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VIKOR METHOD WITH Z-NUMBER APPROACH FOR PORTFOLIO SELECTION DECISION

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ABSTRACT

Investors and decision makers (DMs) have become increasingly interested in portfolio selection in a borderless world in recent years. In real-world market situations, the performance of a great number of portfolios is typically unpredictable due to the presence of uncertainty and unreliable factors in numerous criteria. Therefore, it is essential to increase investor returns and promote an investment strategy through thorough evaluation. This occurrence becomes critical if the DMs employ an unsuitable strategy that fails to handle both aspects in a prudent manner. Due to its importance, this paper implements a VIKOR method with a Z-number approach for selecting the optimal portfolio among the identified alternatives. It is believed that the two components A and B of the Z-number structure, where A is a restriction of the evaluated attribute and B is a degree of certainty of A, deal with uncertainties and reliability issues more effectively. A numerical example from an adopted case study has been provided to demonstrate the effectiveness and viability of the proposed method. The outcome demonstrates that the approach can address the uncertainty of human judgement with greater precision while simultaneously boosting the DMs' confidence throughout the evaluation process. Consequently, the proposed method provides a more dependable and effective method for DMs to make decisions, particularly regarding portfolio selection.

Keywords: *Fuzzy VIKOR, Multi Criteria Decision Making (MCDM), Portfolio Selection, Z-Number.*

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1. Introduction

Good prediction of certain criteria may cause excellent implications in decision making. For instance, in the field of education, to overcome the problem of tracking students' progress for further action to be made, Mahmud *et al*., (2023) used the Multiclass Decision Forest algorithm to develop a model called MathVision that able to predict students' grade based on previous assessments. The model helps educators make decisions for further assistance in the subjects. Meanwhile in portfolio selection, the aim is to maximize the investment returns of investors. In real-world market situations, the performance of many portfolios is typically unpredictable due to the presence of uncertainty associated with imprecision, loss of information and lack of understanding, as well as unreliable factors in a variety of criteria. This problem affects Decision Maker (DM) perceptions.

Fouladgar *et al*. (2011) discovered six criteria for selecting the optimal project, that have been categorized under two main criteria: (1) cost criteria (risk, and payback period) and (2) benefit criteria (profitability, consistency with corporate goals and objectives, flexibility, and sustainability). Therefore, when designing an optimal allocation of limited funds among several options that primarily emphasize financial criteria, portfolio selection is a challenge (Khalili & Sadi, 2013).

The fundamental issue for portfolio selection is the lack of precision and unpredictability in mapping DM perceptions, which makes choosing an appealing portfolio a difficult multi-criteria decision. Zimonjic′ et al. (2018) found that the VIKOR technique is used more in MCDM to solve problems with multiple conflicting criteria for example in risk management, climate change, multi-attributes ranking index, and energy sector (Yucenur & Demirel, 2012), (Kim & Chung, 2013), (Mousavi *et al*., 2016), (Peleckis, 2022). The VIKOR method uses compromise solutions to rank alternatives based on non-comparable and conflicting criteria and can establish decision performance stability by replacing the compromise solution with initial weights. In establishing decision performance stability, reliability of the DMs should be considered (Shen & Wang, 2018).

To improve decision-making and DM reliability, the method uses Z-number. Zadeh's Z-numbers measure decision-making information's trustworthiness and imprecision (Zadeh, 2011). Z-numbers are ordered pairs $Z = (A, B)$, where A is an imprecise restriction of the evaluated attribute X and B is its reliability (certainty). Information reliability affects decision accuracy. Z-number considers uncertainty in information processing and information dependability, making it a simple and effective decision aid. Z-number considers the probability of relying on expert opinions, results that are closer to reality are more reliable.

Choosing a portfolio is crucial to investing, and many studies have examined ways to overcome its challenges. Markowitz's mean-variance optimization (MVO) research on the trade-off between projected returns and hazards established modern portfolio theory (Marques et al. 2022). Researchers are looking at other strategies because MVO fails to manage uncertainty and competing goals.

Decision makers (DMs) are responsible for making decisions. The research of Fouladgar et al. (2011) examines the portfolio selection practices of DMs with more than five years of experience. The method can rank either quantitative or qualitative criteria measurements. Despite altering multiple values, they discovered that the ranking orders of the evaluation alternatives remain unchanged. Alternatively, Khalili & Sadi (2013) included a project selection framework. Its mathematical formulation and design enhancements constitute achievements and contributions.

The theoretical or fundamentals of portfolio selection in finance, more than half a century ago, was originally written by Markowitz (1952). What started as observations and experiences were then measured accordingly. To obtain optimal portfolio, the results from Mousavi *et al*. (2016) is the ranking or rating index to be used.

Investors and DMs are increasingly fascinated with portfolio selection. Due to the inherent complication of the capital market and investors' irrational behaviour, it is challenging for investors to attain their predetermined objectives. Wang *et al*. (2023) developed a novel three-way decision model, designed a novel fuzzy multi-period portfolio selection model, and enhanced the performance of the portfolio selection model and the Particle Swarm Optimization (PSO) algorithm using experimental data. Consequently, the case study was examined by using the Cumulative Prospect Theory with hesitant fuzzy information for portfolio selection (Zhou *et al*., 2019).

Furthermore, Marques *et al*. (2022) proposed the FITradeoff technique for portfolio decision analysis in the context of possible incomplete information regarding the preferences of DMs. During the portfolio generation process, the proposed technique employs the concept of c-optimal portfolios as well as refinement strategies of feasibility and efficiency, with the aim of keeping computational and cognitive efforts within acceptable limits. This procedure is part of a DSS. Following the execution of multiple tests, the computational results of randomly generated instances indicate that this method performs well in terms of minimizing computational effort and reducing the cognitive effort required of the DMs.

In multi-criteria decision-making scenarios, the VIKOR method is a popular option due to its ease of use and computational efficiency. However, it has limitations when it comes to expressing the partial dependability of decision-makers accurately. To circumvent this limitation, researchers have begun investigating Z-number integration. Armin *et al*. (2021) filtered sustainability indexes using historical data, expert opinion uncertainty, and Z-number dependability. In addition, Jirofti & Najafi (2018) investigated the portfolio selection problem based on the Z-number theory and the utility function. Optimization with Z-number conversion to a classical fuzzy number and optimization without Z-number conversion were employed. The results indicated that the optimal portfolios generated by the two models were comparable, but employing the traditional fuzzy number simplified the computation process.

Finally, uncertainty and competing goals make portfolio choice difficult. Z-numbers improve decision-making by taking reliability and imprecision into account, and the VIKOR technique provides a thorough framework for portfolio assessment. This research seeks to improve portfolio performance and risk by examining the VIKOR method and Z-number strategy for portfolio selection.

This study uses VIKOR and Z-numbers to improve portfolio selection and decision-making. It lists many portfolio selection methods and emphasizes the best project selection criteria. Section 1 showed how VIKOR addresses competing criteria while Z-numbers account for uncertainty and reliability. Section 2 explains how to pick the best portfolio from the options. Section 3 provides a numerical example of the strategy's applicability and viability. Section 4 concludes that the approach improves human judgement uncertainty resolution and makes research recommendations.

2. Methodology

This section starts with the background of VIKOR Method and followed by the proposed approach.

2.1 Preliminaries: Background of VIKOR Method

The ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method, originally introduced by Opricovic (1998) to cater to the decision-making problem, particularly involved multi-criteria and various alternatives within the same range of characteristics.

Based on the ideal decision-making problem solution, the method ranks alternatives from conflicting criteria more precisely. Since then, decision-making, problem-solving, and related applications have used this method. Fuzzy VIKOR can better address uncertainty in human judgement and preferences (Bellmean & Zadeh, 1970). Figure 1 shows the approach's steps.

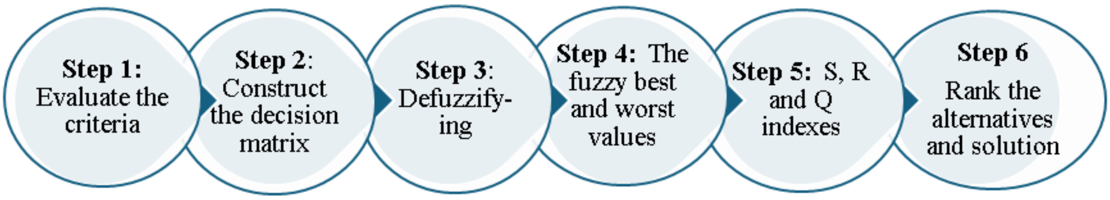

Figure 1. The VIKOR step-by-step with Z-Number approach

In this study, the evaluation of criteria and the importance of the alternatives are based on the linguistic expressions in terms of triangular fuzzy numbers (TFNs), as shown in Table 1. Meanwhile, the degree of certification is defined in four categories, as given in Table 2.

Table 1. Linguistic expression for criteria and importance of alternative

	Linguistic expressions	**Scales (TFNs)**
Criteria	Very high (VH)	(0.7,0.9,1.0)
	High (H)	(0.5,0.7,0.9)
	Medium (M)	(0.3,0.5,0.7)
	Low (L)	(0.1,0.3,0.5)
	Very low (VL)	(0,0.1,0.3)
Alternative	Very good (VG)	(0.6,0.8,1.0)
	Good (G)	(0.4,0.6,0.8)
	Fair (F)	(0.2,0.4,0.6)
	Poor (P)	(0,0.2,0.4)

Table 2. The degree of certainty

	Linguistic expressions	**Scales (TFNs)**
Degree of certainty	Very sure (VH)	(0.6,0.8,1.0)
	Sure (S)	(0.4,0.6,0.8)
	Unsure (US)	(0.2,0.4,0.6)
	Very unsure (VU)	(0,0.2,0.4)

2.2 The Proposed Approach

In this study, the portfolio selection decision was evaluated using the VIKOR method using the Z-numbers approach based on defined criteria and potential alternatives. The steps are:

Step 1: Evaluate the criteria via linguistic expressions based on the identified alternative using the Z-numbers approach (*see* Table 1 and Table 2). Then, quantify entirely the linguistic expressions in the TFN values.

Step 2: Construct the decision matrix (criteria vs. alternatives) and determine the importance weights of each criteria using a Centre of Area (CoA). Meanwhile, the weight of each criterion (*Cj*) can be obtained using the best non-fuzzy performance (*P*) given by Equation (1).

$$P_{ji} = \left[l_j + \left(\frac{(u_j - l_j) + (m_j - l_j)}{3} \right) \right] \tag{1}$$

where $\tilde{U} = (l_j, m_j, u_j)$. (i.e., the left, middle and right foots of TFNs)

Step 3: Defuzzify

The defuzzifying process is needed to obtain the criteria weights and ratings, or preferences, of each alternative. In this paper, the Centre of Gravity (CoG) method was used to calculate criteria weight (w_j) (Equation (2)) and alternative preferences (Equation (3)), respectively, and given by

$$CoG(\tilde{w}_j) = \left[\frac{(w_{j1} + w_{j2} + w_{j3})}{3} \right] \tag{2}$$

$$CoG(\tilde{x}_j) = \left[\frac{(a_{j1} + b_{j2} + c_{j3})}{3} \right] \tag{3}$$

Step 4: Calculate the fuzzy best (F_j^+) and worst values (F_j^-).

$$F_j^+ = \max\{x_{ij}\}, \text{and } F_j^- = \min\{x_{ij}\}; \quad j = 1,...,m \tag{4}$$

j is number of criteria.

Step 5: Calculate *S*, *R* and Q indexes.

For every single alternative, an index S_i which refers to the separation measures based on the best value was calculated using (5), while an index R_i applied which refers to the worst value by (6). Then, the Q_i index (7) using S_i and R_i values.

$$S_i = \sum_{j=1}^{n} w_j \left[\left(\frac{F_j^+ - x_{ij}}{F_j^+ - F_j^-} \right) \right] \tag{5}$$

$$R_i = \max{}_j \, w_j \left[\left(\frac{F_j^+ - x_{ij}}{F_j^+ - F_j^-} \right) \right] \tag{6}$$

$$Q_i = v \left[\frac{S_i - S_{\min}}{S_{\max} - S_{\min}} \right] + (1 - v) \left[\frac{R_i - R_{\min}}{R_{\max} - R_{\min}} \right] \tag{7}$$

where *v* is a weight for maximum utility $\left(v = \frac{n+1}{2n} \right)$, *n* is the number of criteria.

w_i is the weight of criteria, $S_{\min} = \min \, S_i$, $R_{\min} = \min \, R_i$, $S_{\max} = \max \, S_i$, $R_{\max} = \max \, R_i$

Step 6: Rank the alternatives and reach the compromise solution.

From the *S*, *R*, and *Q* values, the alternatives (A_i, i = 1,2,3,4) are ranked in ascending order, and the best alternative can be identified by choosing the one with a smaller *Q*. Please note that this approach focuses only on identifying the best alternative to reach a compromise solution within the considered alternatives available.

3. A Numerical Example and Discussion

This paper provides a numerical example to demonstrate the applicability and practicability of the methodology employed. In the selection of the portfolio selection problem, five criteria were considered based on a previous feasibility study to identify four established potential alternatives, namely equity (A_1), mixed assets (A_2), bond income (A_3), and money market (A_4). Priority (i.e., weights) within the criteria are assigned based on consensus agreement, as determined by the top management. Price per unit (C_1), risk level (C_2), annual return (C_3), track record (C_4), and earnings performance (C_5) are the remaining five criteria considered. Consequently, based on the approach described previously, we have determined the available portfolio options and established a set of criteria. Thus, the subsequent steps have been taken (refer to Figure 1).

Step 1: Evaluate the criteria via linguistic expressions based on defined criteria and identified alternative with Z-numbers approach (*see* Table 1 and Table 2) and obtained as in Table 3. Then, quantify the entire of the linguistic expressions to the TFN values (see Table 4).

Table 3. The linguistic expressions evaluation of each criteria vs each alternative

Alternative / Criteria	A_1	A_2	A_3	A_4
C1	(VH, S)	(M, VS)	(L, S)	(L, VS)
C2	(L, S)	(VH, US)	(M, VS)	(VH, US)
C3	(VH, VS)	(L, VS)	(M, S)	(VH, S)
C4	(L, S)	(VH, S)	(VH, VS)	(M, VS)
C5	(H, US)	(H, S)	(L, US)	(M, S)

Table 4. The quantification of the TFNs values based on linguistic evaluation from Table 3

Alternative / Criteria	A_1	A_2	A_3	A_4
C1	(0.7,0.9,1; 0.4,0.6,0.8)	(0.3,0.5,0.7; 0.6,0.8,1.0)	(0.1,0.3,0.5; 0.4,0.6,0.8)	(0.1,0.3,0.5; 0.6,0.8,1.0)
C2	(0.1,0.3,0.5; 0.4,0.6,0.8)	(0.7,0.9,1; 0.2,0.4,0.6)	(0.3,0.5,0.7; 0.6,0.8,1.0)	(0.7,0.9,1; 0.2,0.4,0.6)
C3	(0.7,0.9,1; 0.6,0.8,1.0)	(0.1,0.3,0.5; 0.6,0.8,1.0)	(0.3,0.5,0.7; (0.4,0.6,0.8)	(0.7,0.9,1; 0.4,0.6,0.8)
C4	(0.1,0.3,0.5; 0.4,0.6,0.8)	(0.7,0.9,1; 0.4,0.6,0.8)	(0.7,0.9,1; 0.6,0.8,1.0)	(0.3,0.5,0.7; 0.6,0.8,1.0)
C5	(0.5,0.7,0.9; 0.2,0.4,0.6)	(0.5,0.7,0.9; 0.4,0.6,0.8)	(0.1,0.3,0.5; 0.2,0.4,0.6)	(0.3,0.5,0.7; 0.4,0.6,0.8)

Step 2: Construct the decision matrix (criteria vs alternatives)
Determine the importance weights of each criterion, and the weight of each criterion (C_j) was calculated using Best Non-Fuzzy Performance (*P*) using Equation (1) (see Table 5).

Table 5. The importance and weights of each criterion via TFNs

	$\tilde{w_i}$	**BNP (P_j)**
C1	(0.4,0.6,0.79)	0.60
C2	(0.4,0.6,0.78)	0.59
C3	(0.48,0.68,0.85)	0.67
C4	(0.48,0.68,0.85)	0.67
C5	(0.33,0.53,0.71)	0.52

Step 3: Defuzzifying
The defuzzify process uses (2) to derive the criteria weights and ratings or preferences of each alternative (see Table 6).

Table 6. Crisp values for criteria weight and alternative preferences

	CoG	A_1	A_2	A_3	A_4
C1	0.60	0.73	0.65	0.45	0.55
C2	0.59	0.45	0.63	0.65	0.63
C3	0.67	0.83	0.55	0.55	0.73
C4	0.67	0.45	0.73	0.83	0.65
C5	0.52	0.53	0.65	0.35	0.55

Step 4: Calculate the fuzzy best and worst values based on Table 6 using Equation (4).

Table 7. Crisp values of criteria based on best and worst

	F_j^+	F_j^-
C1	0.73	0.45
C2	0.65	0.45
C3	0.83	0.55
C4	0.83	0.45
C5	0.65	0.35

Step 5: For each alternative, an index S_i was calculated using (5), while an index R_i applied which refers to the worst value by (6). Then, the Q_i index (7) derived using S_i and R_i values.

Table 8. Values of *S*, *R* and *Q*

	S	***R***	***Q***
A_1	1.468	0.670	0.489
A_2	1.169	0.670	0.400
A_3	1.790	0.670	0.006
A_4	1.175	0.386	0.706

Step 6: Rank the alternatives and reach the compromise solution.
The results of ranking of alternatives based on S, R and Q are obtained as follows.

S_j: $A_3 > A_1 > A_4 > A_2$ R_j: $A_3 = A_1 = A_2 > A_4$ Q_j: $A_3 > A_2 > A_1 > A_4$

where the symbol '>' means 'is preferred or superior to'.

Based on the results obtained, it was observed that the alternative A_3 is the best compromise solution. Hence, the bond income is highly recommended to the top management for investment in portfolio selection based on the identified criteria. The VIKOR method with the Z-number that has been proposed is clearly applicable to dealing with uncertain information with ease and effectiveness. Meanwhile, the Z-number structure that is equipped with the VIKOR method provides an extra measurement in terms of reliability or degree of confidence during the evaluation process. Besides that, the linguistic variables in terms of TFNs for both components (i.e., *A and B*) in the Z-number will provide an easier approach to the evaluation process without dealing directly with the numeric values as usual. Lastly, the index *Q* derives from both the *S* and *R* indices, which measure a separation from the best and worst values, respectively, and is able to provide the precision approach needed to reach the compromise solution.

4. Conclusion

In this study, we utilized the advantages of the Z-numbers approach in the VIKOR method to evaluate the portfolio selection decision problem. Due to the existence of uncertainty and unreliable aspects in the selection process, a restriction on the *A* component has the advantage of being able to minimize the ambiguity of the evaluated criteria versus alternatives. Besides that, the degree of certainty in component *B* has been observed to be very convincing for decision-makers to evaluate at every evaluation stage. From the numerical example, the approach could address the uncertainty of human judgement with more precision. Furthermore, because each evaluation has a component of certainty, this approach can generate a confident feeling among decision-makers throughout the evaluation process. Thus, this approach offers a more reliable and efficient way that can lead to a compromise in terms of the evaluation process to decide, particularly in portfolio selection. For the next effort, this study will explore combining the VIKOR method with the neutrosophic sets in the selection decision problem, which is believed to deal with the uncertainty and unreliability aspects with more precision and convincingness in terms of the decision evaluation process.

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Author Contribution

Author1 wrote the research methodology, performed fieldwork, and interpreted the results. Author2 prepared the literature review and oversaw the article writing. Author3, Author4, Author5 prepared the literature review, and Author6 collaborative writing from Indonesia.

Conflict of Interest

The authors have no conflicts of interest to declare.

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