

Voltage Stability Assessment via Continuation Power Flow Method

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¹Abstract — This paper presents a potential algorithm for continuation power flow method in voltage stability assessment. Modified continuation power flow method termed as MCBF was developed which solved the solution of bifurcation point at the Q-V curve. The solvability of power flow at the bifurcation point beyond the maximum loadability point of a system was achieved through the implementation of predictor-corrector technique. QV curve was automatically plotted once the bifurcation point has been reached. Voltage stability analysis was subsequently conducted utilizing the MCPF solutions. Comparative studies performed with respect to the conventional power flow technique revealed the strength of the proposed MCPF; which was validated on a standard IEEE reliability test (RTS) system.

Index Terms — Bifurcation, continuation power flow, predictor-corrector, voltage stability index.

I. INTRODUCTION

Voltage reduction in a power system network can be monitored when reactive power loading is increased accordingly. This phenomenon can be achieved by performing load flow or power flow study on a power system network. Nonetheless, conventional load flow studies have demonstrated failure in giving solution at its bifurcation point. This is

due to the singularity of Jacobian matrix formed during the load flow study. This problem can be solved by using continuation power flow, which remains well condition at the saddle node bifurcation point. In recent years, an instability usually termed as voltage instability has been observed to be responsible for several major networks collapses in many countries [1-2]. Voltage stability has become critical issue since the continuous load varies along with economical and environmental constrains which has led the power systems to operate close to their limits, along with stability margin reduction [1]. At this point any unexpected rise in the load level can cause voltage collapse phenomena. This phenomenon has made the voltage stability condition as a crucial aspect in the power system operation and planning. The solution curve is an important element in voltage stability assessment, which can be computed by continuation power flow method. The continuation power flow methods are powerful and useful tools for obtaining solution curves for general non-linear algebraic equation by automatically changing the value of a parameter [1]. This solution curve indicates the critical point of voltage stability limit, which is at the nose of the curve. Voltage stability limit is the maximum loading point (MLP) which can be computed by many tools or continuation method. Several continuation power flow techniques have been reported in published literature. Among the previous techniques are continuation power flow (CPF), CPFLOW (the revision of continuation power flow) [2], and Homotopy method. The Homotopy method constructs a Homotopy from an augmented load flow equation and a polynomial with known solutions and the load flow solutions are obtained by tracing the homotopy curves starting from the known solutions of the

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polynomial [3]. The Homotopy type of continuation is considered not efficient method since it required large computational scale while the singularity of the Jacobian matrix is still remained [3]. On the other hand, the CPFLOW Continuation method is slightly a modified CPF technique; which uses the same continuation parameter in their techniques to search equilibrium solution. The difference is the adoption of Secant-base Predictor and pseudo arclength-based corrector method in their algorithm. This method is one of the best continuation methods as reported by Hiroyuki Mori [4] based on its contribution in speeding up of the computation time significantly along with straightforward algorithm. Tiranuchit and Thomas [5] had reported work on a posturing strategy against voltage collapse instabilities in power system. This work is rather concentrating the efforts for voltage instability avoidance scheme. On the other hand, the work conducted by Musirin and Rahman in [6] in deriving a new voltage stability index has revealed the importance of having an indicator for the identification of maximum loadability point at a selected load bus. Its flexibility in terms of applications to voltage stability evaluation and other exploratory studies implied the merit of the developed index. Voltage instability has been profoundly associated with voltage collapse and cascading outages as reported in [7-12]. The failure of achieving solutions utilizing the conventional load flow can be alleviated by the introduction of continuation power flow (CPF) as discussed earlier. However, many other studies have also concerned on the CPF solving technique as reported in [13-16]. One of the popular CPF techniques is the one developed by Ajarapu in [3].

This paper presents the development of modified continuation power flow (MCPF) to solve the failure in convergence experienced in the conventional power flow. The modified CPF technique was based on the technique proposed by Ajarapu [3]. The developed technique has also identified the nose point of Q-V curve, which was discovered to be the failure in the conventional power flow. Comparative studies have been performed with ordinary voltage stability analysis and results

have revealed the merit of MCPF over conventional power flow. Subsequent studies by means of voltage stability assessment utilizing a pre-developed voltage stability index [6] were also conducted. The developed algorithm was validated on a standard test system which has yielded promising results. Comparative studies performed with respect to conventional load flow technique revealed the strength of the proposed MCPF technique.

II. CONTINUATION POWER FLOW

The predictor-corrector continuation method uses predict and correct scheme [3]. The predictor method forecasted the next value of the parameter of the load flow when the load parameter initially varies from the base condition. As shown in Fig. 1, the predictor starts from the known solution and predict the next equilibrium solution. The corrector method corrects the value of the predicted solution. The objective of this scheme is to find a path of equilibrium solution from the Jacobian matrix starting at base condition at stable equilibrium point (SEP).

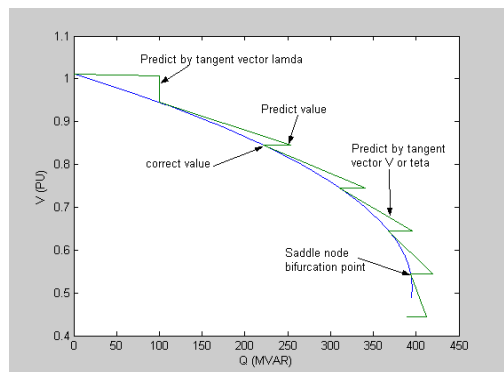


Fig 1. An illustration of predictor-corrector continuation

A. Modified Continuation Power Flow Algorithm

The MCPF algorithms are explained in detail step-by-step as follows:-

Step 1:

The continuation power flow technique starts from base condition which using the conventional Newton-Raphson's load-flow solution to compute the base parameter.

Step 2:

Reformulate the load-flow equation to contain a load parameter λ and write the reformulated load-flow equation into matrix form known as Jacobian matrix $[J]$ as in equation (7).

Step 3:

Calculate the Jacobian matrix of the reformulated load-flow equation (eqn 7) base on the base case value of the load-flow parameter.

Step 4:

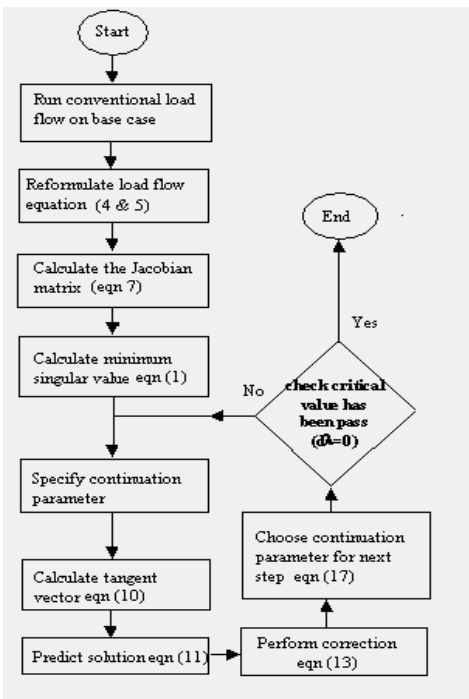
Compute the minimum singular value by using equation below: -

$$\sigma_{\min} = \|[J]^{-1}\|^{-1} \quad (1)$$

Step 5:

Specify the continuation parameter by choosing from one of the state variables of base case value or from equilibrium solution. In this paper the continuation parameter for base value is λ equals to zero and the next continuation parameter is chosen by step 9.

Fig..2. Modified continuation power flow algorithm



Step 6:

Calculate the tangent vector element base on the tangent vector of the continuation parameter used.

Step 7:

Compute the predicted value base on the step size chosen.

Step 8:

Correct the error found in the predictor by using modified Newton-Raphson's load-flow solution.

Step 9:

Choose the next continuation parameter based on the tangent element intersection.

Step 10:

Check the critical point. The process is continued from step 4 until step 9 until the critical point has been passed (the tangent component $d\lambda$ is equal to zero or passing zero for upper portion of the curve and load parameter λ is equal to zero or passing zero for lower portion of the curve).

The flow chart of the continuation power flow is shown in Fig. 2.

A1. Reformulation of Load-Flow Equation Algorithm

The load-flow equations are reformulated in order to explain the step-by-step algorithm for the usage in the MCPF.

Step 1:

Consider the conventional load-flow equation defined as: -

$$P_{gi} - P_{di} - P_{ci} = 0 \quad (2)$$

$$Q_{gi} - Q_{di} - Q_{ci} = 0 \quad (3)$$

Where; $P_{ci} = \sum_{j=1}^n V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \theta_{ij})$

and $Q_{ci} = \sum_{j=1}^n V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \theta_{ij})$

For each bus “i” of an n bus system, the subscripts g , d and c denote the generation, load or demand and injection bus respectively. The voltage at bus i and j are $V_i \angle \theta_i$ and $V_j \angle \theta_j$ respectively and the admittance matrix element Y_{bus} of the $(i,j)^{th}$ element is denote by $Y_{ij} \angle \theta_{ij}$.

Step 2:

The value of λ is inserted into load-flow equation corresponding to weak bus (for this paper bus 3 in the 5-bus test system) and the equation becomes: -

$$P_{gi} - P_{di} - P_{ci} = 0 \quad (4)$$

$$Q_{gi} - Q_{di} - Q_{ci} - \lambda = 0 \quad (5)$$

Where the value of λ is varied from 0 to maximum variation and state as follows: -

$$0 \leq \lambda \leq \lambda_{critical} \quad (6)$$

Step 3:

Compute the Jacobian matrix of the reformulated load-flow equation (equation 3 and 4 for each PV and PQ bus) base on base case parameter value. The vector function of several vectors of the Jacobian matrix $[J]$ from reformulated load-flow equation can be written as: -

$$\begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \dots & \frac{\partial P_2}{\partial \theta_n} & \frac{\partial P_2}{\partial V_{m+2}} & \dots & \frac{\partial P_2}{\partial V_n} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial P_n}{\partial \theta_2} & \dots & \frac{\partial P_n}{\partial \theta_n} & \frac{\partial P_n}{\partial V_{m+2}} & \dots & \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_{m+2}}{\partial \theta_2} & \dots & \frac{\partial Q_{m+2}}{\partial \theta_n} & \frac{\partial Q_{m+2}}{\partial V_{m+2}} & \dots & \frac{\partial Q_{m+2}}{\partial V_n} & \frac{\partial Q_{m+2}}{\partial \lambda} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_n}{\partial \theta_2} & \dots & \frac{\partial Q_n}{\partial \theta_n} & \frac{\partial Q_n}{\partial V_{m+2}} & \dots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \quad (7)$$

A2. Tangent Vector Computation Algorithm

The procedures for the tangent vector computation algorithm are given in the following step-by-step technique:-

Step 1:

Consider the whole set of load-flow equations defined as: -

$$G(\theta, V, \lambda) = 0 \quad (8)$$

Step 2:

Take the derivative of both side of equation (8) and the equation becomes: -

$$G_\theta d\theta + G_V dV + G_\lambda d\lambda = 0 \quad (9)$$

Step 3:

Factorize equation (9) and the equation becomes: -

$$\begin{bmatrix} G_\theta & G_V & G_\lambda \end{bmatrix} \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = 0 \quad (10)$$

Where $d\theta, dV$ and $d\lambda$ denoted the direction tangent to the solution path and G_θ, G_V and G_λ denoted the partial derivative G with respect to θ, V and λ and it is equal to Jacobian matrix equation (7).

Step 4:

Assigned the tangent vector component corresponding to the continuation parameter used as +1 or -1 depends on how the solution curve changing. In this paper the tangent vector $d\lambda$ equals +1 (solution curve increase) and dV equals -1 (solution curve decrease).

Step 5:

Insert the tangent vector component into equation (10) and compute the other tangent vector value.

A3. Predictor Algorithm

Predictor algorithm was implemented as one of the components in MCPF. The algorithm is given by the following step-by-step procedures:-

Step 1:

Choose the suitable step size. In this study the step size is assigned a constant value 0.001.

Step 2:

Predict the next equilibrium solution by the equation below:

Predict the next equilibrium solution by the equation below:

$$\begin{bmatrix} \theta_p \\ V_p \\ \lambda_p \end{bmatrix} = \begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix} + h \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} \quad (11)$$

Where θ_p , V_p and λ_p denoted the predicted values. θ , V and λ denoted the current parameter values, h denoted the step size and $d\theta$, dV and $d\lambda$ denoted the tangent vector value.

A4. Corrector Algorithm

The corrector algorithm is explained in detail according to the following step-by-step procedures:-

Step 1:

Assign the predicted value corresponding to the continuation parameter used as the correct value. This value is constant throughout the corrector process.

Step 2:

Correct the error found in the predictor process (the predicted value other than the correct value in step 1) by using modified Newton-Raphson's load-flow solution expressed as follows: -

$$\begin{bmatrix} G_\theta & G_V & G_\lambda \\ & X_k & \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \\ n \end{bmatrix} \quad (12)$$

Where $\Delta\theta$, ΔV and $\Delta\lambda$ denoted the changes in each parameter θ , V and λ respectively, ΔP and ΔQ denoted the change in active power and reactive power respectively, n denoted the correct value corresponding to the continuation parameter and X_k denoted the row vector with all element equal to zero except the element corresponding to continuation parameter which equals one.

Step 3:

Correct the value of each parameter, θ , V and λ using the following equation: -

$$\begin{bmatrix} \theta_c \\ V_c \\ \lambda_c \end{bmatrix} = \begin{bmatrix} \theta_p \\ V_p \\ \lambda_p \end{bmatrix} + \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta\lambda \end{bmatrix} \quad (13)$$

Where θ_c , V_c and λ_c denoted the corrected value, θ_p , V_p and λ_p denoted the predicted value and $\Delta\theta$, ΔV and $\Delta\lambda$ denoted the changes in each parameter.

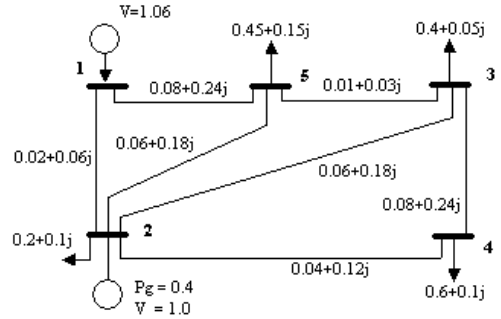


Fig. 3: The 5-bus test system

Step 4:

Check all the changes of $\Delta\theta$, ΔV , ΔP and ΔQ , if not less than the specified accuracy the step 2 is continued again.

A5. Choosing Next Continuation Parameter Algorithm

In order to perform the chosen of the next continuation parameter, the following step-by-step procedures are implemented:-

Step 1:

Compute the distance of λ from the point of Jacobian matrix singularity by using the equation below: -

$$D_\lambda = \sigma_{\min} - \lambda \quad (14)$$

Step 2:

Compute the component of tangent vector by the equation below: -

$$t_\theta = \theta_c - \theta_o \quad (15)$$

$$t_V = V_c - V_o \quad (16)$$

Where t_θ and t_V denoted the component of tangent vector corresponding to PV and PQ bus, θ_c and V_c denoted the correct equilibrium solution and θ_o and V_o denoted the base case value.

Step 3:

Check the maximum value of the tangent vector component and $D\lambda$ by the equation below: -

$$t_{\max} = \{|t_{\theta 2}|, |t_{\theta 3}|, \dots, |t_{\theta n}|, |t_{v_{m+2}}|, \dots, |t_{v_n}|, |D\lambda|\} \quad (17)$$

If the maximum value is one of the tangent vector components, the next continuation parameter should be changed to the corresponding maximum tangent vector, otherwise if the maximum value is $D\lambda$, the next continuation parameter should be λ .

III. VOLTAGE STABILITY INDEX

Voltage stability condition is consequently implemented in this study incorporating the developed MCPF algorithm. The instrument is a line-based voltage stability index termed as *FVSI*, which was developed by I. Musirin and T.K.A. Rahman in [6]. The index equation is given by: -

$$FVSI_{ij} = f(Z, Q_j, V_j, X) \quad (18)$$

Where Z , X , Q_j , and V_j denote the line impedance, line reactance, reactive power at the receiving end and sending end Voltage respectively. This index indicates the voltage stability condition of a line in a system. The line with *FVSI* value closest to 1.0 implies that the line is unstable which could cause entire system instability leading to system collapse.

A. Algorithm for Voltage Stability Assessment

Voltage stability condition of a power system is assessed by evaluating the proposed line-based voltage stability index, *FVSI*. Several steps are implemented in order to carry out the voltage stability analysis. The following procedures were implemented in the voltage stability analysis:-

- i. Run the load flow program (Newton-Raphson) at the base case.
- ii. Use the results from the load flow solution to compute the line index, *FVSI*.
- iii. If the index is smaller than 1.00, increase the reactive load power and repeat steps [ii] and [iii] until it

reaches 1.00 or solution fails to converge.

- iv. Record the highest index and the corresponding line.
- v. Plot individual graph for line index (*FVSI*) versus reactive load variation at the tested load bus. This will identify the sensitive line with respect to the load bus.
- vi. Repeat the whole process, i.e., steps [i] to [v] for other load buses in the system.
- vii. Plot the curves for bus voltages versus reactive load variation on the same axis. This will estimate/determine the voltage at the stability limit for each load bus and hence the weak bus will be identified.

The whole algorithms are simplified in the form of flow chart appeared in Fig. 4.

IV. RESULTS AND DISCUSSION

The developed MCPF has been tested on the standard 5-bus system, with its single line diagram shown in Fig. 3. Bus 3 was taken as the test bus for the implementation of MCPF. The base condition reactive power at bus 3 was varied gradually until a sharp point of the Q-V curve is obtained. The variation of Q at bus 3 along with observation of the bus voltage is illustrated in Fig. 5.

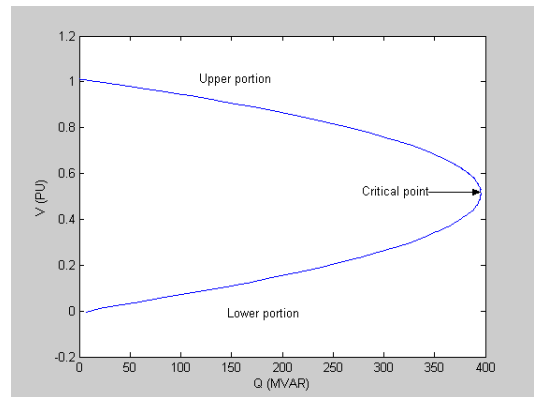


Fig. 5. QV curve performed using MCPF with bus 3 loaded

From Fig. 5, it is observed that the increment of reactive power at bus 3 through the MCPF has reduced the bus voltage. This process continues until the critical point is found as indicated in the Fig. 5. At this nose point, the MCPF is able to find the load flow solution, which implies that the Jacobian matrix is non-singular. A singular Jacobian matrix that was normally produced by the conventional load flow will lead to the divergence of load flow as shown in Fig. 6. The lower portion produced of the Q-V curve as in the Fig. 5 are computed by the MCPF, which cannot be obtained in the conventional load flow. The smoothed Q-V curves for both portions are computed by the implementation of predictor and corrector in MCPF.

From Fig. 5, the upper portion of the Q-V curve implies the stable operating region for the system. On the other hand, the lower portion indicates unstable operating region, which requires high current to operate as also reported in [5].

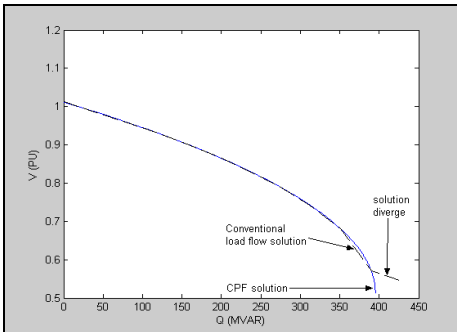


Fig. 6. Solution for MCPF and conventional load flow solution at the nose point

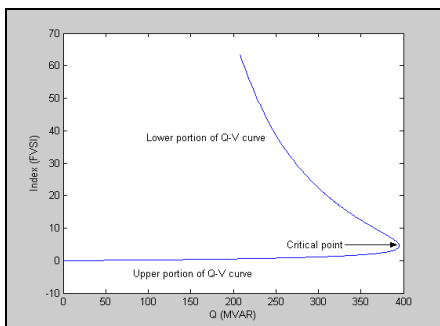


Fig. 7. The profile of *FVSI* obtained using MCPF

The profile of *FVSI* with the reactive load variation is illustrated in Fig. 7 and Fig. 8. The accelerated increase of the curve showed in the Fig. 6 represent the lower portion of the nose Q-V curve as in Fig. 5 which require very high current to operate. The voltage stability condition and the critical line referred to a particular bus are determined by the *FVSI* value close to 1.00 while the weak bus is determined by the maximum permissible load for the individual bus in the system [14]. The result of the stable voltage referred to line is shown in Fig. 7. From Fig. 7, it is observed that MCPF, in which it is impossible to be achieved using the conventional load flow techniques. From the figure, *FVSI* value increased abruptly on the second manifold of QV curve. It means that, the lower portion of the QV curve will correspond to the *FVSI* profile beyond the critical point. This region has been profoundly identified as the unstable region of the voltage stability condition. On the other hand, Fig. illustrates the profile of *FVSI* with respect to the Q variation. Apparently, above the Q_{max} , *FVSI* value shoots up to above unity implying unstable condition of the system.

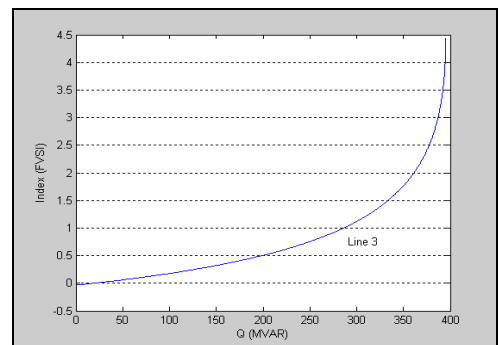


Fig. 8. The critical line correspond to the weakest bus

V. CONCLUSION

Voltage stability assessment via continuation power flow technique has been presented in this paper. An algorithm for a modified continuation power flow (MCPF) utilizing predictor, corrector and tangent techniques was developed. This has overcome the burden in reaching the solution experienced in the traditional power flow technique. It was

discovered that the CPF outperformed conventional load flow in identifying the bifurcation point. This has also solved the insolvability in the conventional load flow. Comparative studies performed with respect to the conventional load flow technique have highlighted the merit of the proposed technique.

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