the students apply this algebraic equation to solve the problems, as one will expect from college students.

DISCUSSION

The goal of the written assessment test for this study was to draw on the habits of thinking developed by college students over the various exposures to mathematics courses in their studies rather than on specific procedures they had learnt earlier. There were 64 subjects in this study where they had taken various mathematics courses since high school such as Modern Mathematics, Additional Mathematics, Mathematical Logic and Proving Techniques, Calculus 1 and Calculus 2. With the acumen of their mathematic achievement as shown in Table 1, it was expected that these students, would have a good grasp in the understanding of fundamental mathematical concepts. However, the result of this study shows that they have learnt how to do numerical computation at the expense of learning how to think and solve problems.

The data for these problems seem to show that for most students in this study, college mathematics instruction was procedural without sense-making: one learns to read the problem, extract the relevant numbers and the operation to be used, to perform the operation, and to write down the result—without even thinking about what it all means. This was depicted in *Question 2* where a majority used an algorithmic cross multiplication approach and could not apply logical thinking where more people would take lesser time to finish the job! Utilizing this algorithmic approach in solving problems simply becomes an act of symbolic manipulation without requiring that an individual makes sense of what they are doing. Students can develop structural understanding if given experiences that create a solid foundation for these concepts (Kieran, 1992).

The grades and their exposure to mathematics courses in High school (Modern Mathematics and Additional Mathematics) and College (Mathematical Logic and Proving Techniques, Calculus 1 and Calculus 2) does not indicate their mathematical thinking prowess. Evidence from these findings clearly show the "mathematical deficiency" when they ar ein college. There are many reasons for this. In some instances, students have not had the opportunity to learn important mathematics. In other instances, the curriculum offered to students does not engage them to think. Sometimes students lack a commitment to learning. The quality of mathematics teaching is highly variable.

There will be people reading this paper who might question the lack of statistical depth in the analysis used. Many thoughtful people might be critical of this but as one reads the tables of statistical data, one will ask "so what!". We strongly feel that that vital questions go unanswered while means, standard deviation, t-tests, and regression analysis pile up. There is too much reliance on statistics nowadays, and a deep look at processes is avoided. Statistics are valuable in their place and they can suggest hypotheses in preliminary studies and help to test these in well-designed experimental studies. But, if we want to understand what goes on in anyone's head when they solve problems, we have to watch them solving problems (Schoenfeld, 1992) as we attempted in this study.

There is evidence that these college students have many of the same conceptual and reasoning difficulties that are common among High School students. There seems to be little change in the conceptual understanding before and after formal instructions on college mathematics courses taken. Although we tend to promote critical and analytical thinking, but at the end of the day, the assessment propels the learning rather than the other way round which tends to inhibit the development of both this skills. It is alarming to see that large number of students taking mathematics courses are just not learning but, to put it eloquently, actively suffering. He further noted that 'We spend a lot of time avalanching students with the answers to things that they wouldn't think of asking'(p.3).

CONCLUSION

Mathematics courses in colleges should be re-engineered where the focus of doing mathematics should be inclined towards "teaching students to think". This is in line with Polya's (1973) assertation "How to think" which is a theme that underlies much of genuine inquiry and problem solving in mathematics. One of the major aims of mathematical learning is the development of mathematical thinking and there is a widespread agreement that it should be taught as a thinking activity (Devlin, 2014; Chapman, 2011; Stacey, 2007). Consequently, the emphasis in instruction should be shifted from learning the rules for operations to understanding of mathematical concepts. One possible solution is to encourage the transition by providing students with "problem solving tools" that would permit them to be accommodative to changing needs (Treffinger, 2008). However, care must be taken so that effort to teach students "how to think" in mathematics problem solving do not get transformed into teaching "what to think" or "what to do". This is, in particular, a by-product of an emphasis on procedural knowledge about problem solving as seen in the linear framework of college mathematics settings.

In concluding this paper, we can surmise that the effectiveness of mathematics teaching to develop mathematical thinking in colleges can and should be improved substantially. If we go with the definition of teaching as an interaction process between instructors and students over a content in an environment (Cohen, 2002), it signifies that the current mode of teaching mathematics is not only ineffective but also seriously stunts the growth of students' mathematical thinking and problem-solving skills. We strongly believe that, colleges place over emphasis on the procedures rather than the processes. So, when students "practice" these problems, they are practicing to get the correct answer. In other words, they ignore things like context, structure and situations, and students do not have the occasion to generate the "richly inter connected spaces" that Cooper (1988) has identified as being crucial for constructing mathematical knowledge. They end up with islands of superficial knowledge without a canoe to get from one to the other.

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