

# The Integral Iterative Method for Approximate Solution of Newell-Whitehead-Segel Equation

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Article Info

### ABSTRACT

Article history:	Most numerical methods require very extensive calculations and use very large computer memory. The resulting numerical solution is also very sensitive to any small changes in the parameters present in the differential equation. This paper presents research finding that has been conducted with two objectives. First objective is to solve the Newell-Whitehead-Segel (NWS) equation using integral iterative method (IIM). Second objective is to determine the accuracy, reliability and efficiency of IIM by compared to the exact solution and other	
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Keywords:		
Newell-Whitehead-Segel Integral Iterative method Nonlinear differential equation Integral operator	existing results obtained by other methods such as New Iterative Method (NIM), Adomian decomposition method (ADM) and Laplace Adomian decomposition method (LADM). This iterative method was calculated based on the integral operator, that is the inverse of the differential operator in the problem under consideration. The analytical solution of the equation was calculated in the form of power series solution. Results of this research has identified that the method is simpler in its computational procedures and needs shorter time to be completed than the other methods. It does not require discretization, linearization or any restrictive assumption in order to provide analytical and approximate solution. The technique provides in this research introduced a straightforward and powerful mathematical tool for solving various differential equations.	

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### 1. Introduction

The Newell-Whitehead-Segel (NWS) equation is an important model in fluid mechanics. This equation, which is a kind of nonlinear partial differential equation (PDE) is used for some problems





in various system such as Faraday instabilities, biological systems, and Rayleigh-Bernard convection. The equation of NWS [1][2] is described in equation (1) and equation (2).

$$u_t(x,t) = a u_{xx}(x,t) + b u(x,t) - c u^m(x,t)$$
(1)

$$u(x,t) = f(x) \tag{2}$$

where *b*, *c* are real numbers and *a*, *m* are positive integers.

Due to its wide range of applications, NWS equation has attracted much intention by researchers to find the accurate and efficient method for solving the equation. Some of these methods are Adomian decomposition method (ADM) [3],[4], Laplace-Adomian decomposition method (LADM) [5], homotopy perturbation method (HPM) [6], variational iterative method (VIM) [7] and new iterative method method (NIM) [8]. A comparative study between reduced differential transform method (RDTM) and ADM for solving NWS equation was conducted by Saravanan and Magesh [9]. Meanwhile, Jassim [10] applied the HPM using Laplace transform to solve the NWS equation. Recently, Busyra et al. [11] successfully applied the semi analytical iterative method (SAIM) to approximate solution of NWS equation. Also, Almousa et al. [12] presented the application of the Mahgoub Adomian decomposition method while Elgazery [13] applied fractional calculus to solve NWS equation.

The purpose of this article is to solve numerically the NWS equation by applying an iterative method called integral iterative method (IIM). This method, proposed by Hemeda [14] to solve nonlinear integro-differential and systems of integro-differential equations. Hemeda and Eladdad [15] successfully applied the IIM to solve the Fokker-Planck equation. It is worth to mentioned that IIM may be considered as a new approach for Picard method, [16],[17]. To the best of our knowledge, the study of NWS using IIM has not been done.

#### 2. Literature Review

#### 2.1 Integral Iterative Method

Consider the general partial differential equations of arbitrary order are [14]:

$$\frac{\partial^n u(x,t)}{\partial t^n} = A(u,\partial u) + B(x,t), n \in N$$
(3)

$$\frac{\partial^k u(x,0)}{\partial t^k} = h_k(x), \quad k = 1, 2, \dots, n-1$$
<sup>(4)</sup>

where *A* is a nonlinear function of  $u, \partial u$  (partial derivatives of *u* with respect to *x* and *t*) and *B* is a nonhomogeneous term. In view of integral operators, the initial value problem in equation (3) and equation (4) is equivalent to the following integral equation (5).

$$u(x,t) = \sum_{k=0}^{n-1} h_k(x) \frac{t^k}{k!} + I_t^n B + I_t^n A = f + N(u)$$
(5)

where  $f = \sum_{k=0}^{n-1} h_k(x) t^k / k! + I_t^n B$ .  $N(u(x)) = I_t^n A$  and  $I_t^n$  is an  $n^{th} - order (n - fold)$  integral operator. The required solution u(x, t) for equation (6) and hence equation (3) and equation (4) can be obtained recurrently by employing the simple recurrence relation:

$$u_0 = f, (6)$$

$$u_{r+1} = u_0 + N(u_r), \ r = 0, 1, 2, \dots$$
 (7)

where  $u(x, t) = \lim_{r \to \infty} u_r(x, t)$ .

The IIM works directly on problems with initial condition and boundary condition after transferring these conditions to initial conditions. For mixed condition, the IIM can be made for works by choosing the initial solution as a function depending on the given mixed conditions such that this initial solution satisfies almost all the given mixed conditions [14].

#### 3. Research Method

In this section the NWS equation will be solved by using IIM. equation (1) can be written equivalently as:

$$u = f(x) + N(u) \tag{8}$$

where

$$N(u) = \int (au_{xx}(x,t) + bu(x,t) - cu^{m}(x,t))dt$$
(9)

then by using IIM, we can obtain the recurrence relation:

$$u_0 = f, \tag{10}$$

$$u_{r+1} = f + \int (a(u_r)_{xx}(x,t) + bu_r(x,t) - cu_r^m(x,t))dt$$
<sup>(11)</sup>

where r = 0, 1, 2, ... and so on.

The required solution u(x, t) for equation (8) which is also the solution for equation (1) and equation (2) can be obtained from the recurrence relation of equation (10) and equation (11).

#### 3.1. Illustrative Example

In this section, three numerical examples are considered to be solved by IIM to reveal the reliability and accuracy of the method.

#### Example 1

Given the NWS equation as follows

$$u_t(x,t) = 5u_{xx}(x,t) + 2u(x,t) + u^2(x,t)$$
(12)

with initial condition

$$u(x,0) = \lambda$$
 (13)  
where the exact solution is

$$u(x,t) = \frac{2e^{2t}\lambda}{2 + (1 - e^{2t})\lambda} \tag{14}$$

By using IIM, the equivalent integral equation of (4.1) is

$$u = \lambda + \int_0^t (5u_{xx} + 2u + u^2)dt$$
(15)

Thus, the first a few iterative solutions are,

$$u_{0}(x,t) = \lambda$$

$$u_{1}(x,t) = \lambda + (\lambda^{2} + 2\lambda)t$$

$$u_{2}(x,t) = \lambda + 2\lambda t + (\lambda^{2} + 2\lambda)t^{2} + \frac{1}{3}\frac{(\lambda + (\lambda^{2} + 2\lambda)t)^{3}}{\lambda^{2} + 2\lambda}$$
(16)
$$u_{3}(x,t) = \lambda + 2\lambda t + 2\lambda t^{2} + \frac{2}{3}(\lambda^{2} + 2\lambda)t^{3} + \frac{1}{6}\frac{(\lambda + (\lambda^{2} + 2\lambda)t)^{4}}{(\lambda^{2} + 2\lambda)^{2}} + \frac{1}{63}(\lambda^{2} + 2\lambda)^{4}t^{7} + \frac{1}{9}(\lambda^{2} + 2\lambda + \lambda(\lambda^{2} + 2\lambda))(\lambda^{2} + 2\lambda)^{2}t^{6} + \frac{1}{5}(\frac{2}{3}(\lambda^{2} + 2\lambda)^{3} + (\lambda^{2} + 2\lambda + \lambda(\lambda^{2} + 2\lambda))(\lambda^{2} + 2\lambda)^{2}t^{5} + \frac{1}{4}(\frac{2}{3}(\lambda + \frac{1}{3}\frac{\lambda^{3}}{(\lambda^{2} + 2\lambda)})(\lambda^{2} + 2\lambda + \lambda(\lambda^{2} + 2\lambda))(\lambda^{2} + 2\lambda + \lambda(\lambda^{2} + 2\lambda)))t^{4} + \frac{1}{3}(2(\lambda + \frac{1}{3}\frac{\lambda^{3}}{(\lambda^{2} + 2\lambda)})(\lambda^{2} + 2\lambda + \lambda(\lambda^{2} + 2\lambda)) + (\lambda^{2} + 2\lambda)^{2}t^{3} + (\lambda + \frac{1}{3}\frac{\lambda^{3}}{(\lambda^{2} + 2\lambda)})(\lambda^{2} + 2\lambda)t^{2} + (\lambda + \frac{1}{3}\frac{\lambda^{3}}{(\lambda^{2} + 2\lambda)})(\lambda^{2} + 2\lambda)t^{2} + (\lambda + \frac{1}{3}\frac{\lambda^{3}}{(\lambda^{2} + 2\lambda)})^{2}t$$

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By letting  $\lambda = 12$ , the fifth iterative solution is:

$$u_{4}(x,t) = 12 + \frac{410460}{2401}t + \frac{8455704}{2401}t^{2} + \frac{115611376}{2401}t^{3} + \frac{28864904}{49}t^{4}$$
(17)  
+  $\frac{1}{71124480}(12 + 168t)^{5} + 3768825984t^{8}$   
+  $\frac{17914080512}{35}t^{7} + \frac{2133965632}{35}t^{6} + \frac{317273472}{49}t^{5}$   
+  $\frac{53293212499968}{5}t^{15} + 12371638616064t^{14}$   
+  $\frac{435108944019456}{65}t^{13} + \frac{11800569102336}{5}t^{12}$   
+  $\frac{173259776987136}{275}t^{11} + \frac{3383696031744}{25}t^{10}$   
+  $\frac{121288308736}{5}t^{9}$ 

In this study, we let  $t \in [0,0.05]$  to execute the solution. The results have been plotted as presented in Figure 1 in the Section five of this paper.

# Example 2

Next, we consider the linear NWS equation:  

$$u_t(x,t) = u_{xx}(x,t) + 2u(x,t) - 3u^2(x,t)$$
(18)

with initial condition

 $u(x,0) = \lambda \tag{19}$ 

where the exact solution is

$$u(x,t) = -\frac{2e^{2t}\lambda}{-2+3(1-e^{2t})\lambda}$$
(20)

In view of IIM, the equivalent integral equation of (4.7) is

$$u = \lambda + \int_{0}^{t} (u_{xx} + 2u - 3u^{2})dt$$
(21)

Thus, the first a few iterative solutions are,

$$\begin{aligned} u_{0}(x,t) &= \lambda \\ u_{1}(x,t) &= \lambda + (-3\lambda^{2} + 2\lambda)t \end{aligned}$$

$$\begin{aligned} u_{2}(x,t) &= \lambda + 2\lambda t + (\lambda^{2} + 2\lambda)t^{2} + \frac{(\lambda + (-3\lambda^{2} + 2\lambda)t)^{3}}{-3\lambda^{2} + 2\lambda} \end{aligned}$$

$$\begin{aligned} u_{3}(x,t) &= \lambda + 2\lambda t + 2\lambda t^{2} + \frac{2}{3}(-3\lambda^{2} + 2\lambda)t^{3} - \frac{1}{2}\frac{(\lambda + (-3\lambda^{2} + 2\lambda)t)^{4}}{(-3\lambda^{2} + 2\lambda)^{2}} \\ &- \frac{3}{7}(-3\lambda^{2} + 2\lambda)^{4}t^{7} + (-3\lambda^{2} + 2\lambda - 3\lambda(-3\lambda^{2} + 2\lambda))(-3\lambda^{2} + 2\lambda)^{2}t^{6} - \frac{3}{5}(-2(-3\lambda^{2} + 2\lambda)^{3} + (-3\lambda^{2} + 2\lambda))(-3\lambda^{2} + 2\lambda)^{2}t^{6} - \frac{3}{5}(-2(-3\lambda^{2} + 2\lambda)^{3} + (-3\lambda^{2} + 2\lambda))(-3\lambda^{2} + 2\lambda)^{2}t^{5} \\ &- \frac{3}{4}(-2\left(\lambda - \frac{\lambda^{3}}{-3\lambda^{2} + 2\lambda}\right)\left(-3\lambda^{2} + 2\lambda)^{2} + 2(-3\lambda^{2} + 2\lambda)(-3\lambda^{2} + 2\lambda - 3\lambda(-3\lambda^{2} + 2\lambda))\right)t^{4} \\ &- (2\left(\lambda - \frac{\lambda^{3}}{-3\lambda^{2} + 2\lambda}\right)(-3\lambda^{2} + 2\lambda - 3\lambda(-3\lambda^{2} + 2\lambda))) \\ &+ (-3\lambda^{2} + 2\lambda)^{2}t^{3} - 3\left(\lambda - \frac{\lambda^{3}}{-3\lambda^{2} + 2\lambda}\right)(-3\lambda^{2} + 2\lambda)t^{2} \\ &- 3\left(\lambda - \frac{\lambda^{3}}{-3\lambda^{2} + 2\lambda}\right)^{2}t \end{aligned}$$

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By letting  $\lambda = 0.1$ , the sixth iterative solution is:

$$\begin{array}{l} u_5 = 0.9999999993788 + 0.17000000018107t + 0.11900000033565t^2 \\ + 0.026633333583805t^3 - 0.021033333283682t^4 \\ - 0.0198163333405179t^5 - 0.00548907333544101t^6 \\ + 0.00614130504512453t^7 + 0.00313447955062534t^8 \\ - 0.000415228058422366t^9 - 0.000874752934866484t^{10} \\ - 0.000188386366019455t^{11} + 0.000144045878042441t^{12} \\ + 0.0000675763321449903t^{13} \\ - 0.0000103564139477513t^{14} \\ - 0.0000140708642079103t^{15} - 2.564993578 10^{-7}t^{16} \\ + 0.00000200797937450533t^{17} + 1.512932613 10^{-7}t^{18} \\ - 2.099346146 10^{-7}t^{19} - 2.229781509 10^{-8}t^{20} \\ + 1.953373900 10^{-8}t^{21} + 1.230349420 10^{-9}t^{22} \\ - 1.530674249 10^{-9}t^{23} + 8.173929604 10^{-11}t^{24} \\ + 6.638546442 10^{-11}t^{25} - 1.221463855 10^{-11}t^{26} \\ - 3.329735922 10^{-13}t^{27} + 3.419411739 10^{-13}t^{28} \\ - 4.554252295 10^{-14}t^{29} + 2.703863174 10^{-15}t^{30} \\ - 6.354701471 10^{-17}t^{31} \end{array}$$

In this example, we let  $t \in [0,0.1]$  to execute the solution. The results will be plotted in Figure 3.

#### Example 3

Consider the linear NWS equation:  $u_t(x,t) = u_{xx}(x,t) - 3u^2(x,t)$ (24)

(25)

subject to the initial condition:

 $u(x,0)=e^{2x}$ 

where the exact solution is:  $u(x,t) = e^{2x+t}$ (26)

In view of IIM, the equivalent integral equation of (4.13) is

$$u = \lambda + \int_{0}^{t} (u_{xx} - 3u^2)dt$$
<sup>(27)</sup>

Thus, the first a few iterative solutions are,  $y_{1}(x, t) = a^{2x}$ 

$$u_{0}(x,t) = e^{2x} + e^{2x}t$$

$$u_{1}(x,t) = e^{2x} + e^{2x}t + \frac{1}{2}e^{2x}t^{2}$$

$$u_{2}(x,t) = e^{2x} + e^{2x}t + \frac{1}{2}e^{2x}t^{2} + \frac{1}{6}e^{2x}t^{3}$$

$$\vdots$$
(28)

Therefore, the  $u_n(x, t)$  solution is given by

$$u_n(x,t) = e^{2x}(1+t+\frac{t^2}{2}+\frac{t^3}{6}+\frac{t^4}{24}+\cdots$$
(29)

by using  $u(x,t) = \lim_{n \to \infty} u_n(x,t)$ , leads to  $u(x,t) = e^{2x+t}$ , which is the exact solution of equation (24).

#### 4. Results and Discussion

In this section we compare the fifth iteration of our result in the first example with the exact solution, fourth iterative solution obtained using NIM by Patade and Bhalekar [8] and LADM by Pue-On [5]. For the second example we compare the sixth iterative of our results with the exact solution, fourth term solution NIM by Patade and Bhalekar [8], ADM by Saravanan and Magesh [9].

Figure 1 presents the comparison solution by IIM, NIM, LADM and exact solution. The value of u(x, t) of IIM which is close to the exact solution compared to NIM and LADM. The results of this example show that the IIM is more accurate than ADM and LADM.

Furthermore, the comparison of magnitude error between IIM, exact solution, NIM and LADM have been depicted in Figure 2. At t = 0, the accuracy of these three methods is similar since the magnitude of error is 0. As t increase the accuracy reduce significantly. However, the magnitude errors of IIM are lower than NIM and LADM. Therefore, the IIM is proved to be more efficient and more accurate.

Figure 3 show the results by IIM, NIM, ADM and exact solution for example 2. At t = 0 or close to 0, all the methods are in good agreement with exact solution. However as t increase the results by IIM is more accurate than NIM and ADM.

It also can be seen in Figure 4 where the magnitude of error by IIM is close enough to zero compared to other methods. This indicates the efficiency and accuracy of IIM in solving NWS equation.



Figure 1. Comparison solution by IIM, NIM, LADM and exact solution for  $\lambda = 12$ 



Figure 2. Comparison of magnitude error between IIM, exact solution, NIM and LADM for  $\lambda = 12$ .



Figure 3. Comparison solution by IIM, NIM, ADM and exact solution for  $\lambda = 0.1$ 



Figure 4. Comparison of magnitude error between IIM, exact solution, NIM and ADM for  $\lambda = 0.1$ .

#### 5. Conclusion

In this work, IIM has been applied successfully for solving 3 examples of NWS equations. In the first example the obtained results are compared with those obtained by both NIM and LADM. On the other hand, for the example 2, the results are compared with NIM and ADM. The comparisons of the results by all the methods shows that IIM provide more accurate results than others. Meanwhile, in example 3, it is apparently seen that IIM is very efficient and powerful to get the exact solutions in a rapid convergent form. From all examples, indicates the reliability, efficiency and accuracy of IIM when implemented to NWS equation. The performance and computer friendly solution procedure of IIM proved that IIM is an excellent tool for system of differential equation.

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### **Conflict of Interest**

The authors declare no conflict of interest in the subject matter or materials discussed in this manuscript.

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