

# Optimal Step-Function Approximation of Load Duration Curve Using Evolutionary Programming (EP)

Eda Azuin Othman

**Abstract**— This paper proposes Evolutionary Programming (EP) to determine optimal step-function approximation of load duration curve (LDC) at minimum error. The EP model optimally discretized a load duration curve based on Malaysia's hourly load data in year 2012 for three and six segments of discretized LDC. The EP is developed using MatLab programming software. Results show that EP technique is able to provide optimum break points of discretized LDC at minimum error. In the analysis, it shows that the 6-step functions of LDC has a lower total error than the 3-step functions of LDC. The EP technique proposed in this paper is also compared with Dynamic Programming (DP) technique. Results show that EP provides a much shorter elapsed time than DP and have a lower total error for 3-step function of LDC. This EP-based model step function approximation of LDC is very useful for the power system planner to develop accurate generation expansion planning.

**Keywords**-Evolutionary Programming (EP), Load Duration Curve, Minimization of Error

## I. INTRODUCTION

In electrical utilities, load can be considered as the total electricity used during a given period time, such as an hour. These loads can be plotted for a day, or a week even for a year, and these curves are known as load curves. However, it is a considerable value to rearrange the loads into a cumulative curve with the hour of highest usage plotted that called as Load Duration Curve (LDC). From the LDC plotted, we can get the approximated load generation curve by a step function normally, of three or six steps. For example, the PIES model of the Department of Energy uses a three-step approximation for a LDC extending over a full year which is 8760 hours. This is based on the concepts of base load and peak load electrical generation, with the remainder being intermediate or cycling generation thus forming three classes of generation.

The step function of LDC is usually produced by sketching or in some other ad hoc manner. This approximated discretized LDC is usually used for planning generation expansion. However, because the expected result

of expansion plant is very dependent on the shape of this discretized LDC, it is necessary to use a more rigorous technique to discretize the LDC. Thus, an optimum and rigorous technique to determine a more accurate step function approximation of LDC is developed in this paper using Evolutionary Programming (EP). This new method is tested to get the optimal step-function approximation for Malaysia's LDC in 2012.

### A. Load Duration Curve

LDC analysis looks at the cumulative frequency of historic load data over a specified period. A load duration curve relates load values to the percent of time those values have been met or exceeded. The y-axis represents the load value associated with the time in a year hourly. LDC development typically uses daily average load used, which are sorted from the highest value to the lowest.

The first attempt was proposed by Loney [2] who used Dynamic Programming with six steps of approximation. The LDC considered the  $F$  and  $T$  as the number of hours the three segments are defined by the break points  $t1$  and  $t2$  and the corresponding heights  $g1$ ,  $g2$  and  $g3$ . Since the area under the LDC is equal to the total electrical generation in the period, the area under the step-function approximation should be equal to the area under the LDC for each step. They also introduced a penalty function,  $p(e(x))$ , to solve the optimization problem where  $p(e(x))$  is the penalty to be paid per unit of mismatch at  $x$  and  $e(x)$  is the amount of mismatch at  $x$ . The authors of [1] extended Loney's to widen the application.

The authors of [3] used the same concept as [1,2] to discretized LDC. Since the price duration curve (PDC) is sensitive to the shape of the LDC and calculated according to each segment of the discretized LDC, an optimal approach to discretize the LDC is introduced prior to the investment evaluation model using dynamic programming.

## II. METHODOLOGY

Evolutionary Programming (EP) is a useful method to minimize the error in approximating the step-function of LDC using MatLab software. The objective of EP is to optimize any fitness which can be represented using mathematical equation. The evolutionary programming consists of three types which are classical, adaptive and

Manuscript received 22<sup>th</sup> April 2014.

E. A. Othman is with the Faculty of Electrical Engineering, Universiti Teknologi MARA, 40450 Shah Alam, Selangor. MALAYSIA.

(e-mail: edaazuin91@yahoo.com.my)

Meta. The mutation technique for each type of EP is different.

#### A. EP-based Optimal Step Function of Load Duration Curve (LDC)

Figure 1 shows a three-step approximation of a typical LDC that is used to illustrate the methodology.

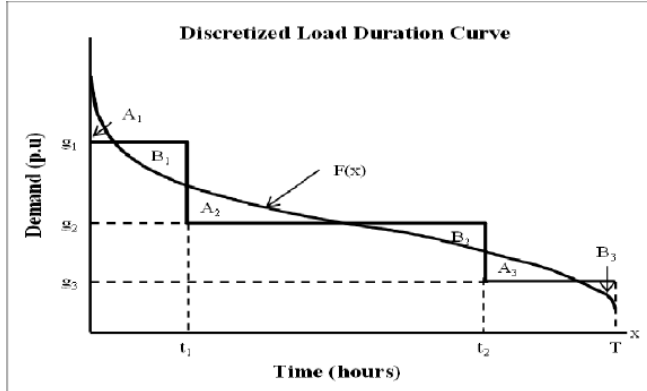


Figure 1: Typical LDC with three step approximations

The LDC is denoted by  $F$  and  $T$  is the number of hours being considered. The three segments are defined by the break points  $t_1$  and  $t_2$  and the corresponding heights  $g_1$ ,  $g_2$  and  $g_3$ . Since the area under the LDC is equal to the total electrical generation in the period, the area under the step-function approximation should be equal to the area under the LDC for each step. Each  $g_i$  can be expressed mathematically as a function of  $t_1$  and  $t_2$  as follows;

$$g_1 = \frac{1}{t_1} \int_0^{t_1} F(x) dx \quad (1)$$

$$g_2 = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} F(x) dx \quad (2)$$

$$g_3 = \frac{1}{T - t_2} \int_{t_2}^T F(x) dx \quad (3)$$

In Figure 1, area  $A_1$  above the first segment and under the LDC can be interpreted as representing a deficit of electrical generation and the area  $B_1$  above the LDC but below the first segment as representing an excess of generation. Areas  $A_2$ ,  $B_2$ ,  $A_3$  and  $B_3$  can be interpreted in the same way.

The optimization problem is solved by minimizing the amount of mismatch  $e(x)$  i.e. the error between the discretized LDC and actual LDC, where  $e(x)$  can be expressed as  $|F(x) - g(x)|$ . The goal of this optimization problem is to find the value of  $t_1$  and  $t_2$  in such a way that

the total mismatch is minimized. This problem can be solved using EP where the amount of mismatch to be minimized is the fitness value and the random  $x$  values is the break points of the optimum discretized LDC.

The simulations were carried out for a three and six steps approximation of an LDC. The hourly load data is from the Malaysia's LDC for the load from 1st January 2012 to 31st December 2012 with 8784 hours.

Flowchart in Figure 2 shows the steps taken in determining the break points  $x$  for the optimum step function approximation of LDC using EP optimization technique.

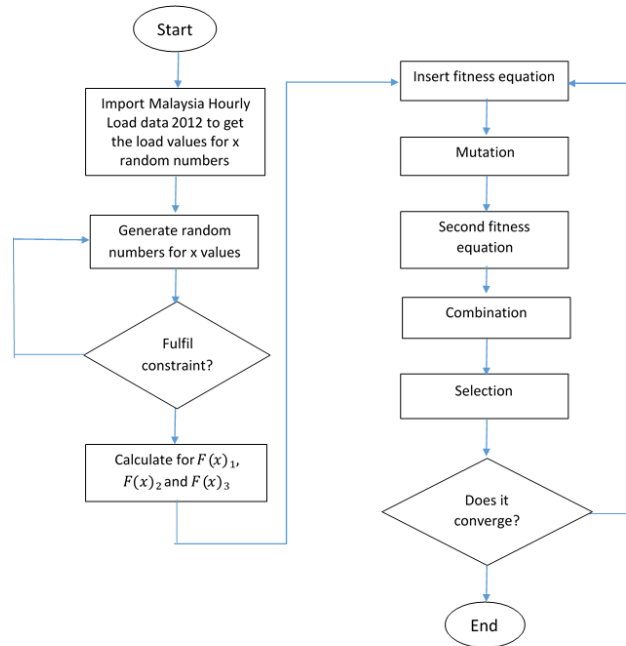


Figure 2: Flowchart of Evolutionary Programming

#### B. Evolutionary Programming (EP)

Evolutionary Programming (EP) is one of the methods that can be used in optimizing the fitness which normally represented in mathematical equations. The evolutionary programming (EP) is a method for simulating evolution and it is similar to evolutionary strategy (ES). In EP, selection is performed using comparison of randomly chosen set of other individuals whereas ES typically uses deterministic selection in which individuals are purged from the population. It is similar to a genetic algorithm, but models only the behavioral linkage between parents and their offspring rather than see the king to emulate specific genetic operators for nature such as the encoding of behavior in a genome and recombination by genetic crossover.

The fitness can either be maximized or minimized depending on the desired output needed. In this paper, the objective function is to minimize the error,  $e(x)$  between the discretized LDC and actual LDC. Below are the steps of EP method based on the pseudo code in MatLab programming;

*i. Initialization*

Initialization is functioning to generate the random numbers. These random numbers are basically the controlled variables in objective function equation. In this EP-based 3-step functions approximation of LDC, the controlled variables are  $x_1$ ,  $x_2$ , and  $x_3$ , where represents the break points of optimum discretized LDC i.e. hours in data from 8760 hours per year. The constraints or the limit range of each variable are set in this phase. The command used to generate random numbers is as follows:

$$Xi = random(x, y) \times (A + B) \quad (4)$$

where:  $x$  : no of row  
 $y$  : no of column  
 $A$  : the offset  
 $B$  : the minimum number

In this step, an initial twenty populations of trial solutions are chosen at random. The populations are generated to meet the constraint set, but no definite answers are available as to how many solutions are appropriate (other than >1). While the random numbers generated does not complies the requirement, the program will keep running until it meets a number that fulfill the constraints. The sets of accepted numbers generated will form a population which will be used later in other steps ahead. In this paper for a 3-steps function approximation of LDC, the generated random numbers are  $x_1$ ,  $x_2$ , and  $x_3$ , where basically these numbers are consider as the parents.

*ii. Fitness*

Next step is fitness which acts as a function or equation to be optimized, it can be a single mathematical equation or a set of sub-program or subroutine. There have two types of fitness which are fitness 1 and fitness 2, but the fitness 2 is calculated after the mutation. Fitness equation can be either a single mathematical equation or a set of sub-program. In this study, the fitness is to minimize the error of discretized load duration curve.

*iii. Mutation*

The mutation function is to generate offspring or children and normally, it use Gaussian Mutation Technique. In mutation process, offspring is produced from the parent

generated in initialization step. There are various obtainable techniques that can be used to carry out the mutation process. The basic Gaussian's formula is shown below:

$$x_{i+m,j} = x_{i,j} + N[0, \beta(x_{jmax} - x_{jmin}) \left(\frac{f_i}{f_{max}}\right)] \quad (5)$$

where:  $x_{i+m,j}$  : offspring  
 $x_{i,j}$  : parents  
 $\beta$  : search step  
 $x_{jmax}$  : max parents  
 $x_{jmin}$  : minparents  
 $f_{max}$  : maxfitness

*iv. Combination*

After the new offspring has been produced, the combination process which combine the parents and offspring in series (by rows) and number of rows will be doubled.

$$Combination = \begin{matrix} parents \\ offsprings \end{matrix} \quad 2mxn = \begin{matrix} mxn \\ mxn \end{matrix} \quad (6)$$

*v. Selection*

The selection process is needed to select the survival of the fittest. One method is elitism and used in the MatLab syntax. This syntax is for objective function which is to minimize the fitness. In the selection process, the survivors from the combination of parent and offspring are determined. The sets of variables are ranked according to their fitness value; ascending order or descending order. In this study, the fitness value is ranked in an ascending order which is from the minimum value to the maximum value.

*vi. New Generation Definition*

New generation definition displays the new sets of variables from the fitness function that have been optimized.

*vii. Convergence Test*

The last stage for EP method is the convergence test which determine the stopping criterion and define the minimum and maximum fitness. If the convergence test success, the programming will be end. The value of accuracy was set to 0.0001 as shown in the equation below:

$$fitness(maximum) - fitness(minimum) = < 0.0001 \quad (7)$$

III. SIMULATION AND RESULTS

A. Before Optimization (Parents)



Table 4: Total error produced by each population after the optimization process for 6-step functions of LDC

x1	x2	x3	x4	x5	g1	g2	g3	g4
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74
258	3316	4301	5061	6491	15245.86	14003.44	12714.32	12060.74

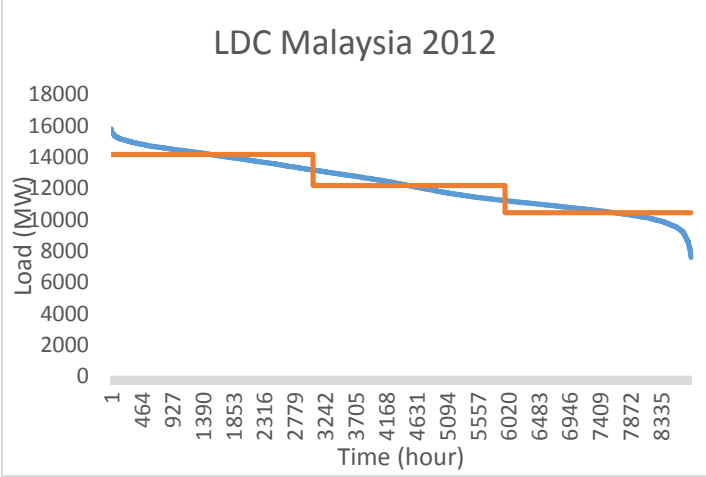


Figure 3: The load duration curve graph for 3-step functions of LDC

g5	g6	y1	y2	y3	y4	y5	y6	Error
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718
11348.91	10322.46	34379.02	1461358	160212.6	116731	260423.3	1003614	3036718

On the other hand, for 6-steps function of LDC, the break points are  $x_1 = 258$  h,  $x_2 = 3,316$  h,  $x_3 = 4,301$  h,  $x_4 = 5,061$  h, and  $x_5 = 6,491$  h with respective load of  $y_1 = 15,245$  MW,  $y_2 = 14,003$  MW,  $y_3 = 12,714$  MW,  $y_4 = 12,060$  MW,  $y_5 = 11,348$  MW and  $y_6 = 10,322$  MW as shown in Figure 4.

C. Optimum Break Points for 3-Step and 6-Step Functions of LDC

The optimum break points for 3-steps and 6-step functions of Malaysia's LDC in year 2012 are shown in Figure 3 and Figure 4 respectively. For 3-steps function of LDC, the break points are  $x_1= 3,055$  h and  $x_2= 5,967$  h with respective load of  $y_1= 14,184$  MW,  $y_2 = 12,195$  MW and  $y_3 = 10,471$  MW.

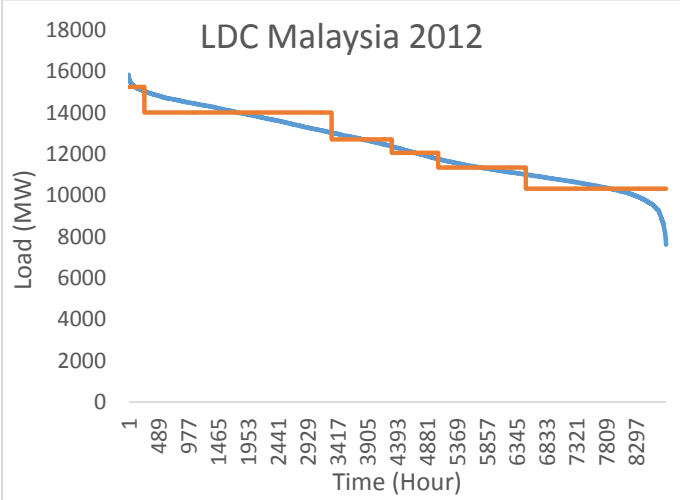


Figure 4: The load duration curve graph for 7-step functions of LDC

D. Comparison Between Evolutionary Programming (EP) and Dynamic Programming (DP)

In this case, the results of discretized LDC using EP is compared with Dynamic Programming (DP) technique as in [3]. Table 7 shows the differences between DP and EP techniques in term of elapsed time, optimum break points and total error. The techniques have been tested using Malaysia's LDC in year 2012.

Results show that, for the 3-step functions of LDC, the optimum break points are comparable. However, EP

provides a lower total error compare to DP. EP also provides a much shorter elapsed time i.e. 29.08s than DP i.e. 766.11 s. On the other hand, for 6-step functions of LDC, DP gives a lower total error compare to EP. However, in term of the elapsed time, EP still shows a much shorter time i.e. 29.07s than DP i.e. 3,463.11s.

Table 5: Comparison between EP and DP

3-segments		DP	EP
	Elapsed time (s)	766.118965	29.084227
	X1	4,398	3,055
	X2	8,201	5,967
	Total Error (MWh)	5,197,020	4,357,002
	Elapsed time (s)	3463.110517	29.071741
6-segments	X1	1,966	258
	X2	3,766	3,316
	X3	5,271	4,301
	X4	7,193	5,061
	X5	8,701	6,491
	Total Error (MWh)	2,566,869	3,036,718

#### IV. CONCLUSION

This study proposes Evolutionary Programming (EP) to determine optimum break points of discretized LDC at minimum error. The EP is developed using MatLab programming software. The proposed EP-based optimal step functions of LDC has been tested on Malaysia's LDC in year 2012 for three and six segments of discretized LDC. Results show that EP technique is able to provide optimum break points of discretized LDC at minimum error. Results also show that the 6-step functions of LDC has a lower total error than the 3-step functions of LDC. The EP technique proposed in this paper is also compared with DP technique. Results show that EP provides a much shorter elapsed time than DP and have a lower total error for 3-step function of LDC.

For future work, a Graphical User Interface (GUI) is recommended to ease user to determine optimal discretized LDC at various segments. With this GUI, user can load their own annual hourly load data and choose the number of segments that they want their LDC to be discretized.

#### ACKNOWLEDGEMENT

First of all, I would like to take this golden opportunity to express my gratitude to my supervisor, Dr. Nofri Yenita Dahlan for her continuous guidance and encouragement during completing this paper. She always be there for me and never give up to support me to finish this paper on desired due date. Her kindness, tireless help and motivation are not be forgotten. I would also like to give a credit to those who involved directly and indirectly in the progression of this paper, especially to all beloved friends for their support and help. Not to forget, thank you to Dr. Zuhaila Mat Yasin who gave me a permission to attend her Artificial Intelligence (AI) class and her kindness of sharing knowledge and ideas about Evolutionary Programming (EP). Last but not least, I would like to give an honorable mention and special thanks to my family, especially to my beloved mother for their spiritual support continuously which meant a lot for me to complete this paper.

#### REFERENCES

1. John Maybee, Paul Randolph, Noel Uri, "Optimal Step Function Approximations to Utility Load Duration Curve," Engineering Optimization 1979. Vol. 4 (2): p. 89-93
2. S.T Loney, "A Dynamic Programming Algorithm for Load Duration Curve Fitting." Numerical Methods for Non-Linear Optimization, Academic Press, 1971: p. 203-208.
3. Nofri Yenita Dahlan, "An Empirical Approach of Modeling Electricity Prices in an Oligopoly Market", Universiti Teknologi MARA.
4. A. Goldberg, "Best Linear Unbiased Prediction in the Generalized Linear Regression Model" J.Aer. Statistical Ass. 57 (1962) 369-375.
5. Anthony Wiskich, "Implementing A Load Duration Curve of Electricity Demand in A General Equilibrium Model", Australian Treasury, Australia.
6. Alain Poulin , Michel Dostie, Michael Fournier, Simon Sansregret, "Load Duration Curve: A Tool for Technico-economic Analysis of Energy Solutions", Laboratoire des Technologies de l'Energie, Institut de Recherche d'Hydro-Quebec, 600 avenue de la Montagne, Shawinigan, Quebec, Canada G9N 7N5.
7. Noel D. Uri, "A Mid-range Forecasting of Load Duration Curve", Department of Energy, Office of Energy Source Analysis, Oil and Gas Analysis Division, Washington D.C. 20461, USA.
8. John S. Maybee, Paul Randolph, Noel D. Uri, "Planning Available Capacity in Electrical Power Generation" Applied Energy, 1981. Vol. 9: p. 23-31.
9. N.Saravanan, David B.Fogel "Evolving Neural Network System", IEEE Intelligent System, vol.10 no.3, pp.23-27, June 1995.
10. Malaysia. Hourly Load Data, cited 2012;