UNIVERSITI TEKNOLOGI MARA

A GENERALIZED CLASS OF CLOSE-TO-CONVEX FUNCTIONS

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ABSTRACT

Let $G(\alpha,\delta,\gamma)$ denote the subclass of analytic univalent functions f defined by $f(z)=z+\sum_{n=2}^{\infty}a_n z^n$ and satisfy the condition

$$\operatorname{Re}\left\{e^{i\alpha}\frac{f'(z)}{g'_\gamma(z)}\right\}>\delta$$

where $z\in D=\{z:|z|<1\}$, $|\alpha|\le\pi$, $0\le\gamma\le1$, $\cos\alpha>\delta$ and $g'_\gamma(z)=\dfrac{1+(1-2\gamma)z}{1-z}$. This class of functions is known as close-to-convex with the concentration on the generalization of the class. The thesis is concerned in finding the extremal properties and radius of convexity for functions in the class $G(\alpha,\delta,\gamma)$. Approximations for the radius of starlikeness, convolution properties are obtained and determination of the second Hankel determinant is discussed.

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CHAPTER 1

PRELIMINARIES

1.1 Introduction

Let A denote the class of functions analytic in the unit disc $D=\{z:|z|<1\}$. Functions f in A are said to be normalized if they have the form

$$f(z)=z+a_2z^2+\cdots+a_nz^n+\cdots=z+\sum_{n=2}^{\infty}a_nz^n,\quad (a_n\in\mathbb{C}),\tag{1.1.1}$$

and $a_2,a_3,\ldots$ are the coefficients of f. Let S be the subclass of A containing normalized univalent functions. Goodman (1983) defined a function $f(z)$ is said to be univalent in a domain D if the conditions $f(z_1)=f(z_2)$, $z_1\in D$, $z_2\in D$, imply that $z_1=z_2$.

Definition 1.1.1. (Duren (1983)). Let $f(z)$ be analytic and univalent in D with $f(0)=f'(0)-1=0$. Then $f(z)$ is called convex in D if, and only if, for $z\in D$,

$$\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right)>0.\tag{1.1.2}$$

We denote by K the class of all convex functions. Functions f satisfying (1.1.2) for a given $0<r<1$ are *convex* on $|z|<r$, and f is *convex* if (1.1.2) holds for all r.