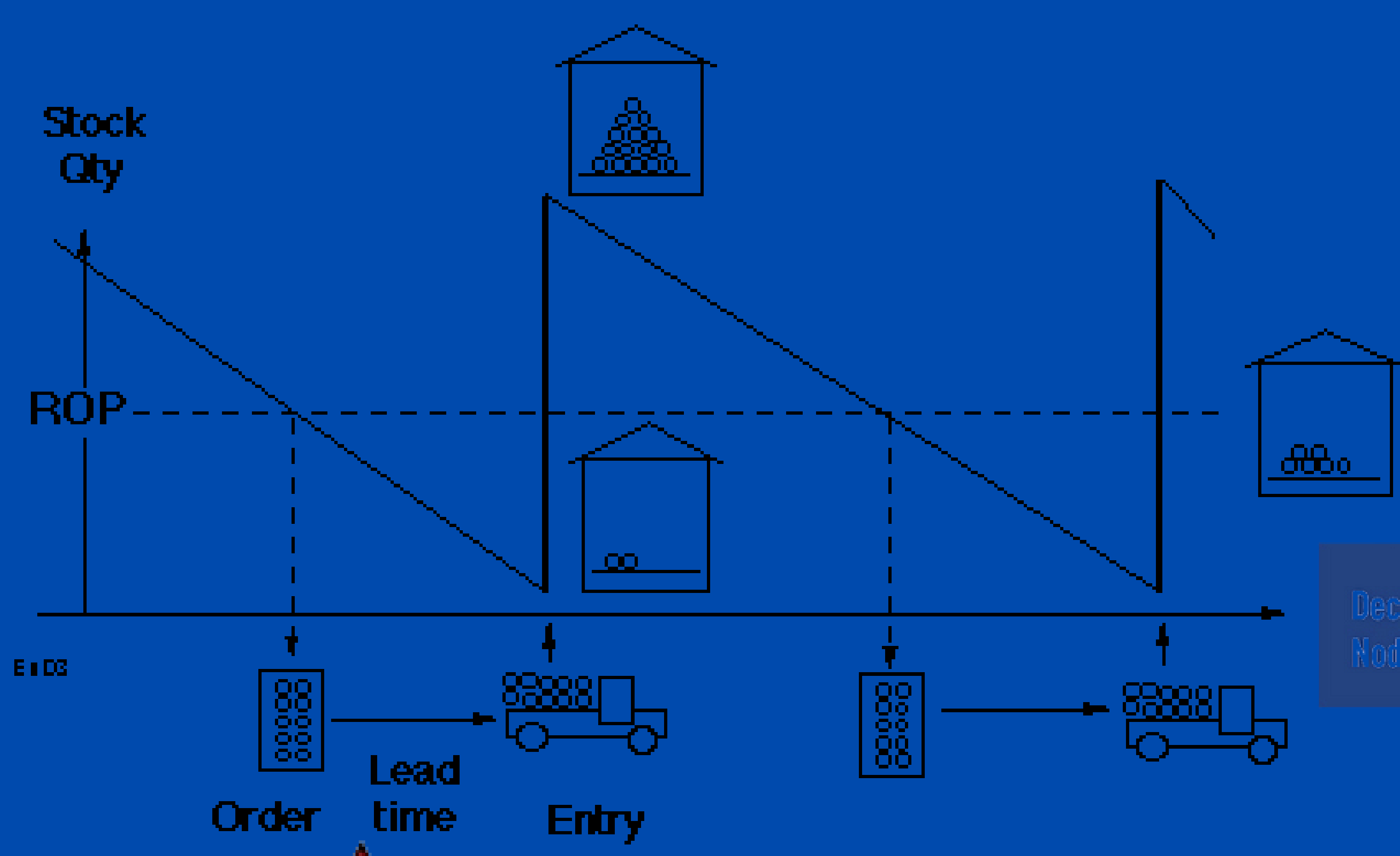
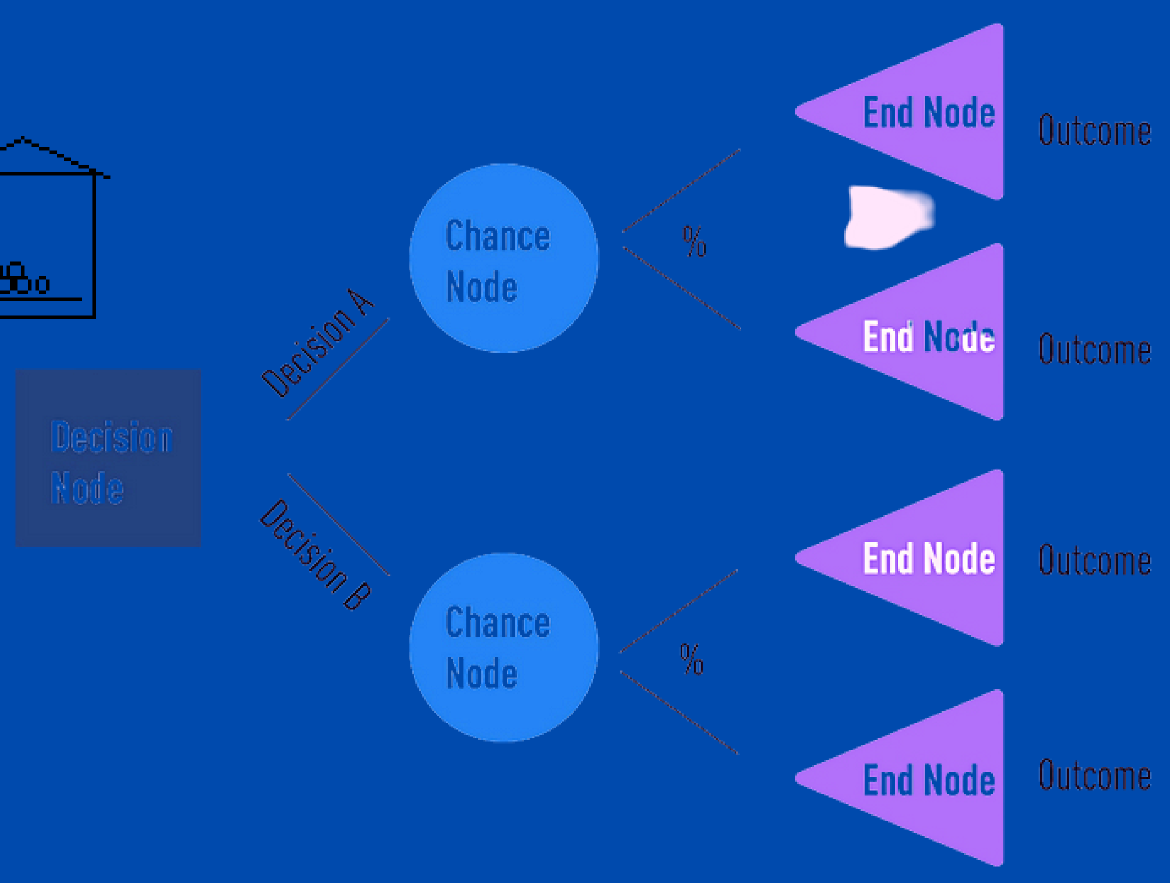
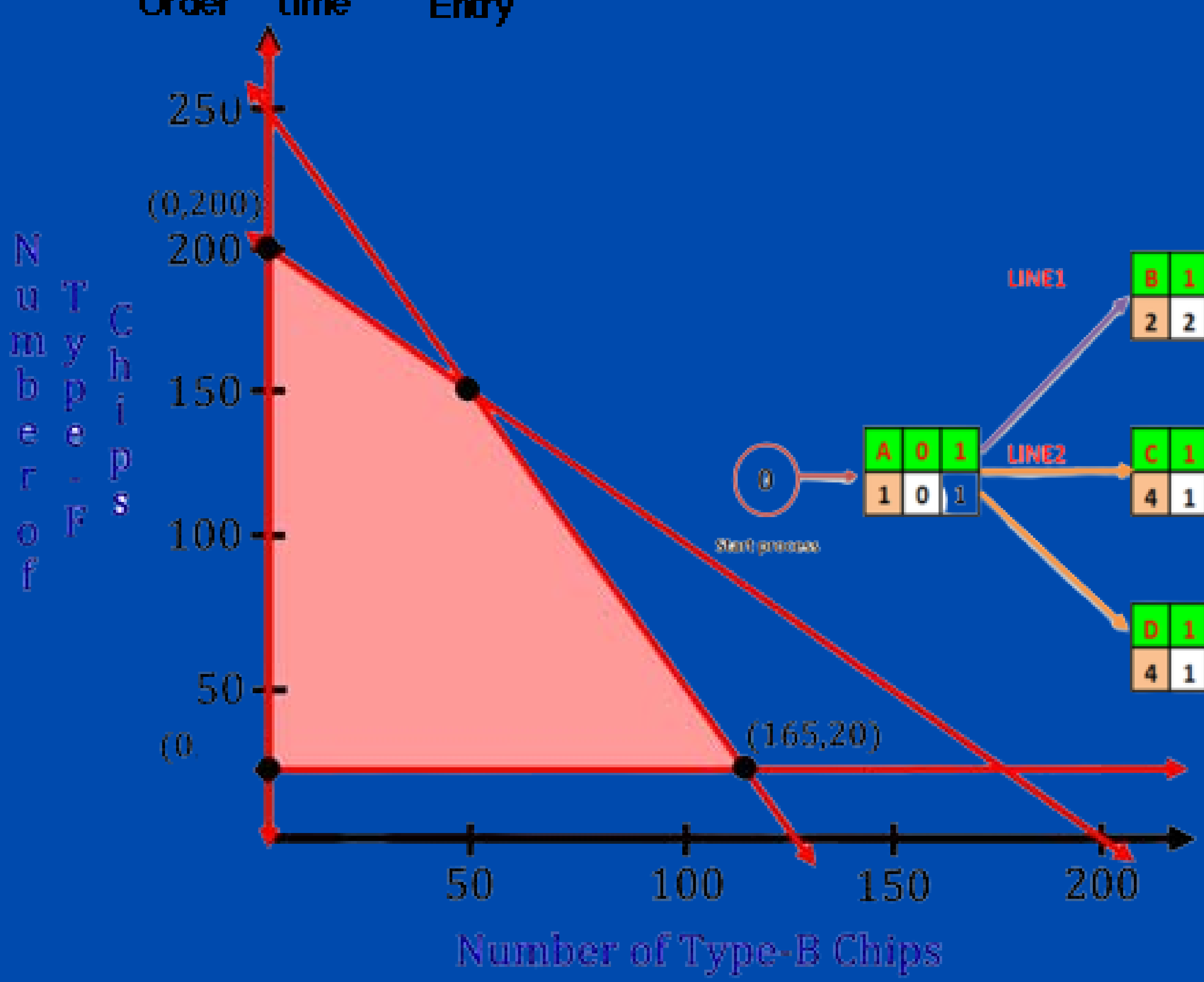
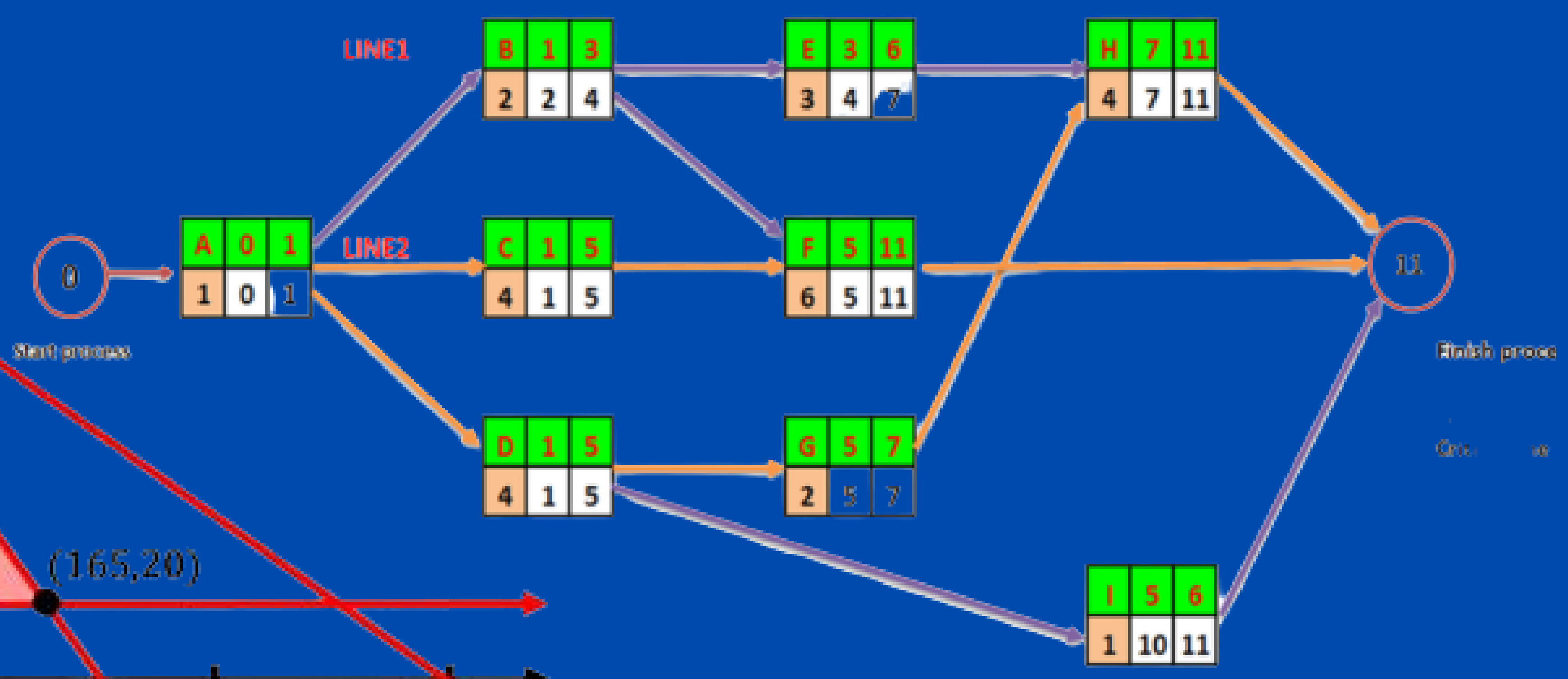
# WORKBOOK FOR QUANTITATIVE BUSINESS ANALYSIS

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FIRST EDITION

# Workbook for Quantitative Business Analysis

First Edition

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# Workbook For Quantitative Business Analysis



e-ISBN: 978-967-19663-8-9

Published by

FACULTY OF COMPUTER AND MATHEMATICAL SCIENCES
UNIVERSITI TEKNOLOGI MARA,
CAWANGAN JOHOR KAMPUS PASIR GUDANG
JALAN PURNAMA, BANDAR SERI ALAM
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# Preface

This Workbook for Quantitative Business Analysis is especially written for all students taking the quantitative analysis subject. The contents of the book are based on the basic topic of quantitative analysis relies on using mathematics to solve business issues with measurements based on verifiable information. This book is suitable for students who are taking diploma or degree in higher institution level.

This book contains seven chapters. In each chapter the introduction, definition, and steps are clearly stated with suitable guided and hands-on examples to ensure students clearly understood the concepts of each topic. At the end of each chapter, exercise are provided with full answer to provide full guidance for students to solve the problem.

The authors would like to express their gratitude and appreciation towards all their colleagues and peers who contributed directly and indirectly in successful publication of this book.

The authors would like to wish all the best to all students undertaking this subject and hopefully they will success in their study.


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# CHAPTER 1: INTRODUCTION TO QUANTITATIVE ANALYSIS


**Learning Outcome (LO):**

At the end of this chapter, students will be able to:

- Understand each process of quantitative analysis
- Understand the definition and type of model.
- Understand the characteristic of building a good model
- Understand the advantages and disadvantages of mathematical modelling.

## 1.1 Introduction

- Mathematical tools have been used for thousands of years
- Quantitative Analysis can be applied to a wide variety of problems
- We must understand the specific applicability of the technique, its limitations and its assumptions

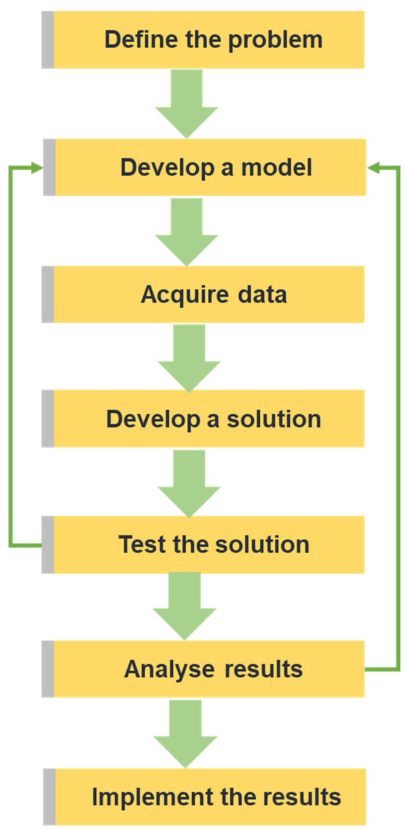
## 1.2 What is Quantitative Analysis?

**Overview of Quantitative Analysis**

- Scientific Approach to Managerial Decision Making
- Data is processed and manipulated into information that is valuable to people making decisions. This type of analysis is a logical and rational approach to making decision.
- We have to consider both Quantitative and Qualitative Factors

## 1.3 The Quantitative Analysis Approach

**The Quantitative Analysis Approach/Process/Steps:**




**A. Define the Problem**

- Problem should be define clearly
- This is a very important stage.
- Must go beyond symptoms to causes
- Problems are related to one another
- Must identify the “right” problem
- May require specific, measurable objectives

## B. Develop a model

- Model is a representation of a situation
- Types of models:

| | | |
|---|---|---|
| **Physical/iconic** | Usually made by architects Eg. Model of a building, car. | 01 |
| **Scale** | Usually made by engineers Eg. Model of a building. | 02 |
| **Schematic** | A picture, drawing or chart of reality eg. Automobiles, gears, fans, typewriters | 03 |
| **Mathematical** | A set of mathematical relationships (expressed in terms of equations and inequalities). It can be divided in to deterministic and probabilistic (stochastic) models.<br>a) Deterministic model: a model where all the input variables are known and have constant values. Eg. EOQ model in Inventory Control.<br>b) Probabilistic model: a model where at least one input variable is uncertain and with values that varies. Eg. Simulation models using Monte Carlo method. | 04 |

- To build a good mathematical model it must have the following characteristics:
  - Solvable – easy to find the solution
  - Realistic – represent a problem or a system under investigation correctly
  - Easy to understand
  - Easy to modified/manipulated

### Advantages & Disadvantages of Mathematical Modeling

**Advantages**

1. Model can accurately represent reality. If properly formulated, a model can be extremely accurate. A valid model is an accurate and correctly represents the problem under investigation.
2. In most cases, using model is faster and less expensive (save time and money) in decision making problem and problem solving than actually trying anew marketing campaign in a real business setting and observing the results.
3. Models can help a decision maker formulate problems – a decision maker can determine the important factors contribute to the problems.
4. Models can give us insight and information – the impact/changes on a certain factors to the problem can be identified.
5. A model may be the only way to solve some large or complex problems in a timely fashion.
6. A model can be used to communicate problems and solutions to others.

**Disadvantages**

1. In some cases, using a model can be expensive and time consuming in developing and testing the model.
2. Model can be often misused or misunderstood due to complexity.

**C. Acquire days**

- Accurate data is essential
- Data can be collected through primary and secondary data.
- Example, data can be collected from:
    - company reports
    - company documents
    - interviews
    - on-site direct measurement
    - statistical sampling

**D. Develop a solution**

- In order to develop a solution, we have to manipulate the model and find the "best" solution
- Criteria for a good solution are the solution must be practical and implementable
- Solution can be in various method. Example:
  a) solution of equation(s)
  b) trial and error
  c) complete enumeration
  d) implementation of algorithm

**E. Test the solution**

- Solution must be test for:
  a) Accuracy
  b) Completeness of input data
  c) collect data from a different source and compare
- Solution that are gained must be tested for :
  a) Input data
  b) Model
- Consistency. One way to check consistency is by answer the question 'Do they make sense?'
- The most importance things are test must be done before we analyze the data.

**F. Analyze the results and perform sensitivity analysis**

- Understand the actions implied by the solution
- Determine the implications of the action
- Conduct sensitivity analysis - change input value or model parameter and see what happens
    - Use sensitivity analysis to help gain understanding of problem (as well as for answers)

**G. Implement the results**

- Incorporate the solution into the company
- Monitor the results
- Use the results of the model and sensitivity analysis to help you sell the solution to management

**How to Develop a QA Model**

$$\begin{aligned}\text{Profits} &= \text{Revenue} - \text{Expenses} \\ &= [(\text{Price per unit}) \times (\text{Number Sold})] - [\text{Fixed Cost} + (\text{Variable Cost/Unit}) \times (\text{Number Sold})]\end{aligned}$$

**QA Models are very importance for managers/owner of a company to :**

a) Gain deeper insight into the nature of business relationships
b) Find better ways to assess values in such relationships.
c) See a way of reducing, or at least understanding, uncertainty that surrounds business plans and actions

**GUIDED EXAMPLE 1**

Please answers the following questions below:

a) Explain the meaning of Quantitative Analysis.

b) What is a model? Give two types of model and explain each of them.

c) Give two advantages of mathematical modeling.

d) Give two advantages of using Quantitative Analysis in problem solving.

**Solution:**

a) Scientific Approach to Managerial Decision Making
b) Model is a representation of a situation
   Types of models:
   i. Physical/iconic: usually made by architects
   ii. Scale: usually made by engineers

c) Model can accurately represent reality
   Model is faster and less expensive (save time and money).

d) Gain deeper insight into the nature of business relationships
   Find better ways to assess values in such relationships.

# CHAPTER 2: DECISION THEORY

**Learning Objective (LO):**

After completing this chapter, students will be able to

- Make decision under decision analysis under uncertainty.
- Make decision under decision analysis under risk using expected monetary value.
- Make decision under decision analysis under risk using expected loss value.
- Calculate expected value of perfect information and explain its meaning.
- Construct decision tree diagram and make decision.

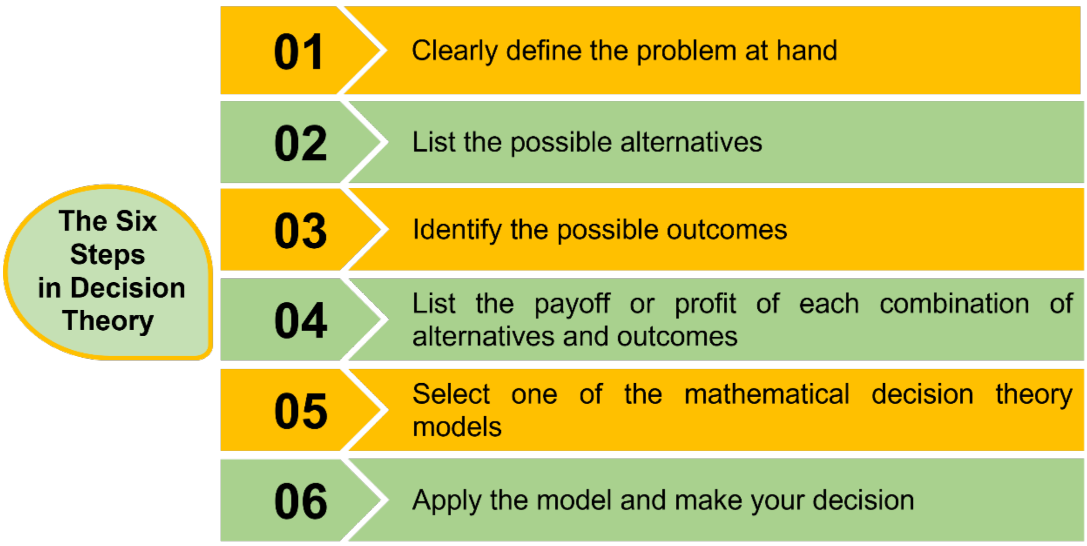
## 2.1 The Elements of Decision Problems

**Introduction:**

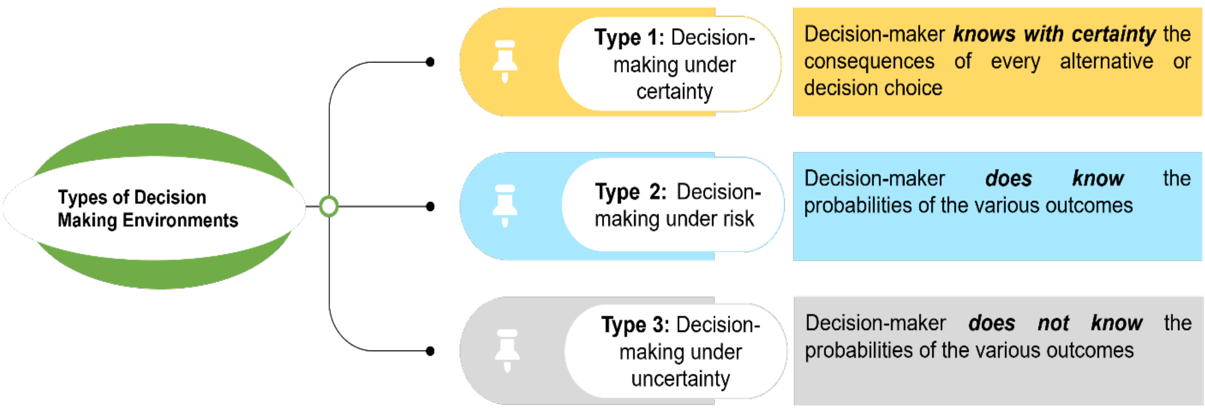
- Decision theory is an analytic and systematic way to tackle problems
- A good decision is based on logic, considers all available data & possible alternatives, & applies the quantitative approach.
- A bad decision is one that is not based on logic, doesn't use all available info doesn't consider all alternatives & doesn't employ appropriate quantitative techniques

**Terminologies:**

- Decision Maker (DM) = a person or a group of people who has to make a decision.
- Alternative (alt) = A set of decision/action/strategy to be chosen by DM. DM has a complete control over an alternative.
- State of Nature (SON) / Outcome = a set of future conditions, an event/outcome over which DM has no control. Eg. Market demand, Weather
- Payoff = profit/revenue/benefit or cost resulting from each combination of alt and SON.
- Decision/Payoff table = a table showing the alt, SON and Payoffs.




## 2.2 Type of Decision Making Environment



## 2.3 Decision Making Under Uncertainty

When several states of nature (SON) exist and manager can't assess the outcome and no probability data available.

**Maximax criterion (optimistic approach)**

- ❑ Locate the highest payoff for each alternative
- ❑ Choose the alternative with highest / maximum output which is maximize the maximum / the "best of the best"

**Maximin criterion (pessimistic approach)**

- ❑ Select the minimum payoff for each alternative
- ❑ Choose the alternative with the maximize the minimum output / the "best of the worst"

**Equally likely / Equal likelihood (Laplace)**

- ❑ Calculate the average payoff for every alternative
- ❑ Choose maximum average

**Criterion of Realism (Hurwicz) - uses the weighted average approach**

- ❑ Select the value of a, the coefficient of realism(CR). When the value is close to 1, the decision maker is optimistic about the future but when the value is close to 0, it is pessimistic about the future.
- ❑ Weighted average for each alternative (CR) $= \alpha(\text{max}\,imum \text{ in a row}) + (1-\alpha)(\text{minimum in a row})$
- ❑ Choose the alternative with highest CR.

**Minimax regret - choose the alternative with the minimum maximum Opportunity Loss(OL)**

- ❑ Find the opportunity loss (OL) table
- ❑ Select maximum OL for each alternative
- ❑ Select the alternative with the minimum value of maximum OL

**GUIDED EXAMPLE 1**

A company is setting the price for one of its products. The marketing department has formulated three pricing schemes, namely, PSA PSB and PSC. The estimated sales volume for three demand levels of high, medium and low are shown in the table below:

| | Levels of Demand | | |
|---|---|---|---|
| Pricing scheme | High (RM) | Medium (RM) | Low (RM) |
| PSA | 18000 | 16000 | 12000 |
| PSB | 16000 | 14500 | 10000 |
| PSC | 14500 | 14000 | 8000 |

Determine which pricing scheme should be chosen using the following decision criteria.

a) Maximax
b) Maximin
c) Minimax regret
d) Hurwicz ($\alpha = 0.2$)
e) Equal likelihood

**Solution:**

| | Levels of Demand | | | Maximax | Maximin | CR(Hurwicz) | Laplace |
|---|---|---|---|---|---|---|---|
| Pricing Scheme | High (RM) | Medium (RM) | Low (RM) | Maximum | Minimum | Weighted Average | Average |
| PSA | **18,000** | **16,000** | **12,000** | **18,000** | **12,000** | 13,200 | 15,333.33 |
| PSB | 16,000 | 14,500 | 10,000 | 16,000 | 10,000 | 11,200 | **40,500** |
| PSC | 14,500 | 14,000 | 8,000 | 14,500 | 8,000 | 9,300 | 36,500 |
| | | | Decision | **18,000** | **12,000** | | |

a) Maximax : the best decision is to choose PSA with the maximum estimated sales volume of RM18,000

b) Maximin : the best decision is to choose PSA with the highest minimum estimated sales volume of RM18,000

| | Levels of Demand | | | Opportunity Loss (LO) |
|---|---|---|---|---|
| Pricing Scheme | High (RM) | Medium (RM) | Low (RM) | Maximum |
| PSA | $18{,}000-18{,}000 = 0$ | $16{,}000-16{,}000 = 0$ | $12{,}000-12{,}000 = 0$ | **0** |
| PSB | $18{,}000-16{,}000 = 2{,}000$ | $16{,}000-14{,}500 = 1{,}500$ | $12{,}000-10{,}000 = 2{,}000$ | 2,000 |
| PSC | $18{,}000-14{,}500 = 3{,}500$ | $16{,}000-14{,}000 = 2{,}000$ | $12{,}000-8{,}000 = 4{,}000$ | 4,000 |

c) Minimax regret : the best decision is to choose PSA with the minimum estimated sales volume of zero.

d) Hurwicz ( $\alpha = 0.2$ )

$$\text{Weighted average PSA} = \alpha(\max imum \text{ in a row}) + (1-\alpha)(\text{minimum in a row})$$
$$= 0.2(18{,}000) + (1-0.2)(12{,}000)$$
$$= RM13{,}200$$

$$\text{Weighted average PSB} = \alpha(\max imum \text{ in a row}) + (1-\alpha)(\text{minimum in a row})$$
$$= 0.2(16{,}000) + (1-0.2)(10{,}000)$$
$$= RM11{,}200$$

$$\text{Weighted average PSB} = \alpha(\max imum \text{ in a row}) + (1-\alpha)(\text{minimum in a row})$$
$$= 0.2(14{,}500) + (1-0.2)(8{,}000)$$
$$= RM9{,}300$$

e) Equal likelihood : the best decision is to choose PSB with the highest average estimated sales volume of RM40,500.

**HANDS-ON EXAMPLE 1**

Kayangan Property, a real estate development firm, is considering several alternative development projects. The financial success of these projects depends on interest rate movement in the next five years. The various development projects and their five-year financial return (RM millions) given that interest rates will decline, remain stable, or increase are shown in the following table.

| Project | Interest Rates | | |
|---|---|---|---|
| | Decline | Stable | Increase |
| Office Building | 0.6 | 1.2 | 4.5 |
| Mall | 1.3 | 1.9 | 2.5 |
| Condominiums | 3.5 | 1.5 | 0.4 |
| Warehouse | 1.7 | 1.5 | 1.0 |

Determine the best investment using the following decision criteria.

a) Maximin
b) Hurwicz ( $\alpha = 0.40$ )
c) Maximax
d) Minimax regret
e) Equal likelihood

**Solution:**

## HANDS-ON EXAMPLE 2

Azman is considering several alternatives with regard to opening Mee Rebus Johor Restaurants. His expected payoff (RM '000) are shown below.

| Alternatives | Market conditions | | |
|---|---|---|---|
| | Good Market | Medium Market | Bad Market |
| **Open 1 restaurant** | 380 | 70 | $-400$ |
| **Open 2 restaurants** | 200 | 80 | $-200$ |

Determine the best decision made under:

a) Maximax criterion

b) Maximin criterion

c) Hurwicz criterion with $k = 0.55$

d) Minimax regret criterion

**Solution:**

Ans: a) & c) open 1 restaurant
b) & d) open 2 restaurant

## 2.4 Decision Making Under Risk

Decision making when there are several possible states of nature (SON) and we know the probabilities associated with each possible state.

Several possible state of nature occur & probabilities are known
A. Expected monetary value (EMV)
B. Expected opportunity loss (EOL)
C. Expected value of perfect information (EVPI)

**A. Expected monetary value (EMV)**

- The weighted sum of possible payoffs for each alternative.
- The expected value or mean value is the long run average value that would result if the decision were repeated a large number of times.
- Most popular method is to choose the alternative with the highest Expected monetary value

EMV (alternative $i$) = (Probability Of 1$^{st}$ SON $\times$ Payoff of 1$^{st}$ SON) $+$
(Probability of 2$^{nd}$ SON $\times$ Payoff of 2$^{nd}$ SON) $+\ldots+$
(Probability of last SON $\times$ Payoff of last SON)

**GUIDED EXAMPLE 2**

Robert, CEO of Megah Plantation Industries, is considering whether to build a manufacturing plant in Segamat. His decision is summarized in the following table:

| ALTERNATIVES | STATE OF NATURE | |
|---|---|---|
| | FAVORABLE MARKET (RM) | UNFAVORABLE MARKET(RM) |
| Build large plant (BLP) | 400,000 | $-300,000$ |
| Build small plant (BSP) | 80,000 | $-10,000$ |
| Don't build (DB) | 0 | 0 |
| Market probabilities | 0.4 | 0.6 |

*negative value = loss

Choose the best alternative using EMV.

**Solution:**

$$EMV(BLP) = (400{,}000)(0.4) + (-300{,}000)(0.6) = -20{,}000$$
$$EMV(BSP) = (80{,}000)(0.4) + (-10{,}000)(0.6) = 26{,}000$$
$$EMV(DB) = (0)(0.4) + (0)(0.6) = 0$$

**Decision:** The highest expected value is RM26,000
Therefore, Robert should elect to build small plant (BSP)

## HANDS-ON EXAMPLE 3

A manufacturing company is trying to set the price for one of its products. The marketing department has formulated three pricing policies, namely Economy (E), Moderate (M), and Expensive (X). The estimated sales volume under three demand levels of high, medium, and low is shown in the following payoff matrix.

| Pricing Policy | Demands Level | | |
|---|---|---|---|
| | High | Medium | Low |
| **Economy (E)** | RM22,000 | RM17,000 | RM14,000 |
| **Moderate (M)** | RM17,500 | RM16,000 | RM12,000 |
| **Expensive (X)** | RM15,500 | RM14,500 | RM16,000 |

The probability for demands level of high, medium and low styles are 0.30, 0.25 and 0.45 respectively. Which pricing policy would maximize the expected payoff?

**Solution:**

Ans: Pricing policy E
EMV=RM17.150

## HANDS-ON EXAMPLE 4

A shop has to decide on the number of a newspaper it should order each day. The daily demand for the newspaper for the past 40 days is as shown in the following table.

| Demand per day (copies) | Number of days |
|---|---|
| 100 | 20 |
| 120 | 10 |
| 140 | 10 |

The shop buys the newspaper for RM2 each and sell them at RM4 each. Any newspaper not sold by the end of the day will be returned to the publisher. In such case, the publisher will return RM0.50 per newspaper to the shop.

a) Construct a payoff table for this problem.

b) Determine how many copies of the newspaper the shop should order per day.

**Solution:**

Ans: order 120 copies per day

| Supply | Demands Level | | |
|---|---|---|---|
| | 100 | 120 | 140 |
| 100 | | | |
| 120 | | | |
| 140 | | | |
| Probability | | | |

## B. Expected opportunity loss (EOL)

The decision maker calculates the expected value of the opportunity loss value for each alternative, and then the minimum EOL will be chosen as the best decision.
Minimize EOL = Maximize EMV

- Expected opportunity loss (EOL) is the cost of not picking the best solution. Opportunity loss(OL) = regret
- Calculating EOL:
  a) Construct opportunity loss table.
  b) Multiply the opportunity loss for each alternative with the probability & add altogether.

Opportunity Loss(OL) $=$ Optimum/best payoff of a SON $-$ Actual payoff received

EOL (alternative $i$) $=$ (Probability Of 1$^{st}$ SON $\times$ Opportunity Loss of 1$^{st}$ SON) $+$
(Probability Of 2$^{nd}$ SON $\times$ Opportunity Loss of 2$^{nd}$ SON) $+ \ldots +$
(Probability of last SON $\times$ Opportunity Loss of last SON)

## GUIDED EXAMPLE 3

Refer to **Guided Example 2**, choose the best alternative using the expected opportunity loss (EOL).

**Solution:**

| ALTERNATIVES | STATE OF NATURE | |
|---|---|---|
| | FAVORABLE MARKET (RM) | UNFAVORABLE MARKET(RM) |
| Build large plant (BLP) | $400{,}000-400{,}000$ <br> $=0$ | $0-(-300{,}000)$ <br> $=300{,}000$ |
| Build small plant (BSP) | $400{,}000-80{,}000$ <br> $=320{,}000$ | $0-(-10{,}000)$ <br> $=10{,}000$ |
| Don't build (DB) | $400{,}000-0$ <br> $=400{,}000$ | $0-0$ <br> $=0$ |
| Market probabilities | 0.4 | 0.6 |

$$EOL(BLP)=(0)(0.4)+(300{,}000)(0.6)=180{,}000$$
$$EOL(BSP)=(320{,}000)(0.4)+(10{,}000)(0.6)=134{,}000$$
$$EOL(DB)=(400{,}000)(0.4)+(0)(0.6)=160{,}000$$

**Decision:** The lowest expected opportunity loss (EOL) value is RM134,000
Therefore, Robert should elect to build small plant (BSP)

## HANDS-ON EXAMPLE 4

Refer to **Hands-on Example 3**, choose the best alternative using the expected opportunity loss (EOL).

**Solution:**

Ans: Pricing policy E
EOL=RM900

**HANDS-ON EXAMPLE 5**

Refer to **Hands-on Example 4**, choose the best alternative using the expected opportunity Loss (EOL).

**Solution:**

Ans: order 120 cpies per day
EOL=RM25

## C. Expected value of perfect information (EVPI)

- Expected value of perfect information (EVPI) is the maximum amount of decision maker would pay for the perfect information and places an upper bound on what to pay for information.
- Expected value with perfect information (EVWPI) is computed by calculating the maximum value of the payoffs associated to each state of nature, and finding the expected value of those maximum values. To calculate EVWPI

EVWPI (alternative $i$) $=$ (Probability Of 1st SON $\times$ Best Payoff of 1st SON) $+$
(Probability Of 2nd SON $\times$ Best Payoff of 2nd SON) $+\ldots+$
(Probability of last SON $\times$ Best Payoff of last SON)

EVPI $=$ EVWPI $-$ maximum EMV

OR

EVPI $=$ minimum EOL

## GUIDED EXAMPLE 4

Refer to **Guided Example 2 & 3**, choose the best alternative using the expected value of perfect information (EVPI). Comment/interpret the value.

**Solution:**

Best payoff of favorable market $= 400{,}000$
Best payoff of unfavorable market $= 0$

$$EVWPI = (400{,}000)(0.4) + (0)(0.6) = 160{,}000$$

Maximum EMV = 26,000 ***(refer Guided Example 2)***
EVPI = EVWPI – maximum EMV $= 160{,}000 - 26{,}000 = 134{,}000$

**OR**

Minimum EOL = 134,000 ***(refer Guided Example 3)***

EVPI = minimum EOL = 134,000

**Comment:**
Based on the EVPI, the most Robert would be willing to pay for perfect information is RM134,000.

## HANDS-ON EXAMPLE 6

Refer to **Hands-on Example 3 & 4**, choose the best alternative using the expected value of perfect information (EVPI). Comment/interpret the value.

**Solution:**

Ans: EVPI=RM900

## HANDS-ON EXAMPLE 7

Green Cosmetic Company wants to strengthen their market power based on these three products. However, due to the limited budget for the R&D phase, the company needs to choose which product that they need to strengthen. The following payoff table indicates the return (RM'000) of sales in three products of cosmetic under Green Cosmetic Company;

| Product | Economic Situation | | |
|---|---|---|---|
| | Good | Stable | Poor |
| **Facial Night Cream** | 8500 | 6500 | −3000 |
| **Foot Cream** | 4500 | 3500 | 3500 |
| **Lotion Body** | 9200 | 4500 | 4000 |

a) Find the best decision using $\alpha = 0.7$.
b) If the probability of good market condition is 0.6, and the probability portion of the stable and poor market condition is equal, what is the best decision?
c) By using the same probability market condition, what is the best decision based on the expected opportunity loss value?
d) Determine how much Green Cosmetic Company would be willing to pay a market research firm to gain better information about the probability of market condition outcome?

**Solution:**

Ans: a), b) & c) Lotion Body product (7640, 7220 & 400), d) EVPI=400

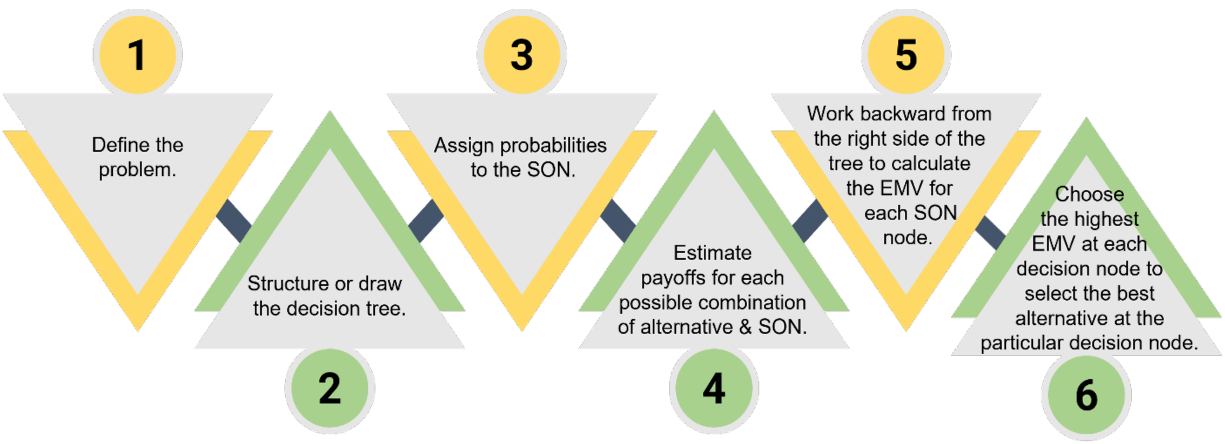
## 2.5 Decision Tree

- A decision tree is a specific type of flow chart used to visualize the decision-making process by mapping out different courses of action, as well as their potential outcomes.
- Any problem that can be presented in a decision table can be presented in decision tree.
- Contain Decision Points *(Decision Nodes)* & State of Nature(SON) Points *(State of Nature Nodes).*

Square ❯ A decision node from which one of several alternatives may be chosen.

Circle ❯ A state of nature node.

### A. 6 Steps of Decision Tree Analysis

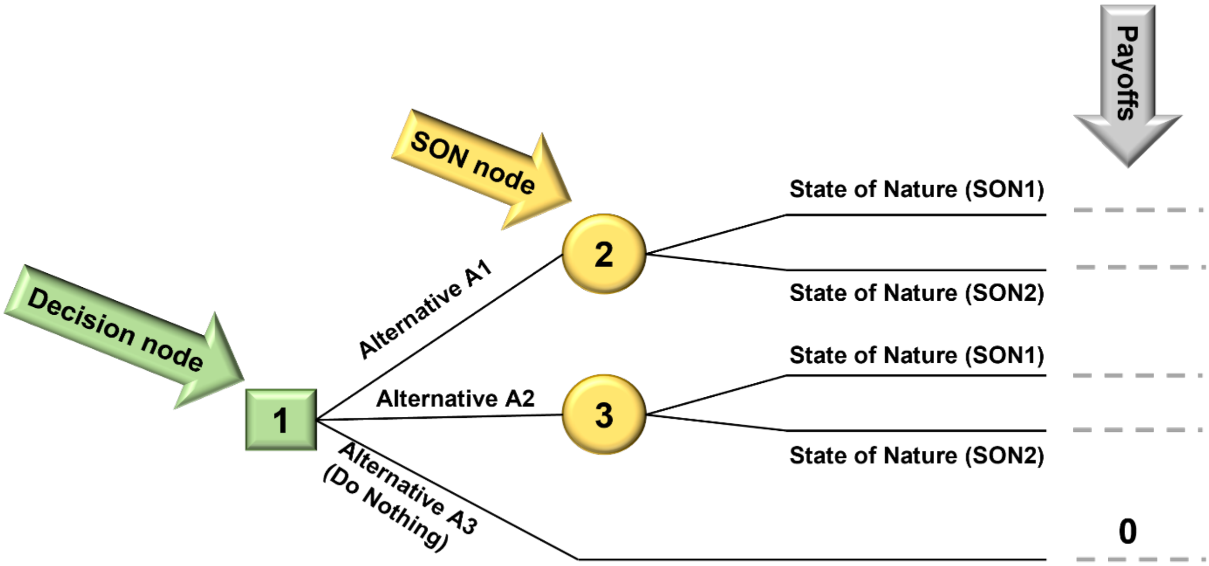


## B. Drawing a Decision Tree

**Tips**

1. Begin drawing from the left to the right.
2. Lines or branches from squares represent alternatives.
3. Lines or branches from circles represent state of nature(SON).

## C. Structure of a Decision Tree




## D. Completing the Structure of a Decision Tree

1. Payoffs are placed at the right side of each tree's branches.
2. Probabilities are shown in parentheses next to each SON.
3. Calculate the EMV for each node.
4. The branch leaving the decision node leading to the SON node with the highest EMV will be chosen.

## E. Expected Value of Sample Information (EVSI)

- Measures the value of sample information (how far the value of survey will help in making decision).
- EVSI is calculated based on imperfect information (SON can't be determined with certainty). The imperfect information is usually obtained from sampling and survey/research and the result obtained from survey will be used to update/improve the information/probability found or estimated.
- It can be regarded as the amount the DM is willing to pay for additional information.

EVSI = (EMV with sample info + cost of survey) − (EMV without sample info)

**Note:** If payoff values had not already subtracted the study cost, do not add cost of survey.

EVSI = (EMV with sample info assuming no cost to gather it) − (EMV without sample info)

**GUIDED EXAMPLE 5**

Binary Sdn. Bhd. intends to build either a shopping mall or a hotel in Masai. The company also considers not proceeding with the project. If market is favourable, the company will earn a profit of RM750,000 from the shopping mall and RM450,000 from the hotel. However, if the market is unfavourable, there would be a loss of RM400,000 with the shopping mall and 250,000 with the hotel. The probability of a favourable market is 50%.

Before making the decision on the project, Binary Sdn. Bhd is considering hiring a marketing research firm at the cost of RM120,000 to obtain additional information. The result is either positive or negative. The probability of positive result is 70%. The probability of favourable market given positive result is 0.65 while the probability of unfavourable market given negative result is 0.55.

a) Construct a decision tree for the above problem and advise the company for the best decision.
b) Find the maximum amount that it would be worthwhile for the company to pay for the additional information.

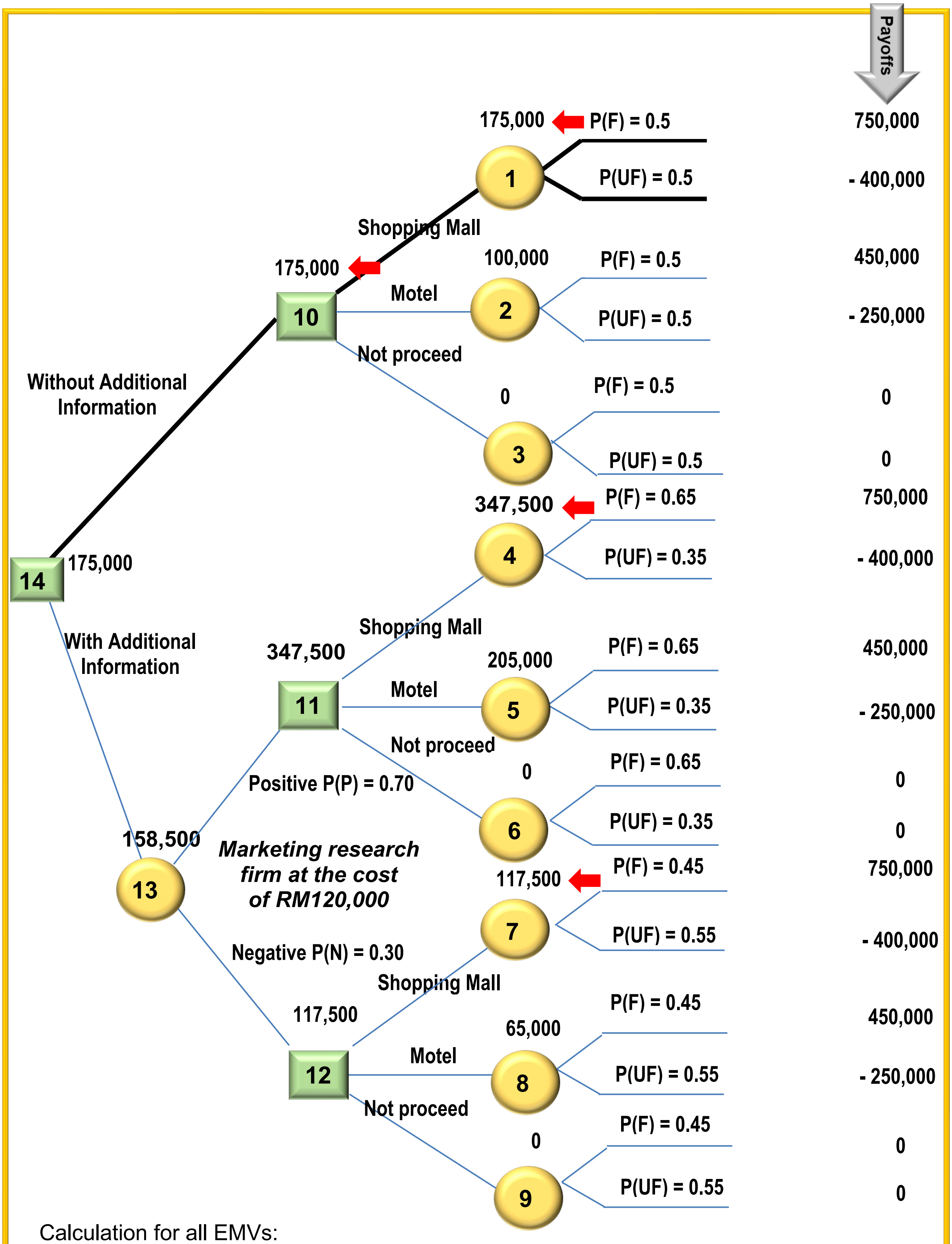
**Solution:**

a)

| Alternative | Profit (RM) | |
|---|---|---|
| | Favourable Market | Unfavourable Market |
| Shopping Mall | 750,000 | −400,000 |
| Motel | 450,000 | −250,000 |
| Not Proceed | 0 | 0 |
| Probability | 0.50 | 0.50 |

Probability (P):

P(Positive), P(P) = 0.70 P(Negative), P(N) = 0.3
P(Favourable/Positive), P(F/P) = 0.65 P(Unfavourable/Positive), P(UF/P) = 0.35
P(Favourable/Negative) P(F/N) = 0.45 P(Unfavourable/Negative), P(UF/N) = 0.55




Calculation for all EMVs:

1 $EMV_1 = [(750{,}000 \times 0.5) + (-400{,}000 \times 0.5)] = 175{,}000$

2 $EMV_2 = [(450{,}000 \times 0.5) + (-250{,}000 \times 0.5)] = 100{,}000$

3 $EMV_3 = [(0 \times 0.5) + (0 \times 0.5)] = 0$

4 $EMV_4 = [(750{,}000 \times 0.65) + (-400{,}000 \times 0.35)] = 347{,}500$

5 $EMV_5 = [(750{,}000 \times 0.65) + (-400{,}000 \times 0.35)] = 205{,}000$

6 $EMV_6 = [(0 \times 0.65) + (0 \times 0.35)] = 0$

7 $EMV_7 = [(750{,}000 \times 0.45) + (-400{,}000 \times 0.55)] = 117{,}500$

8 $EMV_8 = [(450{,}000 \times 0.45) + (-250{,}000 \times 0.55)] = 65{,}000$

9 $EMV_9 = [(0 \times 0.45) + (0 \times 0.65)] = 0$

10 $EMV_{10} = 175{,}000$ is highest EMV value compared to decision node of $100{,}000$ and 0.

11 $EMV_{11} = 347{,}500$ is highest EMV value compared to decision node of $205{,}000$ and 0.

12 $EMV_{12} = 117{,}500$ is highest EMV value compared to decision node of $65{,}000$ and 0.

13 $EMV_{13} = [(347{,}500 \times 0.7) + (117{,}500 \times 0.3)] - 120{,}000 = 158{,}500$

14 $EMV_{14} = 175{,}000$ is highest EMV value compared to decision node of $158{,}500$

∴ Binary Sdn. Bhd. should **not hire a marketing research** firm to obtain additional information because from the result shown that the company **should build shopping mall in Masai**.

b) Since the company is hiring a marketing research firm at the cost of RM120,000 then

$$
\begin{aligned}
EVSI &= EMV_{with} + cost - EMV_{without} \\
&= 158{,}500 + 120{,}000 - 175{,}000 \\
&= RM103{,}500.
\end{aligned}
$$

Hence, the **maximum amount** that it would be worthwhile for the company to pay for the **additional information** is **RM103,500.**

## GUIDED EXAMPLE 6

An oil and gas company are considering of getting a drilling right from the government. If the company wins the rights, the management has three alternatives of extracting the oil and gas. It can develop a new method for extracting the oil and gas, use present process or sell the rights of drilling to another company for RM80 million. The development of a new method, which is highly automated cost RM30 million. The outcomes and probabilities associated with developing the new method are as follows:

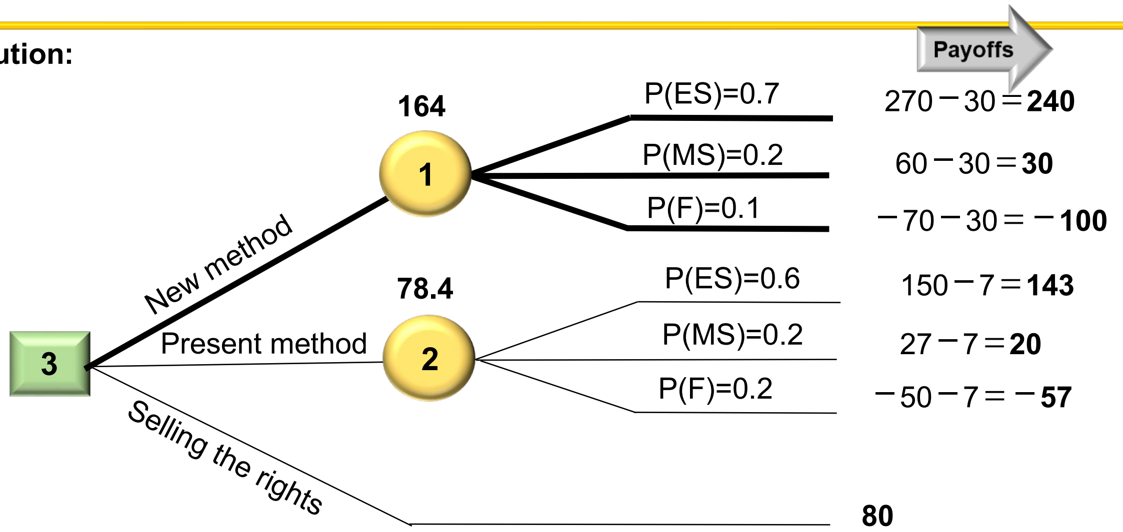
| State of Nature | Probability | Financial outcome (RM million) |
|---|---|---|
| Extremely successful (ES) | 0.7 | 270 |
| Moderately successful (MS) | 0.2 | 60 |
| Failure (F) | 0.1 | $-70$ |

The present method costs RM7 million, and the outcomes and probabilities for this alternative is given as follows:

| State of Nature | Probability | Financial outcome (RM million) |
|---|---|---|
| Extremely successful (ES) | 0.6 | 150 |
| Moderately successful (MS) | 0.2 | 27 |
| Failure (F) | 0.2 | $-50$ |

Construct a decision tree for this problem and determine the optimal decision strategy.

**Solution:**




**1** $EMV_1 = \left[(240\times0.7)+(30\times0.2)+(-100\times0.1)\right]=164$

**2** $EMV_2 = \left[(143\times0.6)+(20\times0.2)+(-57\times0.2)\right]=78.4$

**3** $EMV_3 = 164$ is highest EMV value compared to decision node of 78.4 and 80.

∴ The optimal decision strategy is to develop of a new method.

## HANDS-ON EXAMPLE 8

Farah is thinking about opening a Hijabista Fashion Store. She evaluates two sites: Kuala Lumpur and Johor Bahru. She has done investment analysis on the potential return of the store operation. The estimated profits (RM) of successful and unsuccessful markets at the two locations are in the following table:

| Location | successful | unsuccessful |
|---|---|---|
| **Kuala Lumpur** | 400,000 | −200,000 |
| **Johor Bahru** | 300,000 | −100,000 |

The probability of a successful market without performing a survey is 60% at Kuala Lumpur and 75% at Johor Bahru. She is considering the possibility of hiring a marketing firm to conduct research. It will cost RM2,500. The Previous report showed that there is a 60% chance that the research result will be positive. Furthermore, the revised probabilities for the successful or unsuccessful stores at the two locations given that the research indicates positive or negative results are as follows:

Kuala Lumpur:
*P*(Successful given that research positive) = 0.84
*P*(Unsuccessful given that research negative) = 0.64

Johor Bahru:
*P*(Successful given that research positive) = 0.91
*P*(Unsuccessful given that research negative) = 0.47

a) Draw the decision tree to represent the situation.
b) Determine the expected monetary value (EMV) and give your recommendation to Farah.
c) Compute the expected value of sample information (EVSI). Then, explain.

**Solution:**

Ans: b) $EMV_1$ ….. $EMV_{11}$ = 160,000 …. 224,700, should conduct a survey & open at KL(positive) or open at JB(negative results), c) EVSI =RM27,200

## HANDS-ON EXAMPLE 9

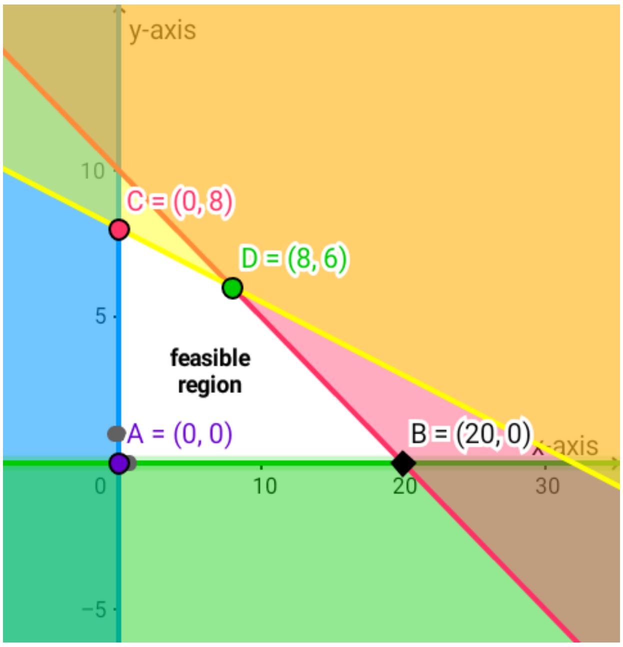
Mr. Chua needs to decide whether to build a petrol station or food court centre on a piece of land that he bought two years ago from his best friend. He also has the option of selling land with a profit of RM50,000. The successful market with probability of 0.70 will profit his RM83,000 for petrol station and RM60,000 for food court centre. With unsuccessful market, he will suffer losses of RM24,000 and RM29,000 for petrol station and food court centre respectively. Before a decision is made, he has the opportunity of conducting a customer survey at a cost of RM18,000 with 0.55 probability of positive result and hence influence the probability of the market. The probability of successful market given a positive survey result is 0.90, while with a negative survey result is 0.40.

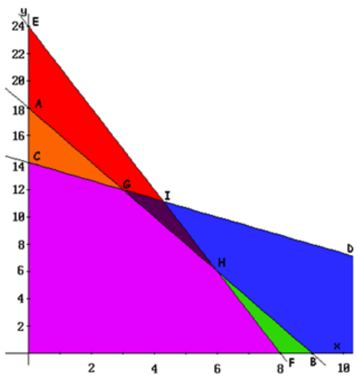
a) Construct a decision tree diagram for this problem.
b) Advice Mr. Chua for the best decision by calculating all expected monetary values (EMV).
c) Determine the expected value of sample information (EVSI).

**Solution:**

Ans: b) $EMV_1$ ….. $EMV_{11}$ = 72,300 …. 50,900, should not conduct a survey & build a petrol station c) EVSI =RM11,365

# CHAPTER 3: LINEAR PROGRAMMING



**Learning Objective (LO):**

After completing this chapter, students will be able to

- Formulate linear programming model
- Solve linear programming problem using graphical method
- Solve linear programming problem (maximation & minimization) using simplex method

## 3.1 Introduction and Requirements of Linear Programming (LP) Problem

Mathematical modeling technique in which a linear function is maximized or minimized when subjected to various constraints. This technique has been useful for guiding quantitative decisions in business planning, in industrial engineering, and to a lesser extent. Linear programming (LP) is a technique that helps in resource allocation decisions

### A. Requirements of LP problem

| | | |
|---|---|---|
| **Decision variables** | Mathematical symbols representing levels of activity of a firm | 01 |
| **Objective function** | A linear mathematical relationship describing an objective of the firm, in terms of decision variables – this function is to be maximized or minimized | 02 |
| **Constraints** | Requirements or restrictions placed on the firm by the operating environment, stated in linear relationships of the decision variables | 03 |
| **Parameters** | Numerical coefficients and constants used in the objective function and constraints. | 04 |

**01**

All problems seek to maximize or minimize some quantity (the objective function).

**02**

The presence of restrictions or constraints, limits the degree to which we can pursue our objective.

**03**

There must be alternative courses of action to choose from.

**04**

The objective and constraints in linear programming problems must be expressed in terms of linear equations or inequalities.

## B. Basic Assumptions of LP

**Proportionality**

- The rate of change (slope) of the objective function and constraint equations is constant
- Exists in the objective & constraints.
- Example: If production of 1 unit needs 3 hours of scarce resources, then producing 10 units will need 30 hours of those resources.

**Additivity**

- Terms in the objective function and constraint equations must be additive.
- The total of all activities equals the sum of the individual activities.
- Example: If an objective is to maximize profit:
  1st product = RM4, 2nd product = RM6
  When product produced, the profit contribution must add up to produce sum of RM10.

**Divisibility**

- Decision variables can take on any fractional value and therefore continuous as opposed to integer in nature.
- The solutions need not be in whole numbers (integers).
- Divisible and may take any fractional value.

**Certainty**

- Value of all the model parameters are assumed to be known with certainty (non-probabilistic).
- Numbers in the objective & constraints are known with certainty & do not change during the period being studied.

**Non-negativity**

- All answers or variables are greater than or equal to ( ≥ ) zero which are nonegative.
- All answers or variables negative values are impossible for physical quantities. For example: tables, pens, pants, cars etc.

# 3.2 Formulating LP problems

## A. LP model

- Involves developing a mathematical model to represent the managerial problem.
- The steps are:

**01** Understand the problem

The goal of a linear programming problems is to find a way to get the most, or least, of some quantity -- often profit or expenses. This quantity is called the objective. The answer should depend on *how much* of some *decision variables* you choose. Your options for how much will be limited by *constraints* stated in the problem.

**02** Describe the objective

What are you trying to optimize? Are you trying to minimize costs? Maximize production quantities? You may have constraints like "you can't spend more than RM1,000" or "you must ship at least 50 tons of product C". These are limits but don't necessarily reflect your employer's top priorities; don't mistake these for suggestions that you minimize costs or maximize production.

**03** Describe the constraints

What are the limits on "how much" your decision variables can be? Look for words like "at least", "no more than", "two thirds of of", "we must fill orders for", etc.

**04** Define the decision variables

The answer to a linear programming problem is always "how much" of some things. What are those things? Choose variables to represent how much of each of those things. For example:
X = number of leadership training programs offered
Y = number of problem solving programs offered.

**05** Write the objective function

Use the variables you just chose to write down an algebraic expression that describes the amount you're trying to minimize. Use the sign $\geq$ or $\leq$.

**06** Write the constraints in terms of the decision variables

For each constraint such as "at least RM500" or "no more than 29" write an inequality using the decision variables. For example: $2X > 500$
$X + Y \leq 29$

**07** Add the nonnegativity constraints

Don't forget to include non-negativity constraints like $Y \geq 0$

**08** Write it up the full formulation

For example:

Maximize profit $Z = 2X + 3Y$

Subject to constraints: $2X > 500$ (leadership training prog. constraint)
$X + Y \leq 29$ (problem solving prog. constraint)
$X, Y \geq 0$

- Most common LP application: product mix problems.

Use LP to decide how much of each product to make, given a series of resource restrictions

## GUIDED EXAMPLE 1

As a farmer, Zoul wants to determine the number of acres of land to be planted with mangos and guavas. He has 80 acres of land and has allocated a capital of RM10,000 to develop the land. The expenditures needed to plant mangos and guavas per acre are RM100 and RM150 respectively. The average yields per acre are 500 kg of mangos and 300kg of guavas.

The fruits are kept in the backyard store before they are sold. The store can only hold 6000kg of fruits at one time. The net profit of each kg of mangos and guavas are RM0.60 and RM0.90 respectively.

Formulate the above problem as a linear programming model to maximize the net profit.

**Solution:**

**Step 2 & 3:** Identify objective and constraints

- Objective: maximize profit
- Constraints:
  - The expenditures needed to plant mangos and guavas per acre are RM100 and RM150
  - The average yields per acre are 500kg of mangos and 300kg of guavas

**Step 4:** Define the decision variables

- $X =$ number of acres of land to be planted with mangos
- $Y =$ number of acres of land to be planted with guavas

  Now we can create the LP objective function in terms of $X$ and $Y$.
  - Maximize profit: $0.60X + 0.90Y$

**Step 5 & 6:** Develop mathematical relationship to describe the 2 constraints.

**The total expenditures**

- Total expenditure = (RM100/acre)(no. of acres of land to be planted with mangos) + (RM150/acre)(no. of acres of land to be planted with guavas)
- 1st constraint can be stated as:
  - Expenditures needed is ≤ capital allocation to develop
  - $100X + 150Y \leq 10{,}000$ (The expenditures needed to plant mangos and guavas per acre)

**The average yields**

- Total yields = (500kg/acre)(The average yields of mangos) + (300kg/acre)(The average yields of guavas)
- 2nd constraint can be stated as:
  - Average yields per acre is ≤ The fruits are kept in the backyard store
  - $500X + 300Y \leq 6{,}000$ (The average yields per acre of mangos and guavas)
- Both constraints represent production capacity restrictions & affect the total profit

**Step 7:** To obtain meaningful solutions, the values of $X$ and $Y$ must be nonnegative numbers.

- $X \geq 0$ (number of acres of land to be planted with mangos is greater than or equal to 0)
- $Y \geq 0$ (number of acres of land to be planted with guavas is greater than or equal to 0)

**Step 8:** Write the full formulation

| | |
|---|---|
| **Maximize profit** | $Z = 0.60X + 0.90Y$ |
| **Subject to constraints:** | $100X + 150Y \leq 10{,}000$ (expenditures needed to plant) |
| | $500X + 300Y \leq 6{,}000$ (average yields per acre) |
| | $X, Y \geq 0$ |

## HANDS-ON EXAMPLE 1

A company makes two products P and Q using two machines A and B. Each unit of P that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Q that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B. Machine A is going to be available for 40 hours and machine B is available for 35 hours. The profit per unit of P is RM25 and the profit per unit of Q is RM30. Company policy is to determine the production quantity of each product in such a way as to maximize the total profit given that the available resources should not be exceeded. Formulate the problem as a linear programming model.

**Solution:**

Ans: $P = 25x + 30y$ , $50x + 24y \leq 2400$ , $30x + 33y \leq 2100$ , $x, y \geq 0$

## HANDS-ON EXAMPLE 2

The Continuing Education Division of the UiTM Community College offers at most 30 courses each semester. The courses offered are usually of two types: practical (such as woodworking, wood processing and car maintenance) and humanistic (such as history, music, and fine arts). To satisfy the demand of the community, at least 10 courses of each type must be offered each semester. The division estimates that the profits of offering practical and humanistic courses are approximately RM1500 and RM1000 per course, respectively. Formulate this problem as a linear programming model.

**Solution:**

Ans: $Z = 1500x + 1000y$ , $y \geq 10$ , $y \geq 10$ , $x + y \leq 30$ $x, y \geq 0$

## 3.3 Graphical solution to LP problem(two-variable)

The graphical solution approach works only when there are 2 decisional variables, but it provides valuable insight into how larger problems are structured. The following below are the steps for graphical solution or method:

Step 1 Formulate the LP Problem

Step 2 Construct a graph and plot the constraint lines

Step 3 Determine the valid side of each constraint line

Step 4 Identify the feasible solution region

Step 5 Find the optimum points

Step 6 Calculate the co-ordinates of optimum points

Step 7 Evaluate the objective function at optimum points to get the required maximum/minimum value of the objective function

## A. Feasible Solution And Corner Point Method/Iso-Profit Line Method/Iso-Cost Line Method

### 1) The Feasible Solution

- Firstly, plot each of the problem's constraint of a graph.
    - Plotting involves finding points at which the line intersects the $X$ and $Y$ axes.
- Find the set of solution points that satisfies all the constraints simultaneously.
    - The feasible region is the overlapping area of constraints that satisfies all the restrictions on resources.

Any point in the region would be a feasible solution to the problem & any point outside the shaded area would represent an infeasible solution

### 2) Corner Point Method

- Looking at the profit at every corner point of the feasible region.
- The mathematical theory behind LP is that the optimal solution must lie at one of the corner points in the feasible region.

**Step 1** Find the coordinates of each corner point that had been labeled as 1, 2….n

**Step 2** Evaluate the objective function at each corner point

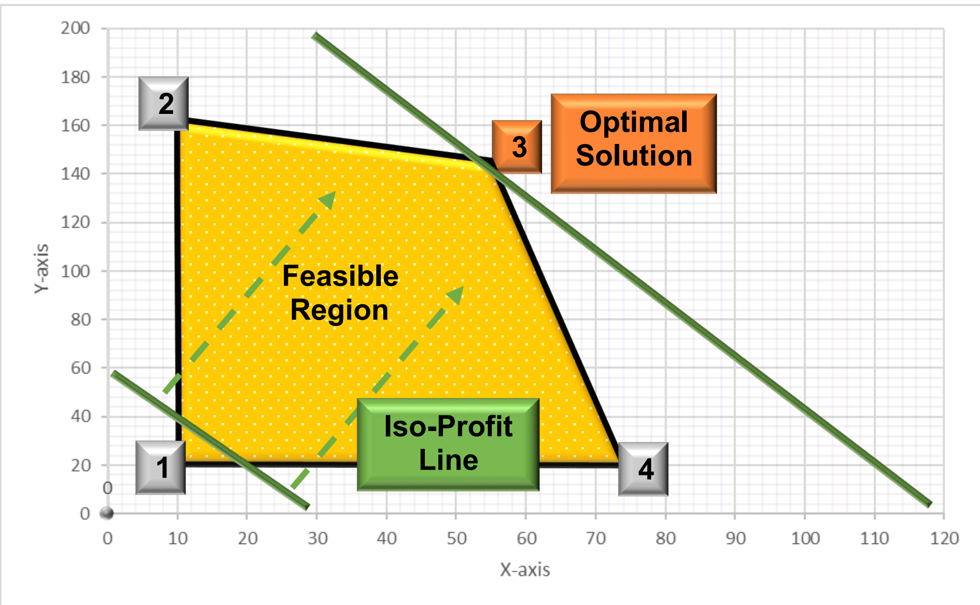
**Step 3** Select the corner point with the best (maximum/highest) value of the objective function to maximize Z (profit) or the best (minimum/lowest) value of the objective function to minimize C (cost).

**Step 4** Find the optimal solution based on the objective function to maximize Z (profit) or to minimize C (cost) according to the best value of point $(x, y)$.

### 3) Iso-Profit (Objective) Line Method

**Step 1** Gives some small amount of profits Z (must be in the feasible region).

**Step 2** Substitute the value of  in **Step 1** into the maximize objective function. Calculate the objective function by replacing $X = 0 \Rightarrow (0, a)$ and $Y = 0 \Rightarrow (b, 0)$



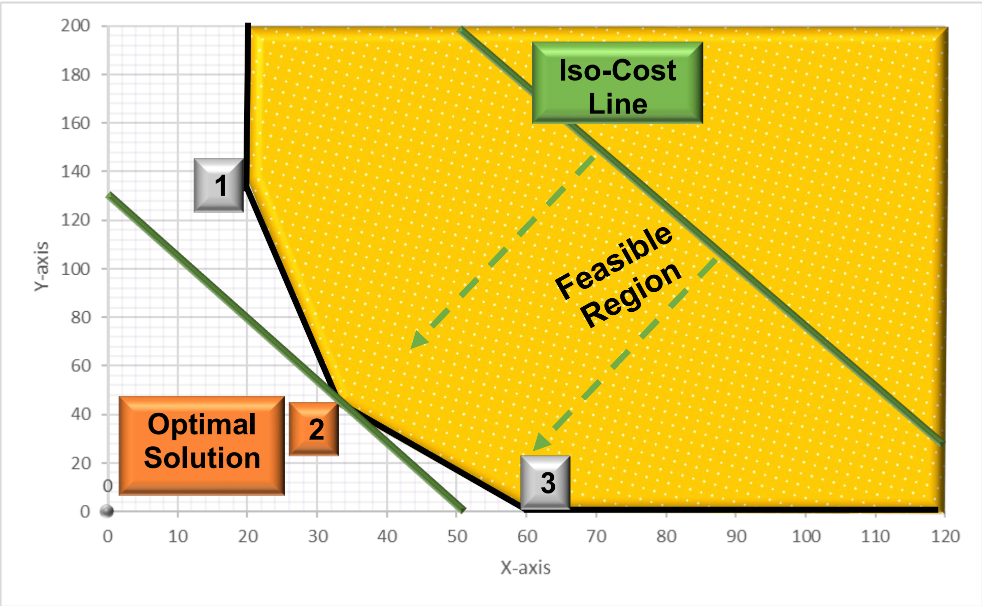
**Step 3**

- Construct the line on the graph with points of $(0,a)$ and $(b,0)$
- If the line does not produce the highest possible profit, try graphing 2 more lines with higher profit than before.
- Make sure that the iso-profit line must be parallel.
- Draw a series of parallel iso-profit lines until we find the highest iso-profit line, that is, the one with the optimal solution.
- Therefore, the highest possible iso-profit line is the one that touches the top of the feasible region at the corner point $(x,y)$ and yields a profit $Z$ for the optimal solution.
- Remember! Move the line toward the upper right (i.e from the origin) and choose the line that the last point touches the feasible region.

## 4) Iso-Cost (Objective) Line Method

**Step 1** Gives some large amount of costs C (must be in the feasible region).

**Step 2** Substitute the value of C in **Step 1** into the minimize objective function. Calculate the objective function by replacing $X=0 \Rightarrow (0,a)$ and $Y=0 \Rightarrow (b,0)$.




**Step 3**

- Construct the line on the graph with points of $(0,a)$ and $(b,0)$.
- If the line does not produce the lowest possible cost, try graphing 2 more lines with lower cost than before.
- Make sure that the iso-cost line must be parallel.
- Draw a series of parallel iso-cost lines until we find the lowest iso-cost line, that is, the one with the optimal solution.
- Therefore, the lowest possible iso-cost line is the one that touches the bottom of the feasible region at the corner point $(x,y)$ and yields a cost C for the optimal solution.
- Remember! Move the line toward the lower left (i.e toward the origin) and choose the line that the last point touches the feasible region.

## B. Solving Maximization and Minimization Problems

### 1) Solving Maximization Problem

- Maximizing an objective such as profit or revenue.
- Example of maximization problem:

- A factory wishes to maximize profits given labour and materials constraint to increase the production of the bowls and mugs.

➢ Maximization problem can be solved graphically by:
  - Setting up the feasible solution region.
  - Use either corner point method or iso-profit line method approach to find the values of decision variables that yield the maximum profit.

## GUIDED EXAMPLE 2

Solve the following linear programming problem by the graphical method.

Maximize profit $Z = 3X + 7Y$

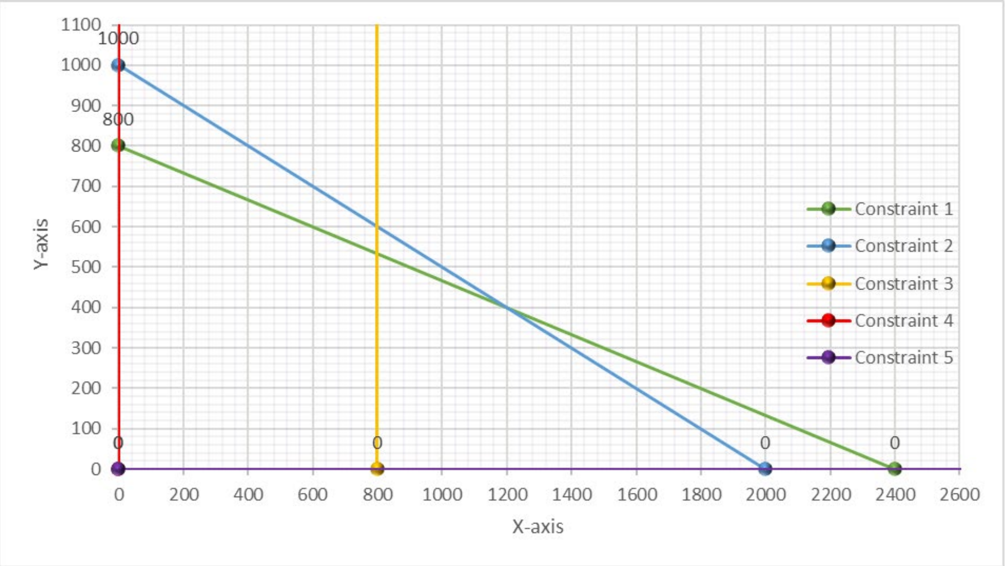
Subject to constraints:
$$0.10X + 0.30Y \leq 240$$
$$X + 2Y \leq 2000$$
$$X \geq 800$$
$$X, Y \geq 0$$

**Solution:**

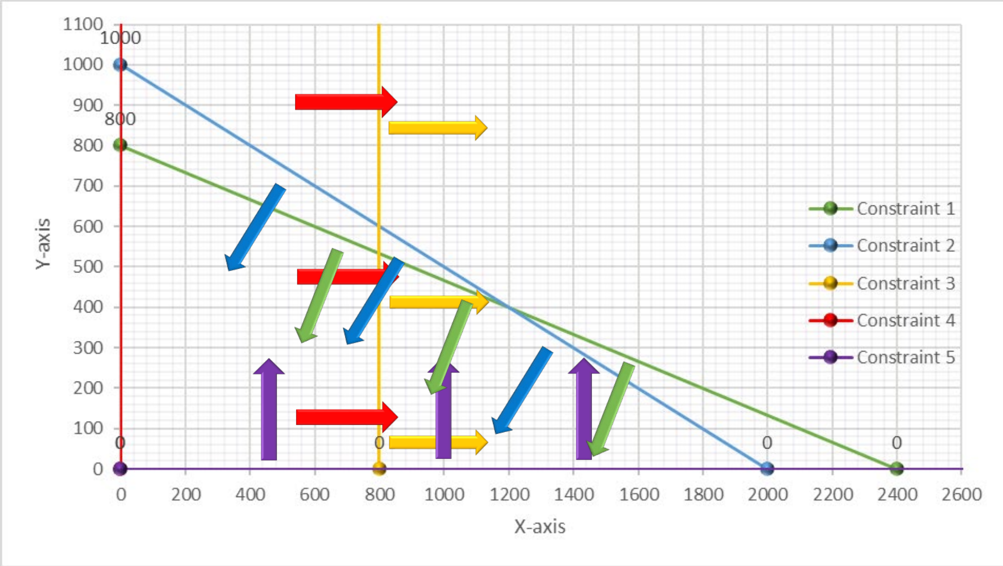
Draw the lines/equations (convert the inequalities sign into the equal sign)

| Equation | When $Y = 0$ | When $X = 0$ | Point |
|---|---|---|---|
| | $X$ | $Y$ | $(X,Y)$ |
| **a)** $0.10X + 0.30Y = 240$ | 2400 | 800 | $(2400,0)$ $(0,800)$ |
| **b)** $X + 2Y = 2000$ | 2000 | 1000 | $(2000,0)$ $(0,1000)$ |

Other lines/equations: $X = 800, X = 0, Y = 0$

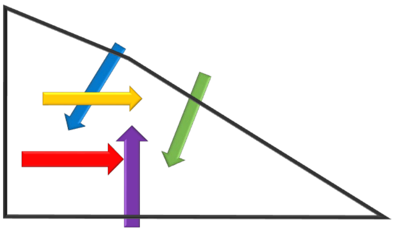


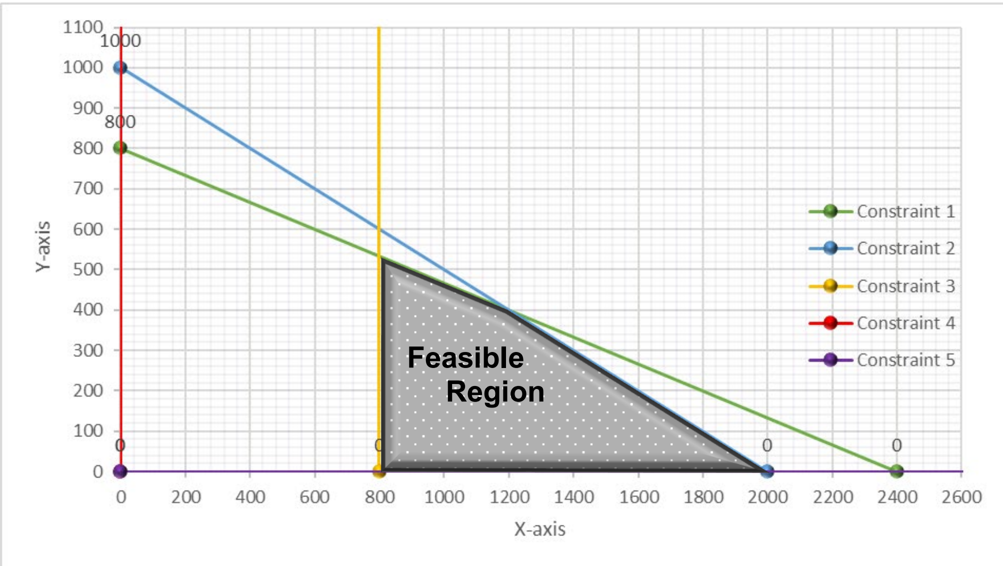

Find the feasible region: check the constraints at origin $(0,0)$




| Constraint | **when $X = 0$ & $Y = 0$** | Result | Area | Colour line/arrow |
|---|---|---|---|---|
| **1.** $0.10X + 0.30Y \leq 240$ | $0 \leq 240$ | True | left/below | green |
| **2.** $X + 2Y \leq 2000$ | $0 \leq 2000$ | True | left/below | blue |
| **3.** $X \geq 800$ | $0 \geq 800$ | True | right | yellow |
| **4.** $X \geq 0$ | $0 \geq 0$ | True | right | red |
| **5.** $Y \geq 0$ | $0 \geq 0$ | True | above | purple |

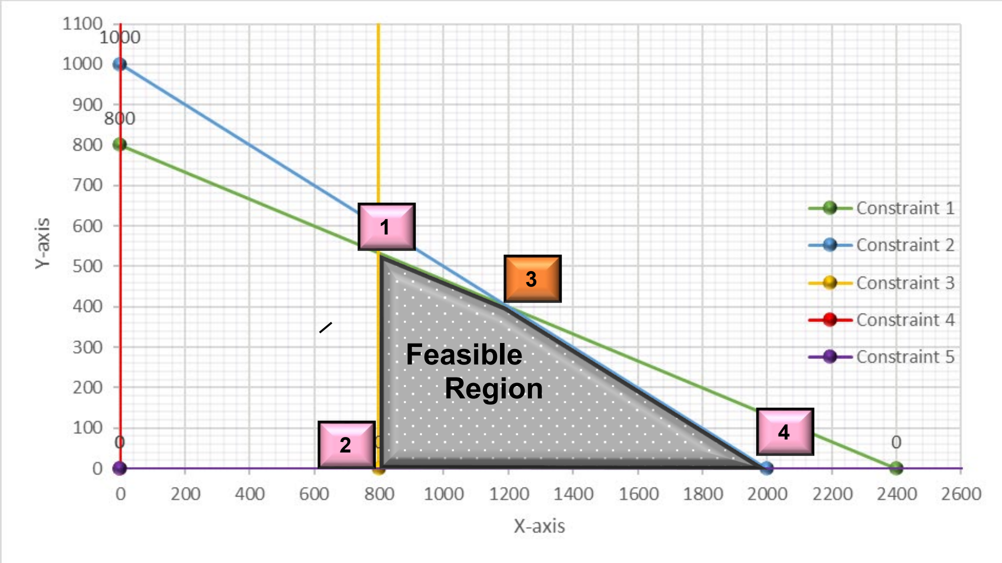
**Note:** *The feasible region consists of all colours (green, blue, yellow, red and purple) constraint on the graph.*





**Method 1: Corner Point Method**

**Step 1:** Find the coordinates of each corner point that had been labeled 1, 2, 3, 4.




***Point 1:** Solve the intersection point, given that $X = 800$, then substitute into

$$0.10X + 0.30Y = 240$$
$$0.10(800) + 0.30Y = 240 \Rightarrow Y = 533.33$$

*** Point 3:** Solve the simultaneous equations:

$$0.10X + 0.30Y = 240 \text{ -----(1)}$$
$$X + 2Y = 2000 \text{-----(2)}$$
$$\therefore X = 1200, Y = 400$$

**Step 2:** Evaluate the objective function at each corner point

| Corner | Point $(X,Y)$ | $Z = 3X + 7Y$ |
|---|---|---|
| 1 | $(800, 533.33)$ | $Z = 3(800) + 7(533.33) = 6133.31$ |
| 2 | $(800, 0)$ | $Z = 3(800) + 7(0) = 2400$ |
| 3 | $(1200, 400)$ | $Z = 3(1200) + 7(400) = 6400$ |
| 4 | $(2000, 0)$ | $Z = 3(2000) + 7(0) = 6000$ |

**Step 3**: Select the corner point with the best (maximum/highest) value of the objective function to maximize Z (profit)

Point 3 produces the highest profit of any corner point. Therefore, the value of $X = 1200$ and $Y = 400$ is the optimal solution. This solution yields a profit of RM6400 per production period.

**Step 4:** Find the optimal solution based on the objective function to maximize Z (profit)

$\therefore$ Optimal solution is at point 3, where $X = 1200$ and $Y = 400$ with maximum profit $Z = RM6400$.

**OR**

**Method 2: Iso-Profit Line Method**

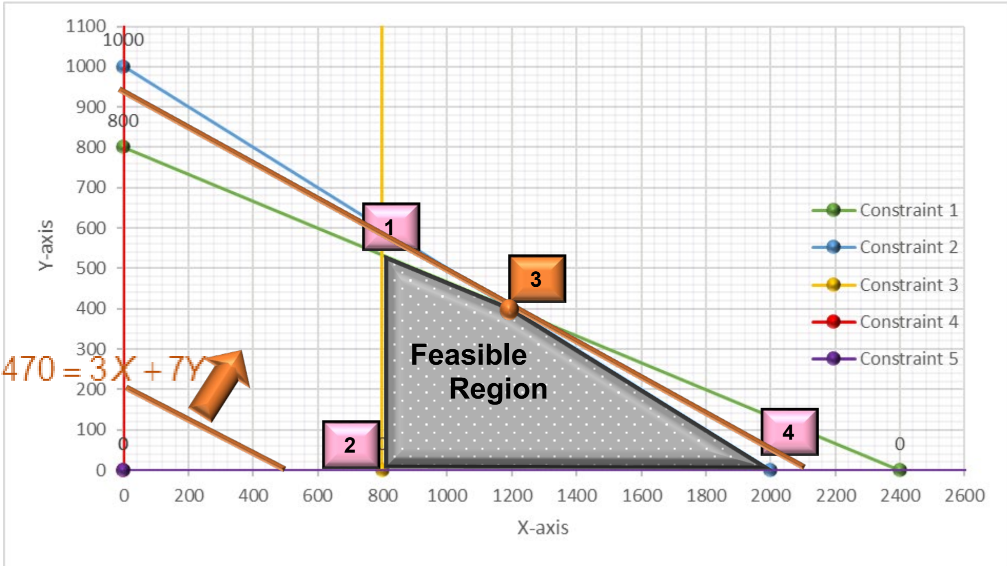
The optimal solution is the point lying in the feasible region that produces the highest profit.

**Step 1:** Gives some small number of profits. For example, $Z = RM1470$ is chosen (must be in the feasible region).

**Step 2:**

Objective function: $1470 = 3X + 7Y$

Calculate the objective function by replacing $X = 0 \Rightarrow Y = 210 \Rightarrow (0,210)$ and $Y = 0 \Rightarrow X = 490 \Rightarrow (490,0)$




**Step 3:**

- Construct the line on the graph with points of $(0,210)$ and $(490,0)$.
- If the line does not produce the highest possible profit, try graphing 2 more lines with higher profit than before.
- Make sure that the iso-profit line must be parallel.
- Draw a series of parallel iso-profit lines until we find the highest iso-profit line, that is, the one with the optimal solution.
- Therefore, the highest possible iso-profit line is the one that touches the tip of the feasible region at the corner point $(1200, 400)$ 3 and yields a profit of $RM6400$.

## GUIDED EXAMPLE 3

From the solution of **Guided Example 1**, solve the following linear programming problem by the graphical method.

Maximize profit $Z = 0.60X + 0.90Y$

Subject to constraints: $100X + 150Y \le 10{,}000$ (expenditures needed to plant)
$500X + 300Y \le 6{,}000$ (average yields per acre)

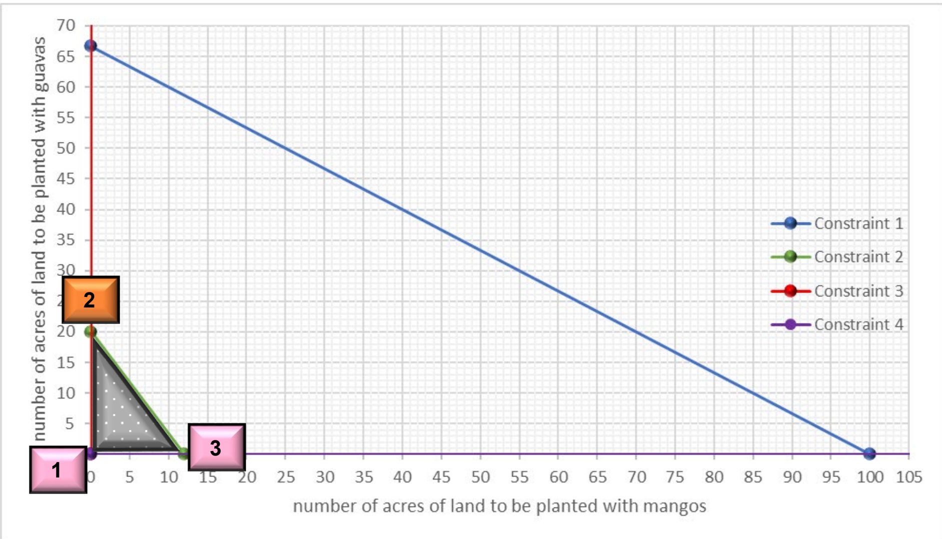
$X,Y \geq 0$

**Solution:**

| **Equation** | **When** $Y=0$ | **When** $X=0$ | **Point** |
|---|---|---|---|
| | $X$ | $Y$ | $(X,Y)$ |
| **a)** $100X+150Y=10000$ | 100 | 66.67 | $(100,0)$ $(0,66.67)$ |
| **b)** $500X+300Y=6000$ | 12 | 20 | $(12,0)$ $(0,20)$ |

Other lines/equations: $X=0, Y=0$

| **Constraint** | **when** $X=0$ **&** $Y=0$ | **Result** | **Area** | **Colour line/arrow** |
|---|---|---|---|---|
| **1.** $100X+150Y \leq 10000$ | $0 \leq 10000$ | True | left/below | blue |
| **2.** $500X+300Y \leq 6000$ | $0 \leq 6000$ | True | left/below | green |
| **3.** $X \geq 0$ | $0 \geq 0$ | True | right | red |
| **4.** $Y \geq 0$ | $0 \geq 0$ | True | above | purple |




By using the corner point method (method 1), to find the optimal solution:

| **Corner** | **Point** $(X,Y)$ | $Z=0.60X+0.90Y$ |
|---|---|---|
| 1 | $(0,0)$ | $Z=0.60(0)+0.90(0)=0$ |
| 2 | $(0,20)$ | $Z=0.60(0)+0.90(20)=18$ |
| 3 | $(12,0)$ | $Z=0.60(12)+0.90(0)=7.20$ |

Point 2 produces the highest profit of any corner point.

$\therefore$ Optimal solution is at point 2, where $X=0$ and $Y=20$ with maximum profit $Z=RM18$

## HANDS-ON EXAMPLE 3

Solve the following linear programming problem using the graphical method.

Maximize $Z = 3X + 2Y$

Subject to:

$$2X - 6Y \le 12$$
$$5X + 4Y \le 40$$
$$X + 2Y \ge 12$$
$$X, Y \ge 0$$

**Solution:**

Ans: optimal soln. $x = 5.33, y = 3.33, z = 22.65$

**HANDS-ON EXAMPLE 4**

Solve the following linear programming problem graphically.

Maximize $z = 8x + y$

Subject to:

$$x + y \le 40$$
$$2x + y \le 60$$
$$x, y \ge 0$$

**Solution:**

Ans: optimal soln. $x = 30, y = 0, z = 240$

## 2) Solving Minimization Problem

- Minimizing an objective such as cost instead of maximizing a profit function.
- Example of minimization problem:
  - A restaurant wish to develop a work schedule to meet staffing needs while minimizing the total number of employees.
- Minimization problem can be solved graphically by:
  - Setting up the feasible solution region.
  - Use either corner point method or iso-cost line method approach to find the values of decision variables that yield the minimum cost.

**GUIDED EXAMPLE 4**

Solve the following linear programming problem graphically.

Minimize $C = 12x + 4y$

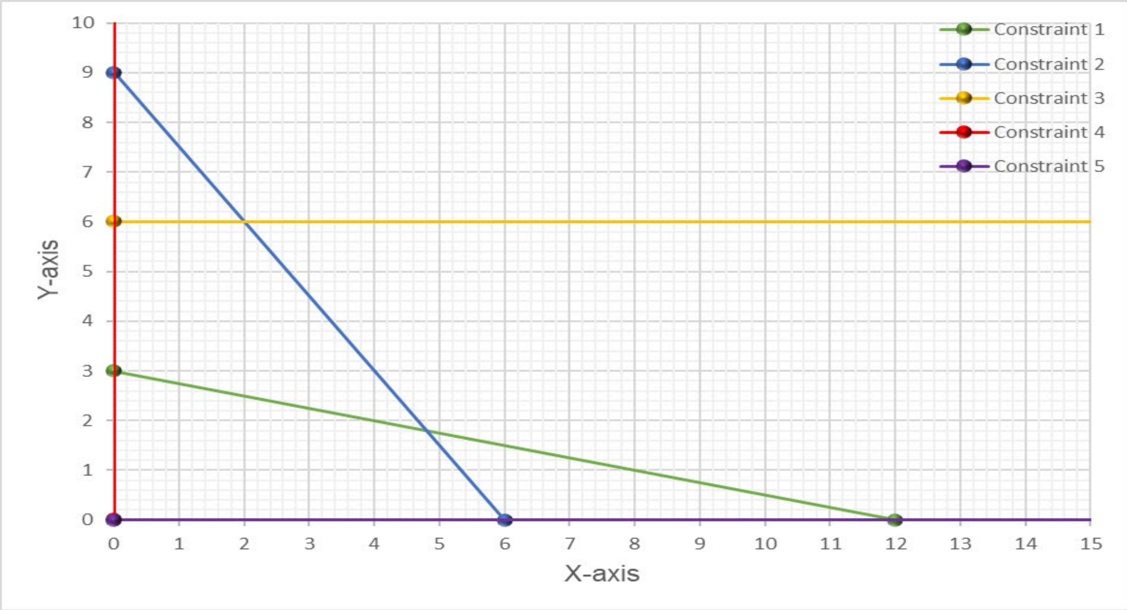
Subject to:

$$x + 4y \geq 12$$
$$3x + 2y \geq 18$$
$$y \leq 6$$
$$x, y \geq 0$$

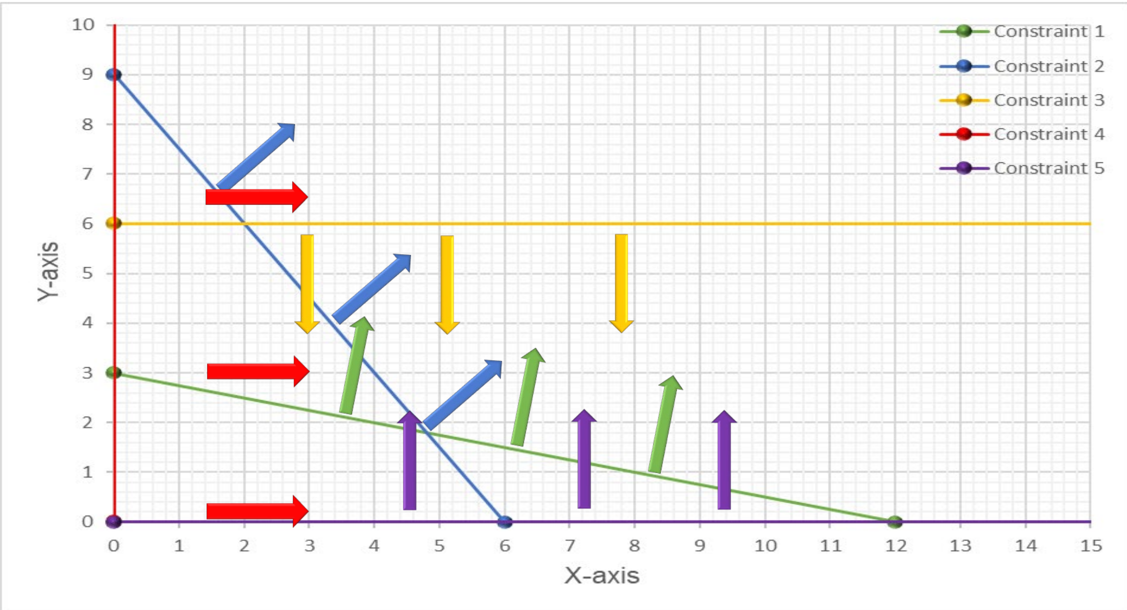
**Solution:**
Draw the lines/equations (convert the inequalities sign into the equal sign)

| Equation | When $Y = 0$ | When $X = 0$ | Point |
|---|---|---|---|
| | $X$ | $Y$ | $(X,Y)$ |
| **a)** $x + 4y = 12$ | 12 | 3 | $(12,0)\ (0,3)$ |
| **b)** $3x + 2y = 18$ | 6 | 9 | $(6,0)\ (0,9)$ |

Other lines/equations: $Y = 6, X = 0, Y = 0$

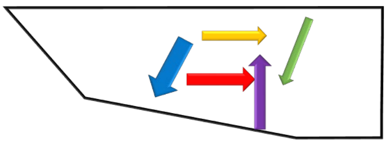


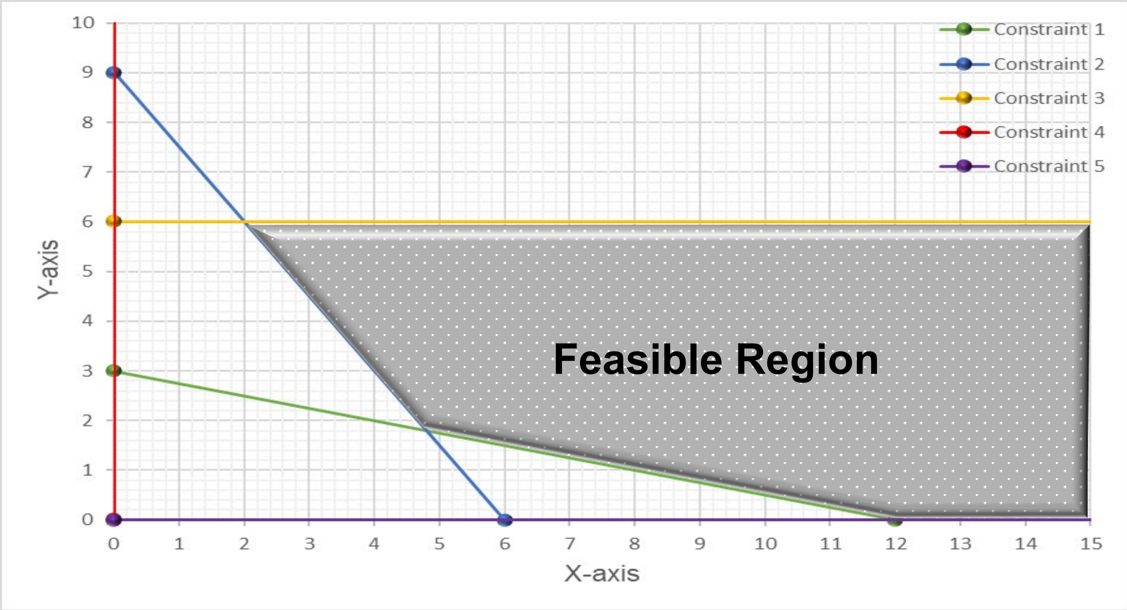

Find the feasible region: check the constraints at origin $(0,0)$




| Constraint | when $X=0$ & $Y=0$ | Result | Area | Colour line/arrow |
|---|---|---|---|---|
| **a)** $x+4y \geq 12$ | $0 \leq 12$ | True | right/above | green |
| **b)** $3x+2y \geq 18$ | $0 \leq 18$ | True | right/above | blue |
| **c)** $y \leq 6$ | $0 \geq 6$ | False | below | yellow |
| **d)** $X \geq 0$ | $0 \geq 0$ | True | right | red |
| **e)** $Y \geq 0$ | $0 \geq 0$ | True | above | purple |

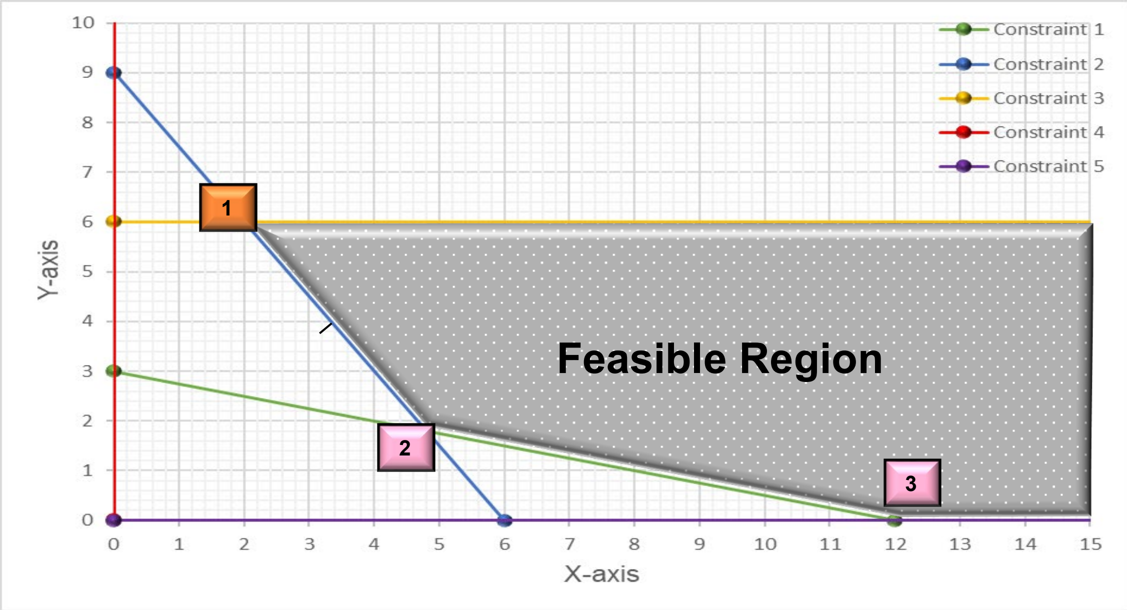
**Note:** *The feasible region consists of all colours (green, blue, yellow, red and purple) constraint on the graph.*




Feasible Region
Constraint 1
Constraint 2
Constraint 3
Constraint 4
Constraint 5
X-axis
Y-axis

**Method 1: Corner Point Method**

**Step 1:** Find the coordinates of each corner point that had been labeled 1, 2 and 3.




***Point 1:** Solve the intersection point, given that $Y = 6,$ then substitute into

$$3x + 2y = 18$$

$$3x + 2(6) = 18 \Rightarrow x = 2$$

*** Point 2:** Solve the simultaneous equations:

$$3x + 2y = 18 \text{ -----(1)}$$

$$x + 4y = 12 \text{-----(2)}$$

$$\therefore x = 4.8, y = 1.8$$

**Step 2:** Evaluate the objective function at each corner point

| Corner | Point $(X,Y)$ | $C = 12x + 4y$ |
|---|---|---|
| 1 | $(2,6)$ | $C = 12(2) + 4(6) = 48$ |
| 2 | $(4.8,1.8)$ | $C = 12(4.8) + 4(1.8) = 64.8$ |
| 3 | $(12,0)$ | $C = 12(12) + 4(0) = 144$ |

**Step 3**: Select the corner point with the best (minimum/lowest) value of the objective function to minimize C (cost)

Point 1 produces the highest profit of any corner point. Therefore, the value of $x = 2$ and $y = 6$ is the optimal solution. This solution yields a cost of RM48 per production period.

**Step 4:** Find the optimal solution based on the objective function to minimize C (cost)

$\therefore$ Optimal solution is at point 1 where $x = 2$ and $y = 6$ with minimum cost $C = RM48$

**OR**

**Method 2: Iso-Profit Line Method**

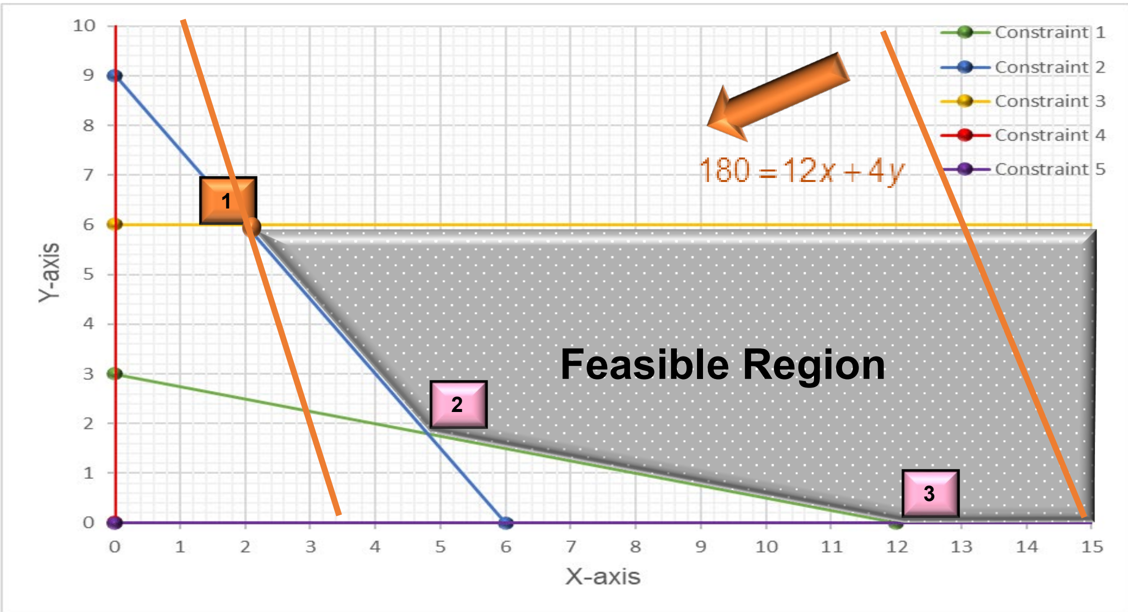
The optimal solution is the point lying in the feasible region that produces the lowest cost.

**Step 1:** Gives some largel number of costs. For example, $C = RM180$ is chosen (must be in the feasible region).

**Step 2:**

Objective function: $180 = 12x + 4y$

Calculate the objective function by replacing $x = 0 \Rightarrow y = 45 \Rightarrow (0,45)$ and $y = 0 \Rightarrow x = 15 \Rightarrow (15,0)$




**Step 3:**

- Construct the line on the graph with points of $(0,45)$ and $(15,0)$.
- If the line does not produce the lowest possible cost, try graphing 2 more lines with lower cost than before.
- Make sure that the iso-cost line must be parallel.
- Draw a series of parallel iso-cost lines until we find the lowest iso-cost line, that is, the one with the optimal solution.
- Therefore, the lowest possible iso-cost line is the one that touches the bottom of the feasible region at the corner point $(2,6)$ 1 and yields a cost of $RM48$.

## HANDS-ON EXAMPLE 5

Solve the following linear programming problem graphically.

Minimize cost $C = 6x + 4y$

Subject to:

$$x + y \geq 4$$
$$5x + 3y \geq 15$$
$$x, y \geq 0$$

**Solution:**

Ans: optimal soln. $x = 1.5, y = 2.5, C = 19$

**HANDS-ON EXAMPLE 6**

Given the following Linear Programming model for a company:

Minimize cost $C = 30x + 50y$

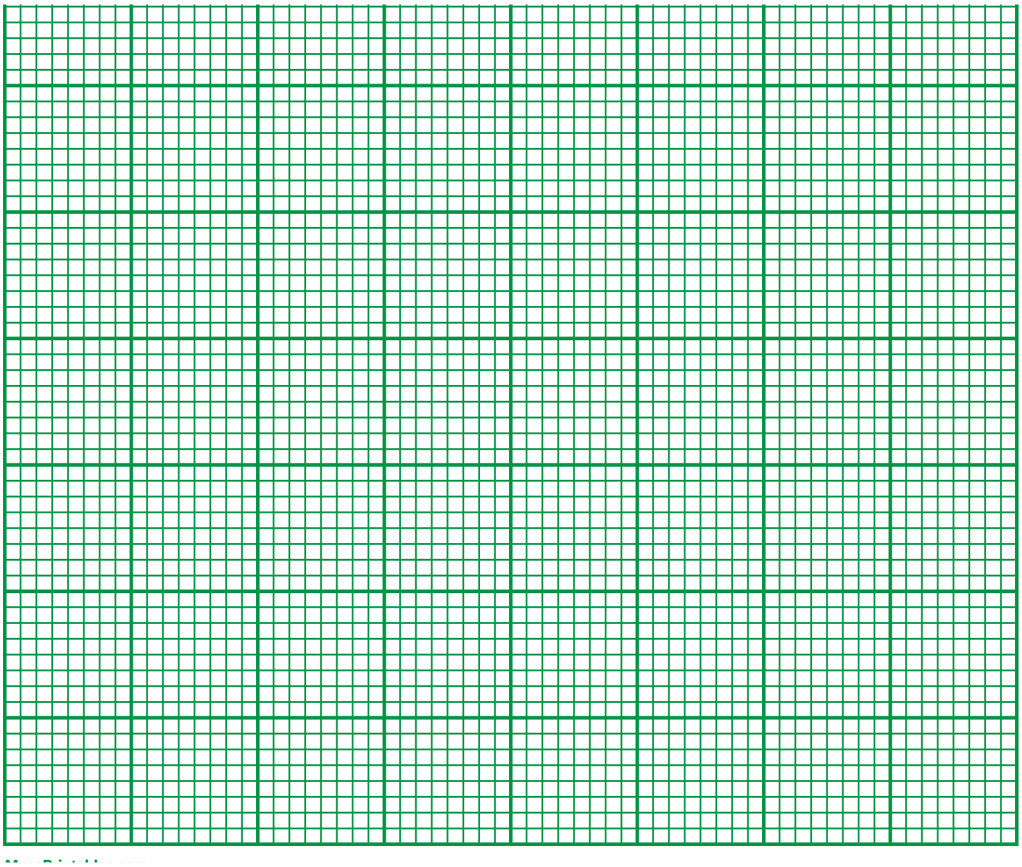
Subject to

$$20x + 10y \le 160$$
$$x + y \ge 4$$
$$2y \le 12$$
$$x, y \ge 0$$

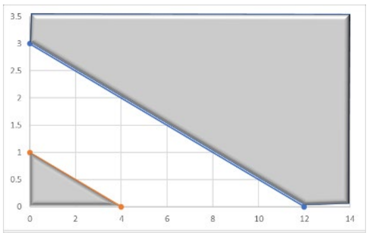
Solve the LP using graphical method.

**Solution:**

Ans: optimal soln. $x = 4, y = 0, C = RM120$

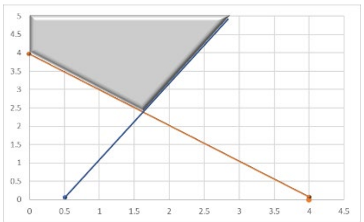


## C. Special Cases in LP



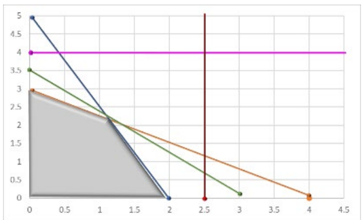
**Infeasibility:**

A situation where there is no solution to the problem. In practice, this would be a situation where conditions on costs, staffing and resources make it impossible to produce any goods.



**Unboundedness:**

This is a situation where there is no limit to the optimum value being found. In practice, this situation is likely to occur if one or more conditions have been omitted from the problem formulation.



**Redundancy:**

A redundancy constraint is one that does not affect the feasible region. This usually occurs in large LP formulation and causes no major difficulties.

**Multiple solutions:**

The optimal value of the objective function occurs at more than 1 extreme points, then the problem has multiple optimal solution. For example: At corner point of $(a,b)$ and $(c,d)$, both value of the objective function $Z = L$ (equal solution).

## 3.4 Simplex Method (LP)

The Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables to finding the optimal solution of an optimization problem. The simplex method systematically examines corner points, using algebraic steps, until an optimal solution is found.

- Having more than two variables- too large for the simple graphical procedure.
- Yields the optimal solution to the variables & the maximum profit or minimum cost.
- The approach provides valuable economic information.

### A. How to set up the Initial Simplex Tableau (maximization and minimization)

Step 1: Set up the Problem in the Standard Form
Step 2: Determine Slack Variables
Step 3: Construct the Simplex Tableau
Step 4: Check Optimality
Step 5: Identify Pivot Variable
Step 6: Create the New Simplex Tableau
Step 7: Identify Optimal Values

### B. Simplex Solution Procedures

The procedure to solve a linear programming model using the Simplex method the following steps are necessary:

**Step 1: Set up the Problem in the Standard Form –** Standard form is the baseline format for all linear programs before solving for the optimal solution and has three requirements:

1) must be a maximization problem,
2) all linear constraints must be in a less-than-or-equal-to inequality,
3) all variables are non-negative.

**GUIDED EXAMPLE 5**

From the solution of **Guided Example 1**, solve the following linear programming problem by the graphical method.

Maximize profit $Z = 0.60X + 0.90Y$

Subject to constraints:
$100X + 150Y \leq 10{,}000$ (expenditures needed to plant)
$500X + 300Y \leq 6{,}000$ (average yields per acre)
$X, Y \geq 0$

**Step 2: Determine Slack Variables** – Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. This is done by adding one slack variable for each inequality. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.

Maximize profit $Z = 0.60X + 0.90Y + 0S_1 + 0S_2$

Subject to :

$$100X + 150Y + 1S_1 + 0S_2 = 10{,}000 \quad \text{(since } 0S_2 = 0 \text{ and } 1S_1 = S_1\text{)}$$

$$500X + 300Y + 0S_1 + 1S_2 = 6{,}000 \quad \text{(since } 0S_1 = 0 \text{ and } 1S_2 = S_2\text{)}$$

$$X, Y, S_1, S_2 \geq 0$$

**Step 3: Construct the Simplex Tableau** - A Simplex tableau is used to perform row operations on the linear programming model as well as to check a solution for optimality. The tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function.

| $C_i$ | Solution mix (basis) | 0.60 | 0.90 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|
| | | $X$ | $Y$ | $S_1$ | $S_2$ | |
| 0 | $S_1$ | 100 | 150 | 1 | 0 | 10,000 |
| 0 | $S_2$ | 500 | 300 | 0 | 1 | 6,000 |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 |
| | $C_i - Z_j$ | 0.60 | 0.90 | 0 | 0 | |

**Step 4: Check Optimality**

**Follow these 5 steps until an optimal solution has been reached:**

1. Determine which variable to enter the solution mix. Variable entering the solution has the largest positive $C_i - Z_j$. This is the pivot column.
2. Determine the solution mix variable to be replaced. Variable leaving the solution mix is determined by the ratio. The following are formula for ratio :

   Ratio = Quantity [Right Hand Side (RHS)] / (corresponding number in pivot column)

   The row with the smallest (nonnegative) ratio will be the pivot row (will be replace in the next tableau). The intersection of the pivot row and pivot column is referred to as the pivot number.

**Ratio** $= \dfrac{\textbf{Quantity}}{\textbf{Pivot Col}} = \dfrac{10000}{150}$

**Pivot Row**

**Pivot Column**

**Pivot Number**

| $C_i$ | Solution mix (basis) | 0.60 | 0.90 | 0 | 0 | Quantity | Ratio |
|---|---|---|---|---|---|---|---|
| | | $X$ | $Y$ | $S_1$ | $S_2$ | | |
| 0 | $S_1$ | 100 | 150 | 1 | 0 | 10,000 | 66.67 |
| 0 | $S_2$ | 500 | 300 | 0 | 1 | 6,000 | 20 |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $C_i - Z_j$ | 0.60 | 0.90 | 0 | 0 | | |

3. Next, setup a new basic simplex tableau. Then, in the new table calculate the new values for the pivot row. To do this, divide every number in the pivot row by the pivot number. This row will become the reference row to compute the new values for each remaining row.

$$\text{New Value Pivot Row} = \frac{\text{Number in the pivot column}}{\text{Pivot number}}$$

4. Calculate the new values for the other row.

   New row = (old row) – (corresponding number in the pivot column)(reference row)

   *Reference row refer to the new value in pivot row that you have calculate in Step 3.

5. Lastly, calculate the value of $Z_j$ and $C_i - Z_j$ this new tableau. If there are any $C_i - Z_j$ number greater than 0, return to step 1. If there are no $C_i - Z_j$ numbers that are greater than 0, an optimal solution has been reached.

Z value is referring to value at **Step 2**:

Max profit
$$Z = 0.60X + 0.90Y + 0S_1 + 0S_2$$
***Note:*** Substitute value of each solution mix for each column to get the value of Z for each column.

**Step 3:** New Value for Pivot Row (reference row)

**Step 4:** New Row

| $C_i$ | **Solution mix (basis)** | **0.60** | **0.90** | **0** | **0** | **Quantity** |
|---|---|---|---|---|---|---|
| | | $X$ | $Y$ | $S_1$ | $S_2$ | |
| 0.60 | $S_1$ | -150 | 0 | 1 | -1/2 | 7,000 |
| 0.90 | $Y$ | 5/3 | 1 | 0 | 1/300 | 20 |
| | $Z_j$ | 0.90 | 0 | 0 | 3/1000 | 18 |
| | $C_i - Z_j$ | 0.90 | 0 | 0 | 3/1000 | |

**Step 5:** This is an optimum table since all values in $C_i - Z_j$ row contains 0 and positive value. Therefore, the company needs to plant 20 guavas to gain a maximum profit of RM18.

## C. Solving Maximization Problems only

**Solution to the Primal LP Problem in the Final Simplex tableau**
Solution to the primal problem can be obtained by looking at the variable of the final simplex tableau in the solution mix and the corresponding values in the RHS.

**Sensitivity Analysis (Shadow Price) with the Simplex Tableau**
The Shadow price (solution to the dual) provides an important piece of economic information regarding which resources can be classified as a *scarce resource (unused resource = 0).* It shows the maximum amount a firm is willing to pay to make one unit of additional scarce resource available.
If the firm is planning to increase any resources, in order to increase the production rate, the scarce resources (the shadow price $\neq 0$ or unused resources $= 0$, refer to the solution) will be considered. The higher the shadow price, the more valuable the resource is.

This concept also shows how the optimal solsution and the value of its objective function change, given changes in various inputs to the problem.

**GUIDED EXAMPLE 6**

Given below is a linear programming model for two different types of can food sold in a supermarket.

Maximize $Z = 2X + 3Y$ (profit in Ringgit Malaysia)

Subject to:
$X + 3Y \leq 180$ (resource 1)
$2X + 3Y \leq 360$ (resource 2)
$X + Y \leq 100$ (resource 3)
$X, Y \geq 0$

a) Set up the initial simplex tableau for the above problem.
b) The incomplete final simplex tableau for the above problem is shown below.
   i. Complete the below simplex tableau.

| $C_i$ | Solution mix (basis) | 2 | 3 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|
| | | $X$ | $Y$ | $S_1$ | $S_2$ | $S_3$ | |
| | $Y$ | 0 | 1 | $1/2$ | 0 | $-1/2$ | 40 |
| | $S_2$ | 0 | 0 | $-1/2$ | 1 | $-3/2$ | 120 |
| | $X$ | 1 | 0 | $-1/2$ | 0 | $3/2$ | 60 |
| | $Z_j$ | | | | | | |
| | $C_i - Z_j$ | | | | | | |

   ii. Determine the optimal solution and the total profit.
   iii. What are the shadow prices of all the resources?
   iv. Is it worthwhile to purchase additional units of resource 3 at a cost of RM2.50 per unit? Why?

**Solution:**

a)

| $C_i$ | Solution mix (basis) | 2 | 3 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|
| | | $X$ | $Y$ | $S_1$ | $S_2$ | $S_3$ | |
| 0 | $S_1$ | 1 | 3 | 1 | 0 | 0 | 180 |
| 0 | $S_2$ | 2 | 3 | 0 | 1 | 0 | 360 |
| 0 | $S_3$ | 1 | 1 | 0 | 0 | 1 | 100 |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | 0 |
| | $C_i - Z_j$ | 2 | 3 | 0 | 0 | 0 | |

b) i.

| $C_i$ | Solution mix (basis) | 2 | 3 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|
| | | $X$ | $Y$ | $S_1$ | $S_2$ | $S_3$ | |
| 3 | $Y$ | 0 | 1 | $1/2$ | 0 | $-1/2$ | 40 |
| 0 | $S_2$ | 0 | 0 | $-1/2$ | 1 | $-3/2$ | 120 |
| 2 | $X$ | 1 | 0 | $-1/2$ | 0 | $3/2$ | 60 |
| | $Z_j$ | 2 | 3 | $1/2$ | 0 | $3/2$ | 240 |
| | $C_i - Z_j$ | 0 | 0 | $-1/2$ | 0 | $-3/2$ | |

ii. Optimal solution: $Y = 40$, $S_2 = 120$, $X = 60$, $S_1 = S_3 = 0$
Total profit $= RM240$

iii. Shadow prices: Resource $1 = RM0.50$
Resource $2 = 0$
Resource $3 = RM1.50$

iv. It is not worthwhile to purchase since the additional cost of resource 3 is RM2.50 more than the shadow price resource 3(RM1.50).

## GUIDED EXAMPLE 7

The linear programming model for the monthly production of $x$, $y$ and $z$ is given below:

Maximize profit $Z = 45x + 60y + 10z$

Subject to

$$x + 3y + 2z \le 1800 \qquad \text{(Item A)}$$
$$4x + 2y + 2z \le 2000 \qquad \text{(Item B)}$$
$$x, y, z \ge 0$$

a) Complete the optimal simplex tableau for the above problem as shown below:

| $C_i$ | Solution mix (basis) | 45 | 60 | 10 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|
| | | $x$ | $y$ | $z$ | $S_1$ | $S_2$ | |
| | $y$ | 0 | 1 | $3/5$ | $2/5$ | $-1/10$ | 520 |
| | $x$ | 1 | 0 | $1/5$ | $-1/5$ | $3/10$ | 240 |
| | $Z_j$ | | | | | | |
| | $C_i - Z_j$ | | | | | | |

b) Determine the optimal solution and the maximum profit
c) Which items are/is fully utilized?
d) If one unit of item B is added, how much will the profit increase?

**Solution:**
a)

| $C_i$ | Solution mix (basis) | 45 | 60 | 10 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|
| | | $x$ | $y$ | $z$ | $S_1$ | $S_2$ | |
| 60 | $y$ | 0 | 1 | $3/5$ | $2/5$ | $-1/10$ | 520 |
| 45 | $x$ | 1 | 0 | $1/5$ | $-1/5$ | $3/10$ | 240 |
| | $Z_j$ | 45 | 60 | 45 | 15 | $15/2$ | 42,000 |
| | $C_i - Z_j$ | 0 | 0 | -45 | -15 | $-15/2$ | |

b) $x = 520 \quad y = 240 \quad s_1 = 0, \quad s_2 = 0$
Maximum profit = RM42,000
c) $S_1$ and $S_2$ are fully utilized
d) If one unit of item B is added, profit will increase by RM7.50

## HANDS-ON EXAMPLE 7

Objective function maximize the profit of $Z = 5x_1 + 3x_2 + 4x_3$

Subject to:

$$3x_1 + 12x_2 + 6x_3 \le 600 \; (\textit{machine hour})$$
$$6x_1 + 6x_2 + 3x_3 \le 1200 \; (\textit{labour hr})$$
$$6x_1 + 9x_2 + 9x_3 \le 900 \; (\textit{components})$$

Where $x_1$, $x_2$ and $x_3$ represent units of products A, B and C.

a) Convert these constraints and the objective function to the proper form for the use in the initial simplex tableau.
b) Set up the initial simplex tableau for this problem.
c) Below is a simplex tableau for the above problem.

| $C_i$ | Solution mix (basis) | 5 | 3 | 4 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | |
| 0 | $S_1$ | 0 | 15/2 | 3/2 | 1 | 0 | -1/2 | 150 |
| 0 | $S_2$ | 0 | -3 | -6 | 0 | 1 | -1 | 300 |
| 5 | $x_1$ | 1 | 3/2 | 3/2 | 0 | 0 | 1/6 | 150 |
| | $Z_j$ | | | | | | | |
| | $C_i - Z_j$ | | | | | | | |

i. Complete the values for $Z_j$ and $C_j - Z_j$ rows.
ii. Is this the final tableau for the problem? Explain your answer.
iii. State the optimal solution.
iv. Which resource(s) is not fully utilized? If so, how much spare capacity is left?
v. Is it worth to spend RM1.00 to buy one unit of a component? Explain.
vi. If one unit of components is added, how much will the profit increase?

**Solution:**

$3x_1 + 12x_2 + 6x_3 + s_1 = 600$

Ans: $6x_1 + 6x_2 + 3x_3 + s_2 = 1200$ , 0 -9/2 -7/2 0 0 -5/6

$6x_1 + 9x_2 + 9x_3 + s_3 = 900$

Yes, because all the entries for $\mathrm{C_j - Z_j}$ row are $\leq 0$,

$Profit = RM750, x_1 = 150, x_2 = x_3 = 0, s_1 = 150, s_2 = 300, s_3 = 0$,

Resources that not fully utilize are machine hours of 150 hours and labour hours of 300 hours. ($S_1 = 150$ *and* $S_2 = 300$), increased by RM0.83

b) Initial simplex tableau:

| $C_i$ | Solution mix (basis) | | | | | | | | Quantity |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $S_1$ | | $S_2$ | $S_3$ | |
| | $S_1$ | | | | | | | | |
| | $S_2$ | | | | | | | | |
| | $S_3$ | | | | | | | | |
| | $Z_j$ | | | | | | | | |
| | $C_i - Z_j$ | | | | | | | | |

| $C_i$ | Solution mix (basis) | 5 | 3 | 4 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | |
| 0 | $S_1$ | 0 | 15/2 | 3/2 | 1 | 0 | -1/2 | 150 |
| 0 | $S_2$ | 0 | -3 | -6 | 0 | 1 | -1 | 300 |
| 5 | $S_3$ | 1 | 3/2 | 3/2 | 0 | 0 | 1/6 | 150 |
| | $Z_j$ | | | | | | | |
| | $C_i - Z_j$ | | | | | | | |

## D. Interpretation of Simplex Solution

**Slack Variable**
Variables added to less than or equal to constraint in order to create an equality to generate the initial solution. It represents a quantity of unused resource.

**Surplus Variable**
The difference between the total value of the true (decision) variables and the number (usually, total resource available) on the right-hand side of the equation. Thus, a surplus variable will always have a negative value.

**Basic Solution**
The feasible corner-point solutions to an LP are basic feasible solutions. The solution can be easily read from the simplex tableau constructed. A basic solution is also called as the basis.

**Basic Variable**
Variables in the basis are the variables that are not set to zero and have non-zero values.

**Non-Basic Variable**
Variables that are set to zero when defining the corner points (solution) are known as non-basic variables or variables that do not appear in the basis.

**Artificial Variable**
Variables to be include in each equality constraint to generate the initial solution. It has no physical meaning and will be eliminated in the process (before the final tableau)

### E. Special cases in LP (simplex method)

**Infeasibility** (in minimization problems only):
A situation where there is no solution to the problem. This happens when LP model is wrongly/improperly formulated.

**Unbounded Solution:**
Unboundedness describes linear programs that do not have finite solution and occurs in maximization problems. A variable can be indefinitely large without violating a constraint. The condition of unboundedness will be discovered before reaching the final tableau that is when trying to decide which variable to remove from the solution mix. The entire ratios turn out to be negative or undefined.

**Degeneracy:**
Degeneracy occurs when a problem contains a redundant constraint that is one or more of the constraints in the formulation make another unnecessary. Degeneracy arises when there is a tie for the smallest ratio. It can lead the situation called cycling that is the algorithm puts a new variable in, then takes it out in the next tableau, puts it back in, and so on. One simple way of dealing with the issue is to select either row arbitrarily. If we are unlucky and cycling does occur, we simply go back and select the other row.

**More Than One Optimal Solution:**
In this case there are a number of potential solutions, all yielding the optimum value for the objective function. Multiple solutions are spotted by looking at the final tableau. In the Cj – Zj row, at least one non-basis variable has zero coefficients. This means that there is an alternative solution with the same amount of profit. The column with zero coefficient will be the new pivot column. After this, continue as before to obtain the alternative solution.

**GUIDED EXAMPLE 8**

The optimal simplex tableau for the linear programming problem is given in the following table:

| $C_i$ | **Solution mix (basis)** | 6 | 9 | 6 | 0 | 0 | 0 | **Quantity** |
|---|---|---|---|---|---|---|---|---|
| | | X | Y | Z | $S_1$ | $S_2$ | $S_3$ | |
| 6 | X | 1 | 0 | -0.5 | 1 | -0.5 | 0 | 40 |
| 9 | Y | 0 | 1 | 1 | -0.5 | 0.5 | 0 | 60 |
| 0 | $S_3$ | 0 | 0 | -3 | 0.5 | -1.5 | 1 | 40 |
| | $Z_j$ | | | | | | | |
| | $C_i - Z_j$ | | | | | | | |

a) Complete the above simplex tableau.
b) State the optimal solution including the total profit.
c) Is there any alternative solution? If yes, find it.

**Solution:**

| $C_i$ | Solution mix (basis) | 6<br>X | 9<br>Y | 6<br>Z | 0<br>$S_1$ | 0<br>$S_2$ | 0<br>$S_3$ | Quantity |
|---|---|---|---|---|---|---|---|---|
| 6 | X | 1 | 0 | -0.5 | 1 | -0.5 | 0 | 40 |
| 9 | Y | 0 | 1 | 1 | -0.5 | 0.5 | 0 | 60 |
| 0 | $S_3$ | 0 | 0 | -3 | 0.5 | -1.5 | 1 | 40 |
| | $Z_j$ | 6 | 9 | 6 | 1.5 | 1.5 | 0 | 780 |
| | $C_i - Z_j$ | 0 | 0 | 0 | -1.5 | -1.5 | 0 | |

Optimal solution: X=40, Y=60, $S_3$=40, Total Profit=780
Alternative solution**:** X=70, Z=60, $S_3$=220, Total Profit=780

**HANDS-ON EXAMPLE 8**

Given the following final simplex tableau of a manufacturing firm:

| $C_j$ | Solution Mix | 150<br>$X_1$ | 130<br>$X_2$ | 250<br>$X_3$ | 0<br>$S_1$ | 0<br>$S_2$ | 0<br>$S_3$ | 0<br>$S_4$ | 0<br>$S_5$ | Quantity |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $S_3$ | 0 | 0 | 0 | 0.25 | 0.25 | 1 | 0 | -0.5 | 56.25 |
| 150 | $X_1$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 250 |
| 250 | $X_3$ | 0 | 0 | 1 | -0.25 | -0.25 | 0 | 0 | 0.5 | 143.75 |
| 0 | $S_4$ | 0 | 0 | 0 | -0.25 | -0.25 | 0 | 1 | -0.5 | 243.75 |
| 130 | $X_2$ | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 375 |
| $Z_j$ | | 150 | 130 | 250 | 67.5 | 87.5 | 0 | 0 | 125 | ***P*** |
| $C_j$-$Z_j$ | | 0 | 0 | 0 | -67.5 | -87.5 | 0 | 0 | -125 | |

a) What is the quantity of each product ($X_1$, $X_2$ and $X_3$) that will be produced?
b) Determine the value of ***P.***
c) Interpret the shadow price for each resource.
d) State the resource(s) that are not fully utilized and state how many units of each resource have not been used.
e) Is there any alternative solution exists? Why?

**Solution:**

Ans: $x_1 = 250, x_2 = 375, x_3 = 143.75, P = RM122{,}187.50$,
$R1 = RM67.50, R2 = 87.50, R3 = R4 = 0, R5 = 125$

## HANDS-ON EXAMPLE 8

A company produces two products ($x_1$ and $x_2$) daily. Each product requires two resources which are resource 1 ($S_1$) and resource 2 ($S_2$). The profits of producing each unit of $x_1$ and $x_2$ are RM50 and RM80, respectively. The final simplex tableau for this linear programming problem is shown in the following table:

| $C_i$ | **Solution mix (basis)** | $x_1$ | $x_2$ | $S_1$ | $S_2$ | **Quantity** |
|---|---|---|---|---|---|---|
| | $x_2$ | 0 | 1 | 0.375 | -0.25 | 5 |
| | $x_1$ | 1 | 0 | -0.25 | 0.50 | 10 |
| | $Z_j$ | | | | | |
| | $C_i - Z_j$ | | | | | |

a) Complete the above simplex tableau.
b) Specify the optimal daily production levels of the two products. What is the total profit?
c) Is there any alternative solution? Explain.

**Solution:**

Ans: $C_i - Z_j = 0, 0, -17.5, -5, \quad x_1 = 10, x_2 = 5, profit = 900$

No alternative solution since $C_i - Z_j$ is 0 for $x_1$ and $x_2$.

Also $x_1$ and $x_2$ are in the solution

| $C_i$ | Solution mix (basis) | | | | | Quantity |
|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $S_1$ | $S_2$ | |
| | $x_2$ | 0 | 1 | 0.375 | -0.25 | 5 |
| | $x_1$ | 1 | 0 | -0.25 | 0.50 | 10 |
| | $Z_j$ | | | | | |
| | $C_i - Z_j$ | | | | | |

## F. Sensitivity Analysis – Right Hand Side Values only

| Changes | Graphical interpretation | Impact |
|---|---|---|
| Change a non-basic decision variable to basic | Change from one corner point to the other | Impair the optimal objective function value (i.e., Z or C) |
| Change the coefficient of a given decision variable in the objective function | Change the slope of the objective function | May or may not change the optimal solution mix depending on the allowable range associated with that decision variable |
| Change the right hand side (RHS) value of a given constraint | shift the constraint | May or may not change the optimal objective function value depending on the allowable range associated with that RHS. |

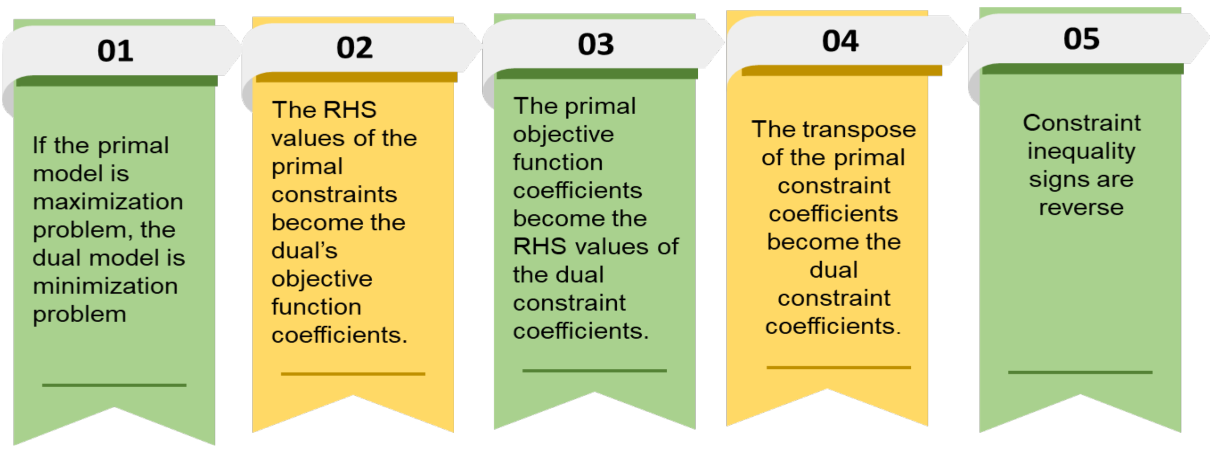
## G. Dual in LP (Interpreting the final tableau)

- In linear programming, duality implies that **each linear programming problem can be analysed in two different ways but would have equivalent solutions**. Any LP problem (either maximization and minimization) can be stated in another equivalent form based on the same data.
- Second way of stating the same problem
- LP primal involves maximizing a profit function subject to less than or equal (≤) to resource constraints
- LP dual involves minimizing total opportunity cost subject to greater than or equal (≥) to product profit constraints

**Solution to the Dual**

As we know, the primal and dual lead to the same solution even though they are formulated differently. In the final simplex tableau of a primal problem, *the absolute values of the numbers in the Cj – Zj row under the slack variables represent the solution to the dual problem*. These numbers are termed shadow prices. Thus the solution to the dual present the **marginal profits of each additional unit of resource** that is the *additional profit obtained for each additional unit resource*.

**Steps to form a dual model from the primal model**




**Relationship between Primal and Dual**

| Primal | Conversion | Dual |
|---|---|---|
| Maximation Problem | | Minimization Problem |
| Minimization Problem | | Maximation Problem |
| Objective Coefficients | | Right Hand Side (RHS) values |
| Right Hand Side (RHS) values | | Objective Coefficients |
| Number of Variables | | Number of Constraints |
| Number of Constraints | | Number of Variables |
| Variables are in terms of $X_n$ | | Variables are in terms of $Y_n$ |

## GUIDED EXAMPLE 9

From the question of **Guided Example 6**, solve the following linear programming problem below by write the dual for the linear programming model.

Maximize $Z = \boxed{2}X + \boxed{3}Y$ (profit in Ringgit Malaysia)

Subject to:

$$\begin{aligned} X + 3Y &\le 180 \quad \text{(resource 1)} \\ 2X + 3Y &\le 360 \quad \text{(resource 2)} \\ X + Y &\le 100 \quad \text{(resource 3)} \\ X, Y &\ge 0 \end{aligned}$$

**Solution:**

Minimize Cost $\boxed{C = 180U + 360V + 100W}$

Subject to:

$$\begin{aligned} &\boxed{U + 2V + W \ge 2} \\ &\boxed{3U + 3V + W \ge 3} \\ &U, V, W \ge 0 \end{aligned}$$

## GUIDED EXAMPLE 9

From the solution of **Guided Example 7**,

Maximize profit $Z = 45x + 60y + 10z$

Subject to

$$\begin{aligned} x + 3y + 2z &\le 1800 \quad \text{(Item A)} \\ 4x + 2y + 2z &\le 2000 \quad \text{(Item B)} \\ x, y, z &\ge 0 \end{aligned}$$

| $C_i$ | **Solution mix (basis)** | 45 | 60 | 10 | 0 | 0 | **Quantity** |
|---|---|---|---|---|---|---|---|
| | | $x$ | $y$ | $z$ | $S_1$ | $S_2$ | |
| | $Z_j$ | 45 | 60 | 35 | 15 | $15/2$ | 42,000 |
| | $C_i - Z_j$ | 0 | 0 | -35 | -15 | $-15/2$ | |

a) Write the dual formulation for the above LP problem.
b) State the optimal solutions for dual.

**Solution:**

a) Minimize cost $C = 1800U + 2000V$

subject to

$$U + 4V \geq 45$$
$$3U + 2V \geq 60$$
$$2U + 2V \geq 10$$
$$U, V \geq 0$$

b) $U = 15, V = 7.5$ and minimum cost RM42,000

## HANDS-ON EXAMPLE 9

Citarasa Company produces three food paste products Rendang, Kuah Kacang and Masak Lemak paste. Three resources required to produce the food paste product are cooking machine time, inspection time, and paste ingredients. The manager has formulated the problem as below.

$X_1$ = number of units of Rendang paste
$X_2$ = number of units of Kuah Kacang paste
$X_3$ = number of units of Masak Lemak paste

Maximise, $Z = 18X_1 + 15X_2 + 10X_3$

Subject to
$X_1 + X_2 + 3X_3 \leq 80$ (resource cooking machine time)
$2X_1 + X_2 + X_3 \leq 40$ (resource inspection time)
$2X_1 + 2X_2 + X_3 \leq 100$ (resource paste ingredients)
$X_1, X_2, X_3 \geq 0$

a) State the dual to this problem.
b) The following is the final simplex tableau of the primal. Complete the tableau.

| $C_i$ | Solution Mix | 18 | 15 | 10 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | |
| | $S_1$ | $-1$ | 0 | 2 | 1 | $-1$ | 0 | |
| | $X_2$ | 2 | 1 | 1 | 0 | 1 | 0 | |
| | $S_3$ | $-2$ | 0 | $-1$ | 0 | $-2$ | 1 | |
| | $Z_i$ | | | | | | | |
| | $C_j - Z_j$ | | | | | | | |

c) Is the solution obtained optimal? Why?
d) State the optimal solution and the total profit.
e) What is the shadow price of resource inspection time?

**Solution:**

Ans: $Minimise\ C = 80U + 40V + 100W$ , $U + 2V + 2W \geq 18$ ,
$U + V + 2W \geq 15$ , $3U + V + W \geq 10$ , $C_i - Z_j = -12, 0, -5, 0, -15, 0$ ,

The solution is optimal as all the values in $C_j - Z_j$ row are non-positive.

$X_2 = 40, S_1 = 40, S_3 = 20$ , $X_1 = X_3 = S_2 = 0$ , total profit: RM600, RM15.00

| $C_i$ | Solution Mix | 18 | 15 | 10 | 0 | 0 | 0 | Quantity |
|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | |
| | $S_1$ | $-1$ | 0 | 2 | 1 | $-1$ | 0 | |
| | $X_2$ | 2 | 1 | 1 | 0 | 1 | 0 | |
| | $S_3$ | $-2$ | 0 | $-1$ | 0 | $-2$ | 1 | |
| | $Z_i$ | | | | | | | |
| | $C_j - Z_j$ | | | | | | | |

**HANDS-ON EXAMPLE 10**

Write the dual formulation for the below linear programming problem.

Maximize profit $Z = x + 6y + 8z$

Subject to

$$x + 2y \le 1200$$
$$2y + z \le 1800$$
$$4x + z \le 3600$$
$$x, y, z \ge 0$$

**Solution:**

Ans: $Minimise\ C = 1200U + 1800V + 3600W$ , $U + 4W \ge 1$, $2U + 2V \ge 6$ , $V + W \ge 8$ ,

# CHAPTER 4: TRANSPORTATION AND ASSIGNMENT MODEL

Warehouse

| | | 1 | 2 | 3 | 4 | Supply | |
|---|---|---|---|---|---|---|---|
| Factory | A | \$2 | \$4 | \$4 | \$1 150 | 150 | |
| | B | 10 | \$3 100 | \$7 100 | \$7 | 200 | |
| | C | \$6 50 | 7 | \$20 50 | \$5 50 | 150 | |
| Demand | | 50 | 100 | 150 | 200 | 500 | 500 |

| | | jobs | | | |
|---|---|---|---|---|---|
| | | $J_1$ | $J_2$ | $J_3$ | $J_4$ |
| persons | $P_1$ | $\boxed{0}$ | 7 | 5 | 9 |
| | $P_2$ | 5 | 1 | $\boxed{0}$ | $\cancel{0}$ |
| | $P_3$ | 8 | $\boxed{0}$ | $\cancel{0}$ | 16 |
| | $P_4$ | 16 | 1 | 1 | $\boxed{0}$ |

**Learning Objective (LO):**

At the end of this chapter, students will be able to:

- Calculate problem involving assignment model
- Differentiate between balanced and unbalanced problem
- Differentiate between maximum and minimum problem
- Develop an initial solution for transportation model
- Solve problem involving transportation model

## 4.1 Assignment Model

- Assignment model is problem based on assigning task for object/people.
- Using this method, the task is assign in order to maximize profit or minimize the cost.
- Method that will be used to solve assignment model is Hungarian Method. This method can ONLY be used for Minimization Problem.
- If we encounter a maximization problem, the problem should be convert into minimization problem before we can solve it using Hungarian Method.

**Approach Of The Assignment Model**

- Develop a table based on the question.
- The row represents the objects or people we wish to assign
- The column represents the tasks or things we want to assign.
- The number in the table are the costs associated with each particular assignment

**A. The Hungarian Method (Flood's technique) - Minimization Problem**

- This method are used to find the optimal solution without having to make a direct comparison of every option
- The first step in this method. We will be doing matrix reduction.
  *Matrix reduction:* by subtracting and adding appropriate number in the cost table or matrix, we can reduce the problem to a matrix of opportunity costs
- After that we will proceed to finding an opportunity costs.
  *Opportunity costs:* show the relative penalties associated with assigning any person to a project as opposed to making the best or least cost assignment.

**Steps in Hungarian Method:**

Before using this step please check for balanced of this problem. Balanced problem means Number Of Row = Number of Column

**01 Find the row opportunity cost for each row (row subtraction)**

Subtracting the smallest number in each row of the original cost table or matrix from every number in that row.

**02 Find the total opportunity cost for each column (column subtraction)**

Subtracting the smallest number in each column of the table from every number in that column.

**03 Testing for optimal assignment**

- Draw the minimum number of vertical and horizontal straight line necessary to cover all zeros in the table.
- If the number of lines equals either the number of rows or columns in the table, an optimal assignment can be made.
- If the number of lines less than the number of rows or columns, we proceed to step 4.

**04 Revise the present table**

- Find the smallest number NOT covered a line (say, X)
- Subtracting the X from every other uncovered number
- This same smallest number is also added to any number (s) lying at the intersection of horizontal and vertical lines
- Return to step 3 and continue the cycle until an optimal assignment is possible.

An Optimal solution can be found by assigning to:
a) Any zero which is unique to BOTH column and a row
b) Any zero which is unique to a column OR a row
c) Assignment which is not in (a) and (b)

**GUIDED EXAMPLE 1**

The manager of Motivation Training Agency has to assign four different facilitators to handle four different programs. These facilitators would be paid at different rates per hour according to their experience. The costs (RM) of assigning the facilitators are shown in the following table. Advice the owner how to make the optimal assignment and calculate the minimum total cost.

| **Facilitator** | **Program** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| **Abu** | 210 | 90 | 180 | 160 |
| **Bakar** | 100 | 70 | 130 | 200 |
| **Ishak** | 175 | 105 | 140 | 170 |
| **Musa** | 80 | 65 | 105 | 120 |

**Solution:**

Row = Column.
Therefore, it is balanced problem, we can proceed to use Hungarian Method.

**Step 1:**
Find the smallest number in a row

| **Facilitator** | **Program** | | | | **Smallest No in a row** |
|---|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** | |
| **Abu** | 210 | **90** | 180 | 160 | **90** |
| **Bakar** | 100 | **70** | 130 | 200 | **70** |
| **Ishak** | 175 | **105** | 140 | 170 | **105** |
| **Musa** | 80 | **65** | 105 | 120 | **65** |

**Row Substraction:**

Substracting every number in a row by the smallest number such as $210-90=120$... $120-65=55$

| **Facilitator** | **Program** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| **Abu** | 120 | 0 | 90 | 70 |
| **Bakar** | 30 | 0 | 60 | 140 |
| **Ishak** | 70 | 0 | 35 | 65 |
| **Musa** | 15 | 0 | 40 | 55 |

**Step 2:**
Find the smallest number in a column from the row substraction

| Facilitator | Program | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| **Abu** | 120 | **0** | 90 | 70 |
| **Bakar** | 30 | **0** | 60 | 140 |
| **Ishak** | 70 | **0** | **35** | 65 |
| **Musa** | **15** | **0** | 40 | **55** |
| **Smallest No in Column** | **15** | **0** | **35** | **55** |

**Column Substraction:**

Substracting every number in a column by the smallest number such as $120-15=105...$ $55-55=0$

| Facilitator | Program | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| **Abu** | 105 | 0 | 55 | 15 |
| **Bakar** | 15 | 0 | 25 | 85 |
| **Ishak** | 55 | 0 | 0 | 10 |
| **Musa** | 0 | 0 | 5 | 0 |

**Step 3:**
Testing for optimal assignment

| Facilitator | Program | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| **Abu** | 105 | 0 | 55 | 15 |
| **Bakar** | 15 | 0 | 25 | 85 |
| **Ishak** | 55 | 0 | 0 | 10 |
| **Musa** | 0 | 0 | 5 | 0 |

The minimum line that cover all zero are 3 lines.
Row = Column = 4 ≠ Minimum line (3)
Therefore, this is not optimal yet. We have to proceed to **Step 4.**

**Step 4:**
Revise the present table

| Facilitator | Program | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| **Abu** | 105 | 0 | 55 | 15 |
| **Bakar** | 15 | 0 | 25 | 85 |
| **Ishak** | 55 | 0 | 0 | 10 |
| **Musa** | 0 | 0 | 5 | 0 |

Smallest Number not Covered in line = 15

**New Revised Table :**

- **Smallest Number NOT covered in line – other number not covered in line**
- **Smallest Number NOT covered in line + intersection**
- **The rest just copy from the previous table**

| Facilitator | Program | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| **Abu** | 90 | 0 | 40 | 0 |
| **Bakar** | 75 | 0 | 10 | 70 |
| **Ishak** | 55 | 0 | 0 | 10 |
| **Musa** | 0 | 0 | 5 | 0 |

**Return to Step 3:**
Testing for optimal assignment

| Facilitator | Program | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| **Abu** | 90 | 0 | 40 | 0 |
| **Bakar** | 75 | 0 | 10 | 70 |
| **Ishak** | 55 | 0 | 0 | 10 |
| **Musa** | 0 | 0 | 5 | 0 |

The minimum line that cover all zero are 4 lines.
Row = Column = 4 = Minimum line (4)
Therefore, this is an optimal solution.

**Decision making:**

Choose the program that contain cost 0 for each of the fasilitator.
Abu - Program II and Program IV
Bakar – Program II
Ishak – Program II and Program III
Musa – Program I, II and IV

Start with the fasilitator that have a less program.

Bakar – Program II
**Since Bakar has been chosen for Program I delete all Program I in other fasilitator.**

Abu - ~~Program II~~ and Program IV
Ishak – ~~Program II~~ and Program III
Musa – Program I, ~~II~~ and IV

Therefore, we will chose
Abu – Program IV
Ishak – Program III
**Now, delete program III and IV from the remaining fasilitator**
Musa – I

Therefore the answer is :
Abu - Program IV
Bakar – Program II
Ishak – Program III
Musa – Program I

Now, calculate the minimum cost for the company :
*Refer to the original table to obtain the cost for each fasilitator based on the program.
Abu - Program IV - RM 160
Bakar – Program II - RM 70
Ishak – Program III - RM 140
Musa – Program I - RM 80
Total Minimum Cost = RM 450

**B. Unbalanced Assignment Problem**

- Number of rows and columns are not equal
- More rows than columns → add dummy columns
- More columns than rows → add dummy rows
- Cost or time is equal to zero

**GUIDED EXAMPLE 2**

Ken is the owner of the Ken Electric Sdn Bhd. He receives 3 items to be repair that is blender, microwave and toaster. Currently, he have 4 repair person that is Ahmad, Ben, Chan and Khalid. This four repair persons have different talents and abilities. The cost of each person repairing the item are differ based on their skill and speed. Advice Ken on how to make the optimal assignment and calculate the minimum total cost.

| Worker | Project (cost RM) | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Ahmad | 11 | 14 | 6 |
| Ben | 8 | 10 | 11 |
| Chan | 9 | 12 | 7 |
| Khalid | 10 | 13 | 8 |

**Solution:**

Row ≠ Column
Therefore add Column in order to balance the table
Cost for the dummy = 0

| Worker | Project | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | Dummy |
| Ahmad | 11 | 14 | 6 | 0 |
| Ben | 8 | 10 | 11 | 0 |
| Chan | 9 | 12 | 7 | 0 |
| Khalid | 10 | 13 | 8 | 0 |

**Row Subtraction**

| Worker | Project | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | Dummy |
| Ahmad | 3 | 4 | 0 | 0 |
| Ben | 0 | 0 | 5 | 0 |
| Chan | 1 | 2 | 1 | 0 |
| Khalid | 2 | 3 | 2 | 0 |

**Column Subtraction :**

| Worker | Project | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | Dummy |
| Ahmad | 3 | 4 | 0 | 1 |
| Ben | 0 | 0 | 5 | 1 |
| Chan | 0 | 1 | 0 | 0 |
| Khalid | 1 | 2 | 1 | 0 |

**Optimal solution:**

| Worker | Project | Cost |
|---|---|---|
| Ahmad | 3 | 6 |
| Ben | 2 | 10 |
| Chan | 1 | 9 |
| Khalid | Dummy | 0 |
| | **Total** | **RM25** |

## C. Maximization assignment problem

- In order to solve maximum problem, convert the original table (maximize) to opportunity cost (minimize) table by subtracting every element in the table from the **largest number in the table**.
- Then, solve it using Hungarian Method.

### GUIDED EXAMPLE 3

Four ship name Garuda, Angkasa, Tiong and Merbok have to ship goods to four different sector that is A, B, C and D. Table below show a profit (RM '0000) the company will gain if he ship the goods in time. Arrange the transportation of the goods in order for the company to gain maximum profit and calculate the profit the company will gain.

| Ship | Sector | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Garuda | 20 | 60 | 50 | 55 |
| Angkasa | 60 | 30 | 80 | 75 |
| Tiong | 80 | 100 | 90 | 80 |
| Merbok | 65 | 80 | 75 | 70 |

**Solutin:**

Convert the maximum problem into minimum problem.
Largest Number in the table (100) – each element in the table.

| Ship | Sector | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Garuda | 80 | 40 | 50 | 45 |
| Angkasa | 40 | 70 | 80 | 25 |
| Tiong | 20 | 0 | 10 | 20 |
| Merbok | 35 | 20 | 25 | 30 |

Solve it using Hungarian Method

**Row Substraction:**

| Ship | Sector | | | |
|---|---|---|---|---|
| | A | B | C | D |
| **Garuda** | 40 | 0 | 10 | 5 |
| **Angkasa** | 15 | 45 | 55 | 0 |
| **Tiong** | 20 | 0 | 10 | 20 |
| **Merbok** | 15 | 0 | 5 | 10 |

**Column Substraction:**

| Ship | Sector | | | |
|---|---|---|---|---|
| | A | B | C | D |
| **Garuda** | 25 | 0 | 5 | 5 |
| **Angkasa** | 0 | 45 | 50 | 0 |
| **Tiong** | 5 | 0 | 5 | 20 |
| **Merbok** | 0 | 0 | 0 | 10 |

**Test Optimality**

| Ship | Sector | | | |
|---|---|---|---|---|
| | A | B | C | D |
| **Garuda** | 25 | 0 | 5 | 5 |
| **Angkasa** | 0 | 45 | 50 | 0 |
| **Tiong** | 5 | 0 | 5 | 20 |
| **Merbok** | 0 | 0 | 0 | 10 |

**Optimal Solution**

| Ship | Sector | Answer Sector | Profit (RM '0000) |
|---|---|---|---|
| **Garuda** | ~~B~~, D | D | 55 |
| **Angkasa** | A, ~~D~~ | A | 60 |
| **Tiong** | B | B | 100 |
| **Merbok** | ~~A,~~ ~~B~~, C | C | 75 |
| **Maximum Profit** | | | **290** |

**HANDS-ON EXAMPLE 1**

Tech Enterprise has four technicians to be assigned to repair four machines for their clients. The four technician are paid hourly. Below are the wages of the four technician, the wages are calculated based on their experience, skill and speed.

| Technician | Machine | | | |
|---|---|---|---|---|
| | A | B | C | D |
| **Ganesh** | 24 | 16 | 28 | 22 |
| **Chandran** | 20 | 18 | 20 | 16 |
| **Maxden** | 28 | 16 | 14 | 22 |
| **Danial** | 12 | 20 | 16 | 18 |

Find the optimal assignment that will minimize the total cost for the company to pay their technician. What is the total hourly profit?

**Solution:**

Ans: Ganesh - C, Chandran - B, Maxden - A, Danial - D, Total Min Cost = RM 92)

## HANDS-ON EXAMPLE 2

Frenz Marketing Research has four project leaders available for assignment to three clients. Find the assignment of project leaders to clients that will minimize the total time to complete all projects. The estimated project completion times in days are as follows:

| Project leader | Client 1 | Client 2 | Client 3 |
|---|---|---|---|
| Azmi | 10 | 15 | 9 |
| Basir | 9 | 18 | 5 |
| Chen | 9 | 14 | 3 |
| Danny | 8 | 16 | 6 |

**Solution:**

Ans: Azmi – Clien 2, Basir-Dummy,Chen – Client 3,Danny- Client 1, Total Min Cost = 26

## HANDS-ON EXAMPLE 3

The Malaysian Royal Navy wishes to assign four warships to patrol four sectors of the Malaysian Costal Areas on the lookout for illegal fishing activities and pirates. The relative efficiencies of these four warships set by the commander are shown in the following table. Find the optimal assignment to maximize efficiencies and calculate the total efficiencies.

| Ship | Sector | | | |
|---|---|---|---|---|
| | A | B | C | D |
| 1 | 90 | 65 | 95 | 40 |
| 2 | 70 | 60 | 80 | 75 |
| 3 | 85 | 40 | 80 | 60 |
| 4 | 55 | 80 | 65 | 55 |

**Solution:**

Ans: Ship 1- Sector C, Ship 2 – Sector D, Ship 3 – Sector A, Ship 4 – Sector B, Total max Cost =335

## 4.2 Transportation Model

- Transportation model are used to find the best way to distribution goods from several points of supply (sources) to a number of points of demand (destinations).
- A product is transported from a number of sources to a number of destinations at the minimum possible cost (to minimize the cost).
- Each source is able to supply a fixed number of units of the product and each destination has a fixed demand for the product
- All constraints are equalities in a balanced transportation model where supply equals to demand.
- Constraints contain inequalities in unbalanced models where supply does not equal to demand
- Solution steps:
  - Define problem
  - Set up transportation tableau
  - Develop initial solution using **least cost method**
  - Find optimal solution using **MODI method** (modified distribution)
- There are 2 step involve in transportation model. The first one is developing Initial solution and the second one is finding an optimal solution.
- Before we start with the calculation, please make sure to always check whether the problem is balanced or unbalanced problem.
- Balanced problem meaning that number of supply = number of demand

**Developing an Initial Solution**

- There are 3 method to find the initial solution
  a) **Method I: Least Cost Method-LCM**
  b) **Method II: Northwest Corner Rule**
  c) **Method III: Vogel's Approximation Method (VAM) (For Minimization Problem)**
- You can choose any suitable method for developing initial solution.

**A. Method I: Least Cost Method-LCM (For Minimization Problem)**

1. Check if the problem is balance (Total Supply SS =Total Demand DD)
   If the problem is unbalance, then either one of the following cases will happen:
   a. Total SS > Total DD, add 1 dummy destination(location)
   b. Total SS < Total DD, add 1 dummy source
2. Set up the transportation table
3. Find the 1$^{st}$ solution using LCM. This method starts by assigning goods to the cell **with the least cost (excluding zero cost assigned to each dummy)**

## GUIDED EXAMPLE 4

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28 | 18 | 32 | 36 | **300** |
| Y | 22 | 16 | 14 | 32 | **420** |
| Z | 32 | 24 | 20 | 36 | **600** |
| **Demand** | **280** | **200** | **360** | **480** | **1320** |

**Solution:**

1. Choose the lowest cost available. That is 14. 14 is in row Y and column C. Now choose that lowest value between number of supply row Y = 420 and number of demand in column C = 360.
   Therefore we will be choose 360. This means that the maximum value you can filled in is 360. Now since we already fill up the box of the lowest cost (14) with value of 360, all of the column C have been filled. Means you cannot filled in other number. So write X to indicidate that this column no longer available.

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28 | 18 | 32<br>X | 36 | **300** |
| Y | 22 | 16 | 14<br>360 | 32 | **420** |
| Z | 32 | 24 | 20<br>X | 36 | **600** |
| **Demand** | **280** | **200** | **360** | **480** | **1320** |

2. Choose the lowest cost from available box. The lowest cost now is 16. 16 located at row Y with supply 420 and column B with demand 200. Now, choose the lowest number between 420 and 200. 200 will be chosen. This means that the maximum value you can filled in is 200. Before you fill the box with cost 16 with 200, take a look at the row. The total supply for row Y is 420 but we already fill in the number of supply for the column C. Therefore the calculation will be 420 – 360 = 60. Now, all row Y have been fulfilled.

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28 | 18<br>140 | 32<br>X | 36 | **300** |
| Y | 22<br>X | 16<br>60 | 14<br>360 | 32<br>X | **420** |
| Z | 32 | 24<br>X | 20<br>X | 36 | **600** |
| **Demand** | **280** | **200** | **360** | **480** | **1320** |

3. Next choose the lowest cost that left. We will choose 18. 18 is located in column B with demand 200 and row X with supply 300. Now, choose the lowest number between 200 and 300. 200 will be chosen. This means that the maximum value you can filled in is 200. Before you fill the box with cost 18 with 200, take a look at the column B. The total supply for column B is 200 but we already fill in the number of supply for the row Y with 60. Therefore the calculation will be 200 – 60 = 140. Now, all column B have been fulfilled.

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28 | 18<br>140 | 32<br>X | 36 | 300 |
| Y | 22<br>X | 16<br>60 | 14<br>360 | 32<br>X | 420 |
| Z | 32 | 24<br>X | 20<br>X | 36 | 600 |
| Demand | 280 | 200 | 360 | 480 | 1320 |

4. Next choose the lowest cost that left. We will choose 28. 28 is located in column A with demand 280 and row X with supply 300. Now, choose the lowest number between 280 and 300. 280 will be chosen. This means that the maximum value you can filled in is 280. Before you fill the box with cost 28 with 280, take a look at the row X. The total demand for column is 280 but we already fill in the number demand for the column B with 140. Therefore the calculation will be 300 – 140 = 160. We can choose the answer. Now, all row X have been fulfilled.

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28<br>160 | 18<br>140 | 32<br>X | 36<br>X | 300 |
| Y | 22<br>X | 16<br>60 | 14<br>360 | 32<br>X | 420 |
| Z | 32 | 24<br>X | 20<br>X | 36 | 600 |
| Demand | 280 | 200 | 360 | 480 | 1320 |

5. Next choose the lowest cost that left. We will choose 32. 32 is located in column A with demand 280 and row Z with supply 600. Now, choose the lowest number between 280 and 600. 280 will be chosen. This means that the maximum value you can filled in is 280. Before you fill the box with cost 32 with 280, take a look at the row X. The total demand for column A is 280 but we already fill in the number of demand for the column A with 280. Therefore the calculation will be 280 – 160 = 120.

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28<br>160 | 18<br>140 | 32<br>X | 36<br>X | 300 |
| Y | 22<br>X | 16<br>60 | 14<br>360 | 32<br>X | 420 |
| Z | 32<br>120 | 24<br>X | 20<br>X | 36 | 600 |
| Demand | 280 | 200 | 360 | 480 | 1320 |

6. Now the only box that have not been filled is cost 36. Fill in the box with the remaining supply.
600 – 120 = 480

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28<br>160 | 18<br>140 | 32<br>X | 36<br>X | 300 |
| Y | 22<br>X | 16<br>60 | 14<br>360 | 32<br>X | 420 |
| Z | 32<br>120 | 24<br>X | 20<br>X | 36<br>480 | 600 |
| Demand | 280 | 200 | 360 | 480 | 1320 |

Total Transportation = (160 x RM 28) + (140 x RM 18) + (60 x RM 16) + (360 x RM 14) + (120 x RM 32) + (480 X RM 36)
= RM34,120.

## B. Method II: Northwest Corner Rule

i. The starting point is the northwest corner of the table with the maximum quantity allocated to the cell.
ii. If the supply at each row is allocated fully, allocation is then moved to the next row. This begins with the first row.
iii. Similarly, if the demand of each column is allocated fully, the demand is then moved to the next column. Again, beginning with the first column
iv. Finally all the supplies and demands are ensured to be satisfied.

**Solution:**

| From/To | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 28<br>**[1]280** | 18<br>**[2]20** | 32<br>x | 36<br>x | 300 |
| Y | 22<br>x | 16<br>**[3]180** | 14<br>**[4]240** | 32<br>x | 420 |
| Z | 32<br>x | 24<br>x | 20<br>**[5]120** | 36<br>**[6]480** | 600 |
| Demand | 280 | 200 | 360 | 480 | 1320 |

Total Transportation = (280 x RM 28) + (20 x RM 18) + (180 x RM 16) + (240 x RM 14) + (120 x RM 20) + (480 x RM 36)
= RM34,120.

## C. Method III: Vogel's Approximation Method (VAM) (For Minimization Problem)

i. Compute opportunity cost (difference between two lowest cost cell of the row/column) for each row/column.
ii. Select the highest opportunity cost and assign maximum units to the lowest cost cell of the row/column. Eliminate the row/column and recalculate the opportunity cost.
iii. Repeat step 1 and 2 until the table is fully filled.

**Solution:**

| From/To | A | B | C | D | Supply | |
|---|---|---|---|---|---|---|
| **X** | 28<br>X | 18<br>[1]**200** | 32<br>x | 36<br>[5]**100** | **300** | [1]**10** 4 8<br>36 [5]**36** |
| **Y** | 22<br>[3]**280** | 16<br>x | 14<br>x | 32<br>[6]**140** | **420** | 2 8 [3]**10**<br>32 32 |
| **Z** | 32<br>x | 24<br>x | 20<br>[2]**360** | 36<br>[4]**240** | **600** | 4 [2]**12** 4<br>[4]**36** x |
| **Demand** | **280** | **200** | **360** | **480** | **1320** | |
| | 6 6 6 x x | 2 x x x x | 6 6 x x x | 4 4 4 4 4 | | |

** [1]200, number 1 represent the first quantity been filled.

Total Transportation = (200 x RM 18) + (100 x RM 36) + (280 x RM 22) + (140 x RM 32) + (360 x RM 20) + (240 x RM 36)
= RM 33 680.

Comparison between the three methods.

| Method | Transportation Cost |
|---|---|
| LCM | RM 34120 |
| Northwest Corner Rule | RM 34120 |
| ***VAM*** | ***RM33680 (lowest)*** |

*** VAM is the best method to choose.**

## D. Finding an Optimal Solution: MODI Method

**Testing for optimal solution**

1. Make sure: No. of occupied cells = no. of rows + no. of column – 1 (to ensure no degeneracy problem)
   Compute the cost value for each row and column, for those squares that are currently used or occupied by setting
   $$R_i + K_j = C_{ij}$$
   where *Ri* = value assigned to row *I*, $K_j$ = value assigned to column *j*, $C_{ij}$ = cost in square *ij*
2. After all the equation has been written, set $R_1 = 0$.
3. Solve the system of equation for all R and K values.
4. Next, calculate the improvement index using formula for **unoccupied/unused** cells only:
   $$\text{Improvement Index}, I_{ij} = C_{ij} - R_i - K_j$$
   Write the improvement index at the bottom left corner of each unused cell.
5. A solution is optimum when all $I_{ij} \geq 0$. If at least one $I_{ij} < 0$, get a better solution.

**Finding a better solution**

1. Find the entering cell - the cell with the largest negative Improvement Index
2. Find the leaving cell.
   a. Make a closed path for the entering cell. (this path must be as simple as possible)
      - Start and end at the entering cell.
      - Use vertical & horizontal line.
      - Lines can change only at occupied cells – not all occupied cells have to be in the closed path.
   b. Place (+) & (-) sign alternately at each corner of the path. Start with the (+) sign at the entering cell.
   c. Choose the lowest number of units from cells containing (-) sign, say Q. This is the leaving cell. (Do the same for maximization problem)
3. Set up a new table.
   - Add Q to any cell with (+) sign and subtract Q to any cell with (-) sign.
4. Compute new improvement indices for this new solution to test either the solution is optimal or not. Repeat all the process until the optimal solution is found.

## E. The Unbalanced Transportation Problem

- When demand exceeds supply, a dummy row is added to the tableau

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| 1 | 6 | 8 | 10 | 150 |
| 2 | 7 | 11 | 11 | 175 |
| 3 | 4 | 5 | 12 | 275 |
| Dummy | 0 | 0 | 0 | 50 |
| Demand | 200 | 100 | 350 | 650 |

- When supply exceeds demand, a dummy column is added to the tableau

| From/To | A | B | C | Dummy | Supply |
|---|---|---|---|---|---|
| 1 | 6 | 8 | 10 | 0 | 150 |
| 2 | 7 | 11 | 11 | 0 | 175 |
| 3 | 4 | 5 | 12 | 0 | 375 |
| Demand | 200 | 100 | 300 | 100 | 700 |

All steps applied in order to obtain optimal solution are same as previous

## F. Alternative Optimal Solution

Multiple solutions are possible when one or more improvement indices in the optimal solution stages equals zero. This means that it is possible to assign alternative transportation routes with the same total transportation cost.

Steps to find the alternative solution are:

i. Choose the cell with $I_{ij} = 0$ as the entering cell.

ii. Use the same procedure as in step 2 and 3 (in finding a better solution)

iii. Write down the new solution

Notes: If there is 1 cell with $I_{ij} = 0$, the problem has only one alternative solution. If there are two cells with $I_{ij} = 0$, the problem has two alternative solutions.

**GUIDED EXAMPLE 5**

Find an alternative optimal solution

| | From/To | $K_A$=6 | $K_B$=7 | $K_C$=10 | Supply |
|---|---|---|---|---|---|
| | | A | B | C | |
| $R_1$=0 | 1 | 6<br>25 | 8<br>+1 | 10<br>125 | 150 |
| $R_2$=1 | 2 | 7<br>0 | 11<br>+3 | 11<br>175 | 175 |
| $R_3$=-2 | 3 | 4<br>175 | 5<br>100 | 12<br>+4 | 275 |
| | Demand | 200 | 100 | 300 | 600 |

## G. Degeneracy

- In a transportation tableau with *m* rows and *n* columns, there must be *m* + *n* - 1 cell with allocations; if not, it is *degenerate*.
- The tableau in the figure does not meet the condition since 3 + 3 -1 = 5 cells and there are only 4 cells with allocations.

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| 1 | 6 | 8<br>100 | 10<br>50 | 150 |
| 2 | 7 | 11 | 11<br>250 | 250 |
| 3 | 4<br>200 | 5 | 12 | 200 |
| Demand | 200 | 100 | 300 | 600 |

- In a degenerate tableau, all the MODI equations cannot be developed
- To rectify a degenerate tableau, an empty cell must artificially be treated as an occupied cell and we place a zero (representing a fake shipment).
- Place zero in one of the lowest cost unused cell and then treat the square as if it were occupied cell. If still unable to solve for R and K, change to the next unused least cost cell.

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| 1 | 6 | 8<br>100 | 10<br>50 | **150** |
| 2 | 7 | 11 | 11<br>250 | **250** |
| 3 | 4<br>200 | 5<br>0 | 12 | **200** |
| **Demand** | **200** | **100** | **300** | **600** |

## GUIDED EXAMPLE 6

Zamcomp Sdn. Bhd. produces one type of computer table at three factories located at different towns. Factory I, II, and III can manufacture 300, 200, and 100 units respectively. These computer tables are distributed to three major outlets, A, B and C. The demands from the outlets are 200, 150 and 250 units respectively. The cost of producing one unit of computer table varies due to different production technologies.

The production and distribution cost per unit (RM) from factories to outlets are as follows.

| FACTORY | OUTLETS | | |
|---|---|---|---|
| | A | B | C |
| I | 37 | 34 | 36 |
| II | 36 | 32 | 35 |
| II | 42 | 35 | 41 |

a) Set up a transportation tableau and determine the initial solution using LCM.
b) Is the initial solution from (a) optimal? Why?
c) Calculate the total production and distribution cost for the initial solution.
d) Solve the optimal solution, list the solution and calculate the minimum total production and transportation cost.
e) Explain why there is an alternative optimal solution, find it and fill in the table below.

**Solution:**

a) Initial Using LCM

- Check for balance/unbalanced problem. Balance problem Demand = Supply

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| I | 37<br>100 | 34<br>X | 36<br>200 | **300** |
| II | 36<br>X | 32<br>150 | 35<br>50 | **200** |
| III | 42<br>100 | 35<br>X | 41<br>X | **100** |
| **Demand** | **200** | **150** | **250** | **600** |

Total Transportation = (100 x RM 37) + (200 x RM 36) + (150 x RM 32) + (50 x RM 35)
(100 x RM 42)
= RM21,650

* check number of quantity that have been filled to make sure we did not encounter degeneracy problem.
Row + Column – 1 = Number of box filled
3 + 3 -1 = 5 √ (not degeneracy problem)

b) Check for Optimality using MODI

| From/To | A<br>$K_1 = 37$ | B<br>$K_2 = 33$ | C<br>$K_3 = 36$ | Supply |
|---|---|---|---|---|
| **I**<br>$R_1 = 0$ | 37<br>100 | 34<br>**1** | 36<br>200 | **300** |
| **II**<br>$R_2 = -1$ | 36<br>**0** | 32<br>150 | 35<br>50 | **200** |
| **III**<br>$R_3 = 5$ | 42<br>100 | 35<br>**-3** | 41<br>**0** | **100** |
| **Demand** | **200** | **150** | **250** | **600** |

$R_i + K_j = C_{ij}$ (for current box filled)

$R_1 + K_1 = 37$ Solve for all *R* and *K* value and fill in the above tableu

$R_1 + K_3 = 36$ $\boldsymbol{R_1 = 0, K_1 = 37, K_3 = 36, R_2 = -1, R_3 = 5, K_2 = 33}$

$R_2 + K_2 = 32$

$R_2 + K_3 = 35$

$R_3 + K_1 = 42$

Then, solve index improvement for each of unused cell. $I_{ij} = C_{ij} - R_i - K_j$

$I_{IB} = 34 - 0 - 33 = \mathbf{1}$

$I_{IIA} = 36 - (-1) - 37 = \mathbf{0}$

$I_{IIIB} = 35 - 5 - 33 = \mathbf{-3}$

$I_{IIIC} = 41 - 5 - 36 = \mathbf{0}$

Since, one of the value of index improvement is not larger and equal to zero. Therefore it is not optimal.

c) Total Transportation = (100 x RM 37) + (200 X RM 36) + (150 X RM 32) + (50 X RM 35)
(100 X RM 42)
= RM21,650

d) Finding a better solution.

| From/To | A<br>$K_1 = 37$ | B<br>$K_2 = 33$ | C<br>$K_3 = 36$ | Supply |
|---|---|---|---|---|
| I<br>$R_1 = 0$ | (+) 37<br>100 | 34<br>1 | (-) 36<br>200 | 300 |
| II<br>$R_2 = -1$ | 36<br>0 | 32<br>150 (-) | 35<br>50 (+) | 200 |
| III<br>$R_3 = 5$ | (-) 42<br>100 | 35<br>-3 (+) | 41<br>0 | 100 |
| Demand | 200 | 150 | 250 | 600 |

- Choose the most negative value for index improvement to be entering cell. Cell with -3 as improvement index are chose.
- Make a close path
- Place (+) & (-) sign alternately at each corner of the path. Start with the (+) sign at the entering cell.
- Choose the lowest number of units from cells containing (-) sign, say Q. This is the leaving cell. (Quantity = 100)
- Set up a new table.
    - Add Q to any cell with (+) sign and subtract Q to any cell with (-) sign.

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| I | 37<br>200 | 34 | 36<br>100 | 300 |
| II | 36 | 32<br>50 | 35<br>150 | 200 |
| III | 42 | 35<br>100 | 41 | 100 |
| Demand | 200 | 150 | 250 | 600 |

Now check for optimality using MODI

| From/To | A<br>$K_1 = 37$ | B<br>$K_2 = 33$ | C<br>$K_3 = 36$ | Supply |
|---|---|---|---|---|
| I<br>$R_1 = 0$ | 37<br>200 | 34<br>1 | 36<br>100 | 300 |
| II<br>$R_2 = -1$ | 36<br>0 | 32<br>50 | 35<br>150 | 200 |
| III<br>$R_3 = 0$ | 42<br>5 | 35<br>100 | 41<br>5 | 100 |
| Demand | 200 | 150 | 250 | 600 |

$R_i + K_j = C_{ij}$ (for current box filled)

$R_1 + K_1 = 37$ Solve for all R and K value and fill in the above tableu

$R_1 + K_3 = 36$ $\boldsymbol{R_1 = 0, K_1 = 37, K_3 = 36, R_2 = -1, R_3 = 0, K_2 = 33}$

$R_2 + K_2 = 32$

$R_2 + K_3 = 35$

$R_3 + K_2 = 33$

Then, solve index improvement for each of unused cell. $I_{ij} = C_{ij} - R_i - K_j$

$I_{IB} = 34 - 0 - 33 = \mathbf{1}$

$I_{IIA} = 36 - (-1) - 37 = \mathbf{0}$

$I_{IIIA} = 42 - 0 - 37 = \mathbf{5}$

$I_{IIIC} = 41 - 0 - 36 = \mathbf{5}$

Since, index improvement is larger and equal to zero. Therefore it is optimal.

**Answer:**

Transport from I to A = 200
Transport from I to C = 100
Transport from II to B = 50
Transport from II to C = 150
Transport from III to B = 100

Total Transportation = (200 x RM 37) + (100 X RM 36) + (50 X RM 32) + (150 X RM 35)
(100 X RM 35)
= RM21,350

e) Yes, it has alternative optimal solution because one of the improvement index is equal to zero.

The method to find alternative optimal solution is the same as finding an optimal solution. The different is we choose index improvement = 0 as our entering cell.

| **From/To** | **A** $K_1 = 37$ | **B** $K_2 = 33$ | **C** $K_3 = 36$ | **Supply** |
|---|---|---|---|---|
| **I** $R_1 = 0$ | (-) 37<br>200 | 34<br>1 | (+) 36<br>100 | **300** |
| **II** $R_2 = -1$ | 36<br>0 (+) | 32<br>50 | (-) 35<br>150 | **200** |
| **III** $R_3 = 0$ | 42<br>5 | 35<br>100 | 41<br>5 | **100** |
| **Demand** | **200** | **150** | **250** | **600** |

- Make a close path
- Place (+) & (-) sign alternately at each corner of the path. Start with the (+) sign at the entering cell.
- Choose the lowest number of units from cells containing (-) sign, say Q. This is the leaving cell. (Quantity = 150)
- Set up a new table.
    - Add Q to any cell with (+) sign and subtract Q to any cell with (-) sign.

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| I | 37<br>50 | 34 | 36<br>250 | **300** |
| II | 36<br>150 | 32<br>50 | 35 | **200** |
| III | 42 | 35<br>100 | 41 | **100** |
| **Demand** | **200** | **150** | **250** | **600** |

**Answer:**

Transport from I to A = 50
Transport from I to C = 250
Transport from II to A = 150
Transport from II to B = 50
Transport from III to B = 100

Total Transportation = (50 x RM 37) + (250 x RM 36) + (150 x RM 36) + (50 x RM 32)
(100 x RM 35)
= RM21,350

**** the total transportation is the same as (d) but the transportation item from factory to outlet is different.***

**Maximization Transportation Problem**

- The optimal solution to a maximization problem has been found when all improvement indices are negative or zero.
- If any index is +ve, the cell with the largest +ve improvement index is selected to be filled using a MODI method.
- This new solution evaluated and the process continues until there are no +ve improvement indices.

**GUIDED EXAMPLE 7**

Refer to **Guided-Example 6**:
Calculate the total profit if the computer tables are sold at RM320 each of the outlets.

**Solution:**

**Initial Using LCM**

- Check for balance/unbalanced problem. Balance problem Demand = Supply

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| I | 283<br>100 | 286<br>X | 284<br>200 | **300** |
| II | 284<br>X | 288<br>150 | 285<br>50 | **200** |
| III | 278<br>100 | 285<br>X | 279<br>X | **100** |
| **Demand** | **200** | **150** | **250** | **600** |

Total Transportation = (100 x RM 283) + (200 x RM 284) + (150 x RM 288) + (50 x RM 285)
(100 x RM 278)
= RM170,350

* check number of quantity that have been filled to make sure we did not encounter degeneracy problem.

Row + Column – 1 = Number of box filled
3 + 3 – 1 = 5 √ (not degeneracy problem)

Check for Optimality using MODI

| From/To | A $K_1 = 283$ | B $K_2 = 287$ | C $K_3 = 284$ | Supply |
|---|---|---|---|---|
| I $R_1 = 0$ | [283] 100 | [286] -1 | [284] 200 | 300 |
| II $R_2 = 1$ | [284] 0 | [288] 150 | [285] 50 | 200 |
| III $R_3 = -5$ | [278] 100 | [285] 3 | [279] 0 | 100 |
| Demand | 200 | 150 | 250 | 600 |

$R_i + K_j = C_{ij}$ (for current box filled)

$R_1 + K_1 = 283$ Solve for all *R* and *K* value and fill in the above tableu

$R_1 + K_3 = 284$ $R_1 = 0$ , $K_1 = 283$, $K_3 = 284$, $R_2 = 1$, $R_3 = -5$, $K_2 = 287$

$R_2 + K_2 = 288$

$R_2 + K_3 = 285$

$R_3 + K_1 = 278$

Then, solve index improvement for each of unused cell. $I_{ij} = C_{ij} - R_i - K_j$

$I_{IB} = 286 - 0 - 287 = -1$

$I_{IIA} = 284 - 1 - 283 = 0$

$I_{IIIB} = 285 - (-5) - 287 = 3$

$I_{IIIC} = 279 - (-5) - 284 = 0$

Since, one of the value of index improvement is larger and equal to zero. Therefore it is not optimal.

Finding a better solution.

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| I | (+) 283<br>100 | 286<br>-1 | (-) 284<br>200 | 300 |
| II | 284<br>0 | (-) 288<br>150 | 285<br>50 (+) | 200 |
| III | (-) 278<br>100 | (+) 285<br>3 | 279<br>0 | 100 |
| Demand | 200 | 150 | 250 | 600 |

- Choose the most positive value for index improvement to be entering cell. Cell with 3 as improvement index are chose.
- Make a close path
- Place (+) & (-) sign alternately at each corner of the path. Start with the (+) sign at the entering cell.
- Choose the lowest number of units from cells containing (-) sign, say Q. This is the leaving cell. (Quantity = 100)
- Set up a new table.
  - ✓ Add Q to any cell with (+) sign and subtract Q to any cell with (-) sign.

| From/To | A | B | C | Supply |
|---|---|---|---|---|
| I | 283<br>200 | 286 | 284<br>100 | 300 |
| II | 284 | 288<br>50 | 285<br>150 | 200 |
| III | 278 | 285<br>100 | 279 | 100 |
| Demand | 200 | 150 | 250 | 600 |

Now check for optimality using MODI

| From/To | A<br>$K_1 = 283$ | B<br>$K_2 = 287$ | C<br>$K_3 = 284$ | Supply |
|---|---|---|---|---|
| I<br>$R_1 = 0$ | 283<br>200 | 286<br>-1 | 284<br>100 | 300 |
| II<br>$R_2 = 1$ | 284<br>0 | 288<br>50 | 285<br>150 | 200 |
| III<br>$R_3 = -2$ | 278<br>-3 | 285<br>100 | 279<br>-3 | 100 |
| Demand | 200 | 150 | 250 | 600 |

$R_i + K_j = C_{ij}$ (for current box filled)

$R_1 + K_1 = 283$ Solve for all R and K value and fill in the above tableu

$R_1 + K_3 = 284$ $R_1 = 0, K_1 = 283, K_3 = 284, R_2 = 1, R_3 = -2, K_2 = 287$

$R_2 + K_2 = 288$

$R_2 + K_3 = 285$

$R_3 + K_2 = 285$

Then, solve index improvement for each of unused cell. $I_{ij} = C_{ij} - R_i - K_j$

$I_{IB} = 286 - 0 - 287 = -\mathbf{1}$

$I_{IIA} = 284 - 1 - 283 = \mathbf{0}$

$I_{IIIA} = 278 - (-2) - 283 = \mathbf{-3}$

$I_{IIIC} = 279 - (-2) - 284 = \mathbf{-3}$

Since, index improvement is not larger and equal to zero. Therefore it is optimal.

**Answer:**

Transport from I to A = 200
Transport from I to C = 100
Transport from II to B = 50
Transport from II to C = 150
Transport from III to B = 100

Total Transportation = (200 x RM 283) + (100 X RM 284) + (50 x RM 288) + (150 x RM 285)
(100 x RM 285)
= RM170,650

## HANDS-ON EXAMPLE 4

A plywood producer manufactures plywood at three factories to meet the demand of three construction sites. The factories are located at different locations. The production capacity at each factory and the demand from each construction site are shown in the following tables:

| Factory | Capacity (units) |
|---|---|
| X | 300 |
| Y | 200 |
| Z | 100 |

| Site | Demand (units) |
|---|---|
| P | 200 |
| Q | 150 |
| R | 230 |

The transportation cost (RM) per unit from each factory to each site is shown in the following table:

| Factory | Site | | |
|---|---|---|---|
| | P | Q | R |
| X | 7 | 4 | 6 |
| Y | 6 | 2 | 5 |
| Z | 12 | 5 | 11 |

a) Find an initial solution for this transportation problem.
b) Determine an optimal solution and the associated total transportation cost.
c) Which factory will have the surplus? State the surplus.
d) Is there any alternative optimal solution to this problem? If yes, find the alternative optimal solution.

**Solution:**

Ans: Total Cost = RM 3430, Total Cost=RM 3190,
Factory Z will have the surplus of 20 units

| From/To | P | Q | R | Dummy | Supply |
|---|---|---|---|---|---|
| X | 7<br>70 | 4 | 6<br>230 | 0 | 300 |
| Y | 6<br>130 | 2<br>70 | 5 | 0 | 200 |
| Z | 12 | 5<br>80 | 11 | 0 | 100 |
| Demand | 200 | 150 | 230 | 20 | 600 |

## HANDS-ON EXAMPLE 5

Sasuki Manufacturing Company has orders for three kinds of product manufactured in their factory located in Pasir Gudang.

| Product | Orders (Unit) |
|---|---|
| A | 1300 |
| B | 800 |
| C | 1200 |

Three machines are available for the manufacturing operations. All three machines can produce all the products at the same production rate. However, due to varying defect percentages of each machine, the unit costs of products vary depending on the machine used. Machine capacities for the next week, and the unit costs, are as follows:

| Machine | Capacity (Units) |
|---|---|
| 1 | 1300 |
| 2 | 1800 |
| 3 | 900 |

| | Product | | |
|---|---|---|---|
| **Machine** | **A** | **B** | **C** |
| 1 | 7 | 6 | 6 |
| 2 | 12 | 7 | 2 |
| 3 | 8 | 5 | 9 |

a) Find the minimum-cost production schedule for the products and machines by using the Stepping-Stone Method.

b) Is the solution unique? Explain.

**Solution:**

Ans: Total Min. Cost = RM15,500, the solution is unique

## HANDS-ON EXAMPLE 6

Ceria Company manufactures tires in two factories XY and PQ and ships them to four distribution centres in Johor Bahru, Penang, Petaling Jaya and Kuala Terengganu. The following table shows the cost (RM) per unit, capacity and demand.

| To / From | Distribution Centre | | | | Capacity |
|---|---|---|---|---|---|
| | **Johor Bahru** | **Penang** | **Petaling Jaya** | **Kuala Terengganu** | |
| **XY** | 7 | 11 | 5 | 12 | **300** |
| **PQ** | 10 | 15 | 8 | 16 | **200** |
| **Demand** | **150** | **120** | **50** | **180** | |

a) Find the optimal solution to the problem.
b) Calculate the total transportation cost.

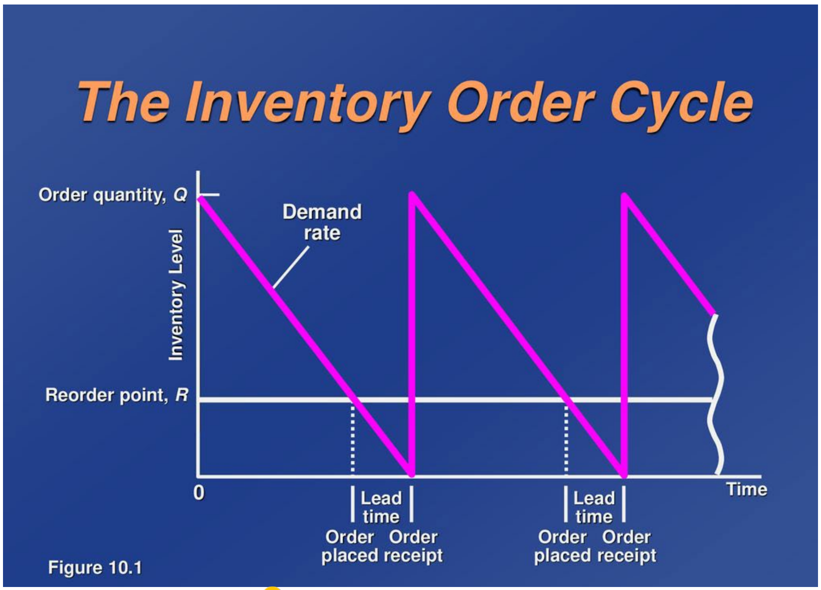
**Solution:**

Ans:

| To / From | Distribution Centre | | | | Capacit |
|---|---|---|---|---|---|
| | Johor Bahru ($K_1$=7) | Penang ($K_2$=11) | Petaling Jaya ($K_3$=5) | Kuala Terengganu ($K_4$=12) | |
| XY ($R_1$=0) | 7 [0] | 11 **(120)** | 5 **(0)**** | 12 **(180)** | 300 |
| PQ ($R_2$=3) | 10 **(150)** | 15 [1] | 8 **(50)** | 16 [1] | 200 |
| Demand | 150 | 120 | 50 | 180 | **500** |

Total transportation=RM5380

# CHAPTER 5: INVENTORY CONTROL




**Learning Outcome (LO)**
At the end of the chapter, students will be able to:

- Understand the importance of inventory control
- Develop the basic EOQ model to determine how much to order
- Calculate the reorder point (ROP) in determining when to order more inventory
- Find the best decision using quantity discount

## 5.1 The Important of Inventory Model

### A. Introduction

- Inventory is any stored resource that is used to satisfy a current or future demand from customer. For examples items for sale, extra table in classrooms, blood in blood bank, money in a saving accounts, doctors on standby and etc.
- Usually, every company will maintain a level of inventory that will meet anticipated/ expected customer demand.
- However, because demand is usually not known with certainty, additional amounts of inventory called safety or buffer stocks are often kept on hand to meet unexpected variations in excess of expected demand.
- Companies can also purchase large amounts of inventory to take advantage of price discounts.

### B. Objective

The objective of inventory models is to develop an inventory control system that will indicate how much should be ordered and when orders should take place to minimize the total inventory costs.

### C. Demand

- Major component of inventory model because inventory exists to meet the demand of customers.
- It is very important to develop an accurate forecast of customer demand in order to determine appropriate level of inventories to be kept.

### D. Importance Terms of Inventory Control

**Decoupling Function**

- decouple manufacturing processes within organization
- If you did not store inventory, there could be many delays and inefficiencies
- Inventory can act as a buffer

**Storing Resources**

- Terms in the objective function and constraint equations must be additive.
- The total of all activities equals the sum of the individual activities.
- Example: If an objective is to maximize profit:
  1st product = RM4, 2nd product = RM6
  When product produced, the profit contribution must add up to produce sum of RM10.

**Irregular Supply and Demand**

- When the supplies or demand for an inventory item is irregular, storing certain amounts in inventory can be important
- You have to make sure that there is enough supply to meet this irregular demand

**Quantity Discounts**

- Many suppliers offer discounts for large orders
- Advantage : Purchasing in larger quantities can substantially reduce the cost of products
- Disadvantages; higher storage costs and higher costs due to spoilage, damage stock, theft, insurance, and etc.

**Avoiding Stock outs and Shortages**

- If you are repeatedly out of stock, customers are likely to go elsewhere to satisfy their needs
- Lost goodwill can be an expensive price to pay for not having the right item at the right time

## 5.2 The Inventory Decision

### Inventory Decisions

- Two fundamental decisions that you have to make when controlling inventory are:
  - How much to order
  - When to order

- There are four basic costs associated with inventory:
  - Holding Cost
  - Ordering Cost
  - Shortage Cost
  - Purchase Cost

### Holding costs

the costs of holding items in storage and these costs vary with level of inventory. The greater the level of inventory over time, the higher the holding cost.

### Ordering costs

the costs associated with replenishing the stock of inventory. These are normally expressed as a RM amount per order and are independent of the order size.

### Shortage costs

occur when demand cannot be met because of insufficient inventory on hand.

### Purchase cost

the actual price of the items.

### Ordering cost

1. **The cost of developing and sending an order**
2. **The costs of transporting he order from the supplier**
3. **The cost of receiving an order and placing it I inventory**
4. **The salaries of employees involved in the placement and replacement and receipt of an order**
5. **All supplies utilized in the ordering process**

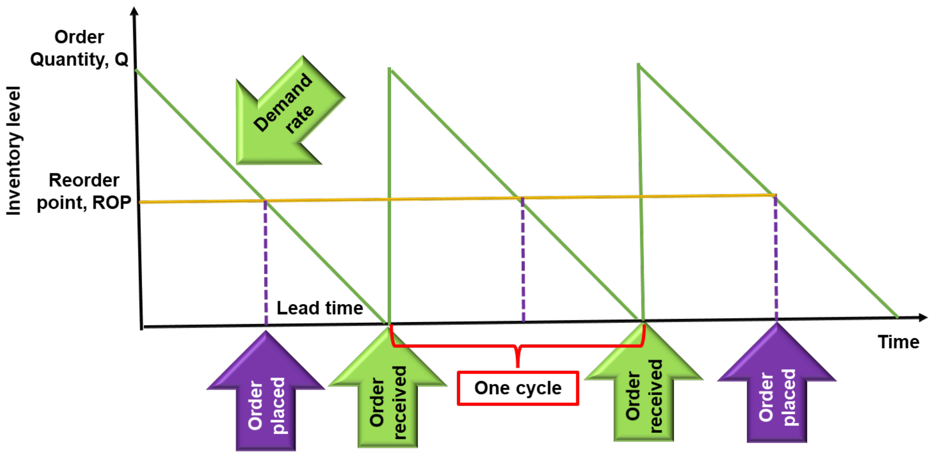
### Holding/Carrying cost

1. **The direct costs such as rental, maintenance, spoilages, theft and the like incurred from storing the product in inventory**
2. **Deferred profit for carrying the product in inventory**
3. **Product obsolescence**
4. **Taxes and insurance**
5. **The salaries of employees involved in inventory**
6. **All supplies utilised in keeping the inventory**

## 5.3 The Economic Order Quantity (EOQ)

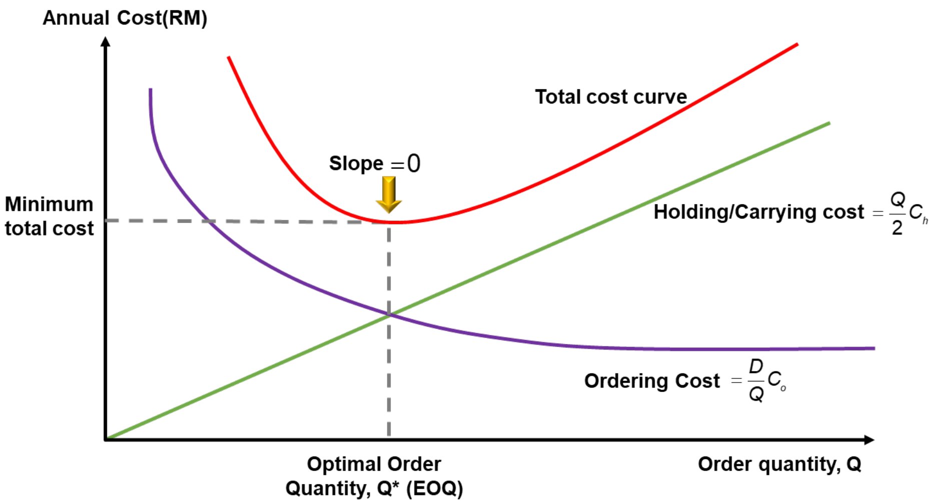
- The function of the EOQ model is to determine the optimal order size that minimizes the total inventory costs.
- This is the simplest form of inventory control models.
- The assumptions of this model are:
  - Demand (*D*) is known with certainty and is relatively constant over time.
  - No shortages are allowed and quantity discounts are not possible
  - Lead time (*L*) is the time between the placement of the order and the receipt of the order, is known and constant
  - The order quantity is received all at once.

The inventory order cycle for basic EOQ model.




**One inventory cycle is the time between successive reorders.*

- From the above figure, we can observe that, an order quantity, Q, is received and is used up over time at a constant rate.
- When the inventory level decreases to the reorder point, ROP, a new order is placed and a period of time, referred as lead time, is required for delivery.
- The order is received all at once just at the moment when the entire stocks of inventory reaches zero, thus allowing no shortages.
- This cycle is continuously *repeated for the same order quantity, reorder point and lead time*.
- Q, the order size, is the order quantity that minimizes the total costs.



a) Optimal order quantity: $Q^* = \sqrt{\dfrac{2DC_o}{C_h}}$

b) Total annual cost = annual ordering cost + annual holding cost + annual purchasing cost

$$TC = \frac{D}{Q}C_o + \frac{Q}{2}C_h + DC$$

$Q =$ number of pieces to order

$EOQ = Q^* =$ optimal number of pieces to order

$D =$ annual demand in units for the inventory item

$C_0 =$ ordering cost of each order

$C_h =$ holding or carrying cost per unit per year

$C =$ cost of item

**GUIDED EXAMPLE 1**

Demand for a particular product at a retail shop is estimated at 6000 units per year. The price per unit for that product is RM50. It costs the owner RM130 every time an order is placed. Since safety is one of the major factors at a retail shop, inspection of the product will cost the owner RM70 once the product arrives. The cost of holding one unit of the product per year is 10% of the price. Placing an order once a month is the current practice of the retail shop.

a) Determine the order quantity that minimizes the total inventory cost

b) Calculate the petrol station's annual savings if the optimum-order quantity policy is used instead of the current practice.

**Solution:**

a) $C = 50 \quad C_o = 130 + 70 = 200 \quad D = 6000$

$C_h = 0.1 \times 50 = 5$,

$$Q^* = \sqrt{\frac{2(6000)200}{5}} = 692.8 \approx 693$$

$$TC_{693} = 6000(50) + \frac{693}{2}(5) + \frac{6000}{693}(200) = RM\,303,464.10$$

b) Current Practice

$Q = 6000/12 = 500$

$$TC_{500} = 6000(50) + \frac{500}{2}(5) + \frac{6000}{500}(200) = RM\,303,650$$

Total saving = 303,650 – 303,464.10 = RM185.90

## 5.4 The Reorder Point (ROP)

A reorder point (ROP) is a specific level at which your stock needs to be replenished. In other words, it tells you when to place an order so you won't run out of stock.

a) Number of orders per year: $N = \frac{D}{Q}$

b) Time between orders (cycle time): $T = \frac{Q}{D}$ year

c) Reorder point: $ROP = d \times L$

Demand per day:

$$d = \frac{D}{Number\ of\ working\ days\ in\ a\ year}$$

$Q =$ number of pieces to order
$D =$ annual demand in units for the inventory item
$L =$ lead time for a new order in days (time taken to receive an order)

**GUIDED EXAMPLE 2**

Chan Electric Sdn Bhd stocks and sells electric kettle. The annual demand is 12,000 units and the company opens 360 days a year. The cost of electric kettle is RM40 and the annual holding cost is 10% of the unit cost. It cost the company RM375 every time it places an order and the lead time is 15 days.

a) Determine
   i) The optimal quantity of order (EOQ)
   ii) The optimal number of order per year
   iii) The level of inventory at which a new order must be placed
   iv) The time between order
b) Draw the inventory cycle for one year.
c) Calculate the minimum total inventory cost.

**Solution:**

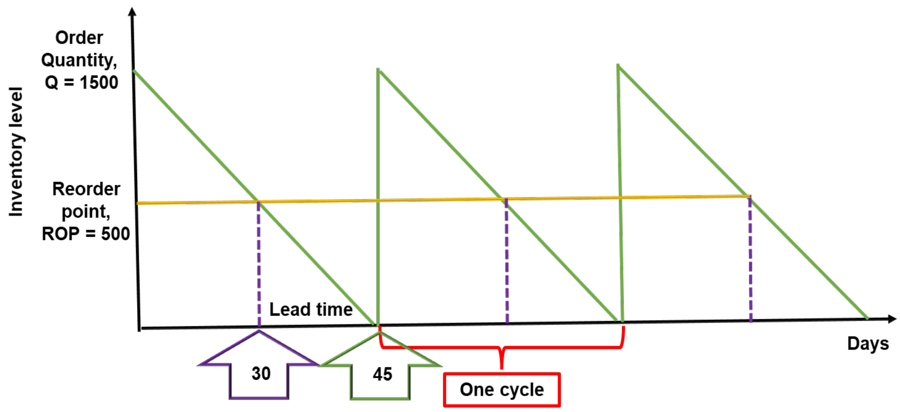
a) i) $\text{EOQ}, Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(12000)(375)}{4}} = 1500$

ii) $N = \frac{D}{Q} = \frac{12000}{360} = 8$

iii) $ROP = d \times L = \left(\frac{12000}{360}\right) \times 15 = 500$

iv) $T = \frac{Q}{D}\text{ year} = \left(\frac{1500}{12000}\right)(360) = 45\text{ days}$

b)




c) $TC = \frac{D}{Q}C_o + \frac{Q}{2}C_h + DC = \frac{12000}{1500}(375) + \frac{1500}{2}(4) + (12000)(40) = 486000$

**HANDS-ON EXAMPLE 1**

A manufacturing company orders a major component from a local supplier. The annual demand for the component is 8,000 units. The cost of placing an order is RM500 while the unit cost is RM800. The unit annual holding cost is 6.25% of the unit cost. Currently the company is ordering 10 times per year and it operates 50 weeks a year.

a) Calculate the order quantity of the current policy.
b) If the lead time is four weeks, determine the reorder point.
c) Calculate the optimal order quantity and the total minimum inventory cost.

**Solution:**

Ans: Q=800, ROP=640, TC=6,420,000

**HANDS-ON EXAMPLE 2**

Puan Hafizah is a store manager at HealthyJuice, a distributor of healthy fruit juice. Recently a supplier introduced her a brand new juice called 'VegeFruity' to improve sales. The supplier offers him RM5 per bottle. The carrying cost is estimated to be 10% of the unit price per annum, while the ordering cost is estimated to be RM20 per order. The demand for the new drink is expected to be constant at a rate of 9600 bottle per year. There are 4 days between the placement of an order and the arrival of the order. The store operates 300 days a year.

a) How many bottles should be ordered to minimize the total annual inventory cost?
b) How many orders should be placed per year?
c) What is the total annual inventory cost?
d) What is the reorder point? Explain the meaning of the value.

**Solution:**

Ans: 876, 11 orders, TC=48,437.93, ROP=128 bottles

**HANDS-ON EXAMPLE 2**

The Simple Electronic sells a printer for RM400. The annual demand is forecasted to be 1100 units. The holding cost is RM20 per unit per year, while the cost of ordering is RM90 per order. Currently, the company is ordering 12 times per year (92 units each time). The company operates 45 weeks per year (6 days per week) and lead time is 8 days.

a) What is the annual inventory cost for the current policy?
b) If the company used the optimal inventory policy, what would be the annual inventory cost?

**Solution:**

Ans: TC(current)=441,996.09, EOQ=99, TC(EOQ)=441,990

## 5.5 EOQ WITH QUANTITY DISCOUNTS

- The EOQ with quantity discounts model is applicable where a supplier offers a lower purchase cost when an item is ordered in larger quantities.
- Assumptions:
  - Demand occurs at a constant rate of $D$ items per year.
  - Ordering cost per order is constant at $C_o$ per order.
  - Holding cost is $C_h = IC_i$, where $I$ is the holding cost as a percentage of the unit cost and $C_i$ is the unit cost.
  - Purchase cost
    i. $C_1$ per item if the quantity ordered is between $0$ and $x_1$
    ii. $C_2$ per item if the quantity ordered is between $x_1$ and $x_2$ and etc.
  - Lead time, $L$ is constant.
  - Planned shortages are not permitted.

- Orders arrive instantaneously.

**Procedure for determining the optimal order quantity (EOQ with quantity discounts model)**

**Step 1**

For each discount price/cost C, calculate the EOQ using the formula:

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \qquad C_h = IC_i$$

**Step 2**

If EOQ < minimum quantity to qualify for the discount, adjust the quantity for the minimum discount.

**Step 3**

For each EOQ or adjusted EOQ, determine the total annual cost:

$$TC = \frac{D}{Q}C_o + \frac{Q}{2}C_h + DC$$

**Step 4**

Choose the quantity (EOQ or adjusted EOQ) with the lowest cost as the optimal order quantity.

Other formulas:

a) Number of orders per year: $N = \frac{D}{Q}$

b) Time between orders: $T = \frac{Q}{D}$ year

## GUIDED EXAMPLE 3

Toy4u Enterprise stocks toy robots. Recently the store was given a quantity discount schedule for the robots.

Quantity Discount Schedule

| Discount Number | Discount Quantity | Discount (%) | Unit Cost (RM) |
|---|---|---|---|
| 1 | 0 to 999 | 0 | 5.00 |
| 2 | 1000 to 1999 | 4 | 4.80 |
| 3 | 2000 and over | 5 | 4.75 |

Furthermore, the ordering cost is RM49 per order, the annual demand is 5000 robots and the inventory holding charge as a percentage of cost, *I*, is 20%. What order quantity will minimize the total inventory cost?

**Solution:**

**Step 1:** Compute EOQ for each discount price

$$EOQ_1 = \sqrt{\frac{2DC_o}{C_{h1}}} = \sqrt{\frac{2DC_o}{IC_1}} = \sqrt{\frac{(2)(5000)(49)}{(0.2)(5.00)}} = 700 \text{ robots per order}$$

$$EOQ_2 = \sqrt{\frac{2DC_o}{C_{h2}}} = \sqrt{\frac{2DC_o}{IC_2}} = \sqrt{\frac{(2)(5000)(49)}{(0.2)(4.80)}} = 714 \text{ robots per order}$$

$$EOQ_3 = \sqrt{\frac{2DC_o}{C_{h3}}} = \sqrt{\frac{2DC_o}{IC_3}} = \sqrt{\frac{(2)(5000)(49)}{(0.2)(4.75)}} = 718 \text{ robots per order}$$

**Step 2:** If EOQ < minimum discount level, adjust Q = minimum discount level
$EOQ_1$ (700) > minimum discount (0), → $Q_1 = 700$
$EOQ_2$ (714) < minimum discount (1000), → $Q_2 = 1000$
$EOQ_3$ (718) < minimum discount (2000), → $Q_3 = 2000$

**Step 3:** Compute Total inventory cost for each Q.

| Discount | Unit Cost | Q | Annual Ordering cost $\left(\frac{D}{Q}C_o\right)$ | Annual Holding Cost $\left(\frac{Q}{2}C_h\right)$ | Annual Purchase cost (DC) | Total Inventory cost |
|---|---|---|---|---|---|---|
| 1 | 5.00 | 700 | 350 | 350 | 25000 | 25700 |
| 2 | 4.80 | 1000 | 245 | 480 | 24000 | 24725 |
| 3 | 4.75 | 2000 | 122.50 | 950 | 23750 | 24822.50 |

**Step 4:** Choose the Lowest Cost Quantity
The order quantity that will minimize the total inventory cost is when $Q = 1000$ with the $TC = RM24,725$.

## GUIDED EXAMPLE 4

a) Kayangan Packaging Company uses 3000 units of boxes per year. These boxes are purchased from a supplier for RM12 each and the lead-time is 3 days. The holding cost per box per year is 10% of the unit cost and the ordering cost per order is RM16.25. There are 300 working days per year.
   i. In minimizing the total inventory cost, how many orders would be made each year?
   ii. What is the total inventory cost?

b) Upon hearing that Kayangan Packaging Company is considering producing the boxes in-house, the suppliers had notified Karim that the purchase price would drop to RM11 per box if Karim will purchase the boxes in lots of 1200. Lead times, however would increase to 4 days.
   i. What is the total cost if Karim buys the boxes in lots of 1200?
   ii. With the purchase, what effect would this have on the reorder point?

c) Given the option of a) and b), what is your recommendation to Kayangan Packaging Company?

**Solution:**

a) Given D = 3000 units C = RM12 L = 3 days
$C_o = RM16.25$ $C_h = 0.1(12) = RM1.20$

i. $$EOQ = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(3000)(16.25)}{1.2}} = 285$$

To minimize the cost, the number of orders should be $\frac{3000}{285} = 10.5$ @ $11$orders.

ii. $TC = \text{Material cost} + \text{Holding Cost} + \text{Ordering Cost}$

$$TC = 3000(12) + \frac{285}{2}(1.2) + \frac{3000}{285}(16.25) = RM36{,}342.05$$

b) $C_o = RM11$, EOQ = 1200 units, $L = 4days$, $C_h = 0.1(11) = RM1.10$

i. $$TC = 3000(11) + \frac{1200}{2}(1.10) + \frac{3000}{1200}(16.25) = RM33{,}700.63$$

ii. ROP has increased to 40 units, i.e increased by 10 units.

c) Kayangan Packaging Company should take the offer by the supplier in b).

## HANDS-ON EXAMPLE 3

Bacang Computer Sdn Bhd offers the following discount schedule for its canon printing cartridges.

| Order | Price |
|---|---|
| 999 units or less | RM25.00 |
| 1000 to 1999 units | RM24.75 |
| 2000 units or more | RM24.25 |

Swee Chee Outlet orders cartridge from Bacang Computer. The cost of ordering is RM90 per order. The holding cost is 20% of the unit cost, and the annual demand is 5000 units. Which discount offer do you recommend for Swee Chee Computer Outlet?

**Solution:**

Ans: Order 2000 units, TC=126,325

## HANDS-ON EXAMPLE 4

The demand for a component for a car is assessed to be a constant 2,000 units per month. The factory pays RM25 for each component to the vendor. The annual holding cost of the component is 15% of the unit cost. The administrative cost of placing an order is RM40 and each order is assumed to arrive instantaneously.
Assume that no shortages are allowed and there are 20 working days in each month.

a) Determine the economic order quantity.
b) Calculate the annual total inventory cost associated with the above order size.
c) The manufacturing firm is considering two more options.

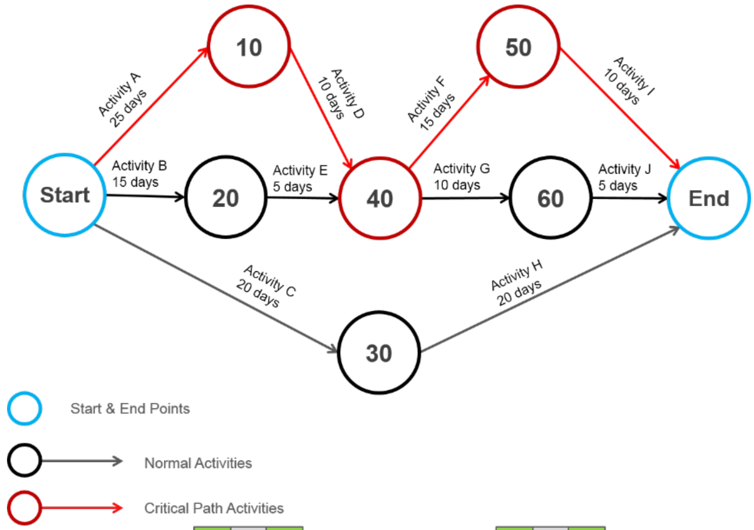
Option 2: Take advantage of a 5% discount on minimum orders of 4000 units per order offered by the current vendor.

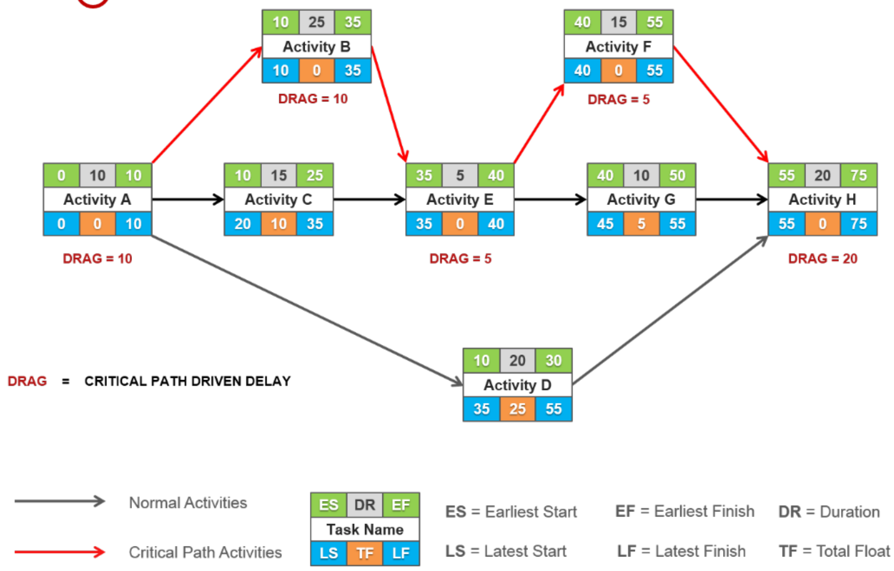
Should the company order from the vendor using the optimal ordering policy or take advantage of the quantity discount (Option 2)?

**Solution:**

Ans: EOQ=715, TC=602,683.28, Option 2: EOQ=734 adjust to 4000, TC = RM577,360, Choose Option 2

# CHAPTER 6: PROJECT MANAGEMENT(PERT/CPM)




**Learning Outcome (LO):**

At the end of this chapter, students will be able :

- To differentiate between CPM and PERT
- To build a project network and find a critical path for the network.
- To use the standard normal table to estimate the probability of project completion time
- To calculate the project crashing time and the minimum cost of the crash project.

## 6.1 Project Management

### A. Introduction

- Project Scheduling are one of the most popular uses of networks is for project analysis.
- Network show how project are organized and are used to determine time duration of the projects.
- The network techniques that are used for project analysis are:
  a) **Critical Path Method (CPM)** – A deterministic network technique that is similar to PERT but allows for project crashing.
  b) **Project Evaluation and Review Technique (PERT)** – A network technique that allows three time estimates for each activity in a project (probabilistic).

### B. By using PERT/ CPM all of the below question will be answered:

- When will the entire project be completed?
- What are the critical activities in the project?
- Which are the non – critical activities?
- How long each activity in the project can be delayed without delaying the completion of the project.
- What is the probability that the project will be completed by a specific date?

### C. Definition

| | |
|---|---|
| **Activities** | Specific jobs or tasks that are components of a project. Activities are represented by nodes in a project network. |
| **Immediate Predecessors** | The activities that must be completed immediately prior to the start of a given activity. |
| **Project Network** | A graphical representation of a project that depicts the activities and shows the relationships among the activities. |
| **Path** | A sequence of connected nodes that leads from the start node to the Finish node. |
| **Critical Path** | The longest path in a project network. |
| **Critical Activities** | The activities on the critical path. |

### D. Steps of PERT and CPM

**Define the project and all of its significant activities**

↓

**Develop the relationships among the activities**

↓

**Draw the network connecting all of the activities**

↓

**Assign time estimates to each activity**

↓

**Compute the longest time through network (critical path)**

↓

**Use network to plan, schedule, monitor and control the project**

## 6.2 Drawing the CPM/PERT Network for the project management problems

**CPM/PERT Network**

- There are 2 common techniques for drawing PERT networks:
  - **Activity-on-node (AON)** because the nodes represent the activities.
  - **Activity-on-arc (AOA)** because the arcs represent the activities.
- In this book, **we will be using AON technique**. There should be one node representing the start of the project and one node representing the finish of the project.
- There will be one node (rectangle) for each activity.
- The arcs are used to show the predecessors for the activities.

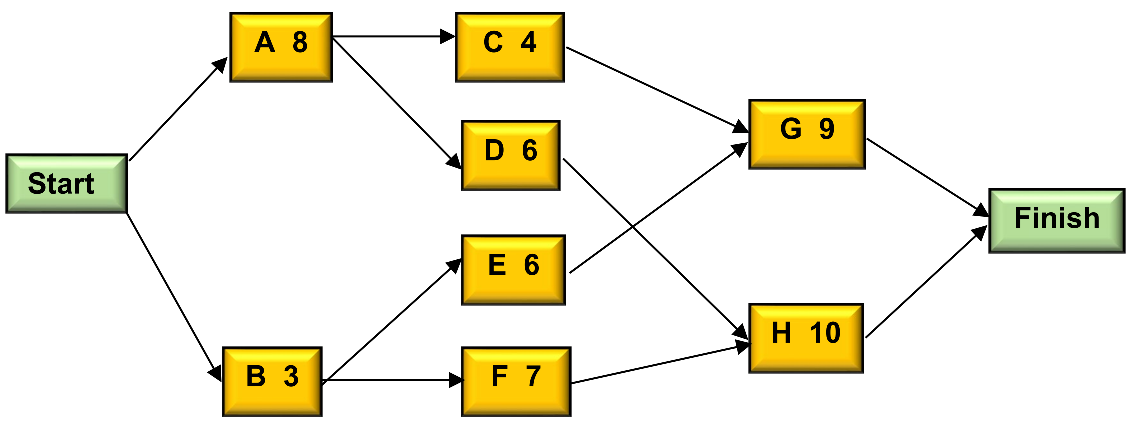
**GUIDED EXAMPLE 1**

Gemilang Production Studio is to begin producing a television advertisement. Major activities involved have been identified and data for activity times (in weeks) for the project are as follows.

| Activity | Immediate Predecessor | Time (in weeks) |
|---|---|---|
| A | - | 8 |
| B | - | 3 |
| C | A | 4 |
| D | A | 6 |
| E | B | 6 |
| F | B | 7 |
| G | C,E | 9 |
| H | D,F | 10 |

Construct the project network for the project.

**Solution:**



**HANDS-ON EXAMPLE 1**

Mega Project Sdn. Bhd. has developed a proposal to construct a compute laboratory. The following is the list of activities that must be accomplished in order to complete the project.

| Activity | Immediate Predecessor | Time(in weeks) |
|---|---|---|
| A | - | 4 |
| B | A | 6 |
| C | B | 3 |
| D | C | 7 |
| E | C | 2 |
| F | E | 8 |
| G | D | 5 |
| H | E, G | 5 |

**Solution:**

## 6.3 Activity Times (Single time estimates; Multiple time estimates)

### A. Activity Times

- In the project network all of the activity time estimates were single values.
- By using only a single activity time estimate, we are assuming that activity times are known with certainty.
- In reality, however it is rare that activity time estimates can be made with certainty.
- The next step in the PERT procedure is to assign estimates of the time require to complete each activity.
- Three time estimates for each activity:
  a) **Optimistic time estimate (a)** – The minimum activity time if everything progresses ideally.
  b) **Most likely time estimate (m)** – The most probable activity time under normal conditions.
  c) **Pessimistic time estimate (b)** – The maximum activity time if significant delays are encountered.

- These three time estimates can subsequently be used to estimate the mean and variance of a beta distribution.
- The mean and variance are computed are as follows:

$$\text{Mean (expected time), } t = \frac{a + 4m + b}{6}$$

$$\text{Variance, } v = \left(\frac{b - a}{6}\right)^2$$

- The expected project completion time ($\mu$) is the sum of the expected time of the critical path activities.
- The project variance ($\sigma^2$) is the sum of the variance of the critical path activities.

**Identifying the Critical Path (CP) and the Project Completion Time (PCT)**

- The critical path is the longest time path route through the network.
- It is the minimum time the network can be completed.
- For those activities on the critical path, ES = EF and LS = LF.
- No slack time (S = 0).
- A delay for any activity that is on the critical path will delay the completion of the entire project.

## B. Activity Scheduling

**Forward Pass**

Earliest start time (ES)

- The earliest time an activity can begin without violation of immediate predecessors' requirements.
- ES = largest EF of immediate predecessors

Earliest finish time (EF)

- The earliest time an activity can end.
- EF = ES + t

**Backward Pass**

Latest start time (LS)

- The latest time an activity can begin without delaying the entire project.
- LS = LF – t

Latest finish time (LF)

- The latest time an activity can end without delaying the entire project.
- LF = smallest LS of following activities

**Slack time**

- The amount of time that an activity can be delayed without delaying the project.
- Slack time exist for those activities not on the critical path, $ES \neq EF$ and $LS \neq LF$.
- Slack, *S*, is computed using either of the following formula:

$\boxed{S = LS - ES}$ **Or** $\boxed{S = LF - EF}$

| Activity | Time(*t*) |
|---|---|
| ES | EF |
| LS | LF |

**GUIDED EXAMPLE 2**

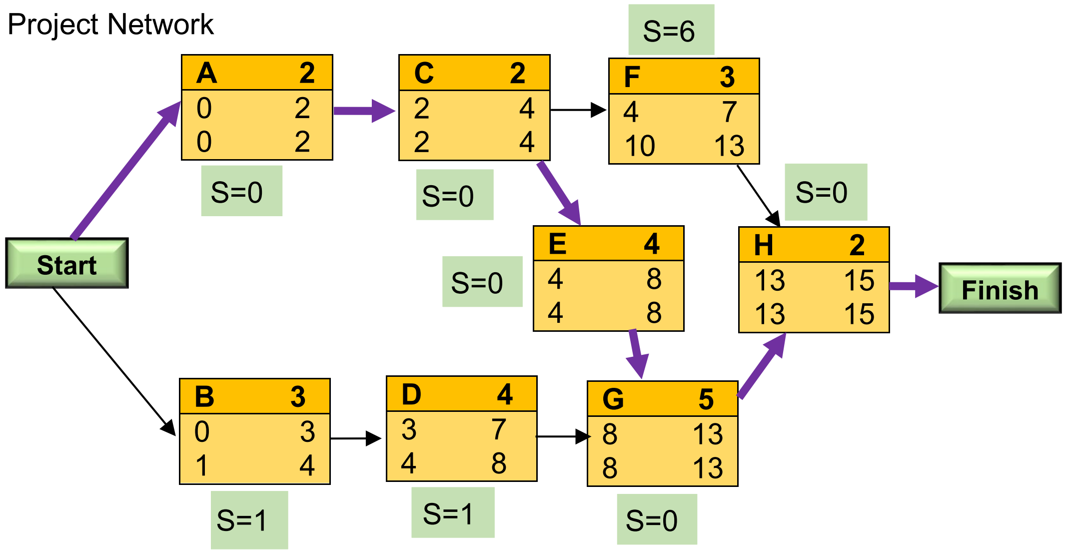
Sahara Construction Sdn Bhd has long been trying to avoid the expense of installing air pollution control equipment. The local environmental protection group has recently given the foundry 16 weeks to install a complex air filter system on its main smokestack. The company was warned that it will be forced to close unless the device is installed in the allotted period. The managing partner, wants to make sure that installation of the filtering system progresses smoothly and on time. All activities involved listed below:

| Activity | Description | Immediate Predecessors | Optimistic Time (week) | Most Likely Time (week) | Pessimistic Time (week) |
|---|---|---|---|---|---|
| A | Build internal components | - | 1 | 2 | 3 |
| B | Modify roof and floor | - | 2 | 3 | 4 |
| C | Construct collection stack | A | 1 | 2 | 3 |
| D | Pour concrete and install frame | B | 2 | 4 | 6 |
| E | Build high-temperature burner | C | 1 | 4 | 7 |
| F | Install control system | C | 1 | 2 | 9 |
| G | Install air pollution device | D, E | 3 | 4 | 11 |
| H | Inspect and test | F, G | 1 | 2 | 3 |

**Solution :**

| Activity | Immediate Predecessors | Time estimates (week) | | | $t = \frac{a+4m+b}{6}$ | $v = \left(\frac{b-a}{6}\right)^2$ |
|---|---|---|---|---|---|---|
| | | a | m | b | | |
| A | - | 1 | **2** | 3 | 2 | **1/9** |
| B | - | 2 | 3 | 4 | 3 | 1/9 |
| C | A | 1 | **2** | 3 | 2 | **1/9** |
| D | B | 2 | 4 | 6 | 4 | 4/9 |
| E | C | 1 | **4** | 7 | 4 | **1** |
| F | C | 1 | 2 | 9 | 3 | 16/9 |
| G | D,E | 3 | **4** | 11 | 5 | **16/9** |
| H | F,G | 1 | **2** | 3 | 2 | **1/9** |

Project Network




Path : ACFH : 9 week
**ACEGH : 15 week** **(critical path : since this path is the longest path)**
BDGH : 14 week

## C. Probability of Project Completion

- To determine the probability of project completion time by specific date.
- Using normal distribution, probabilities can be determined by computing the number of standard deviations ($Z$) a value is from the mean.
- The value , $Z$, is computed using the following formula:

$$Z = \frac{x - \mu}{\sigma}$$

Where, x = due date
$\mu$ = expected project completion time
$\sigma$ = project standard deviation

- This value is then used to find the corresponding probability in *Normal Table*.

**GUIDED EXAMPLE 3**

Refer to **Guided Example 2**,

a) Find the expected time to complete the project and the project variance.
b) What is the probability that the project will be finished on or before 16 weeks?
c) Find the probability that the project can be complete in:
   i. less than 14 weeks
   ii. between 14 weeks to 16 weeks
   iii. more than 17 weeks

**Solution:**

a) The project completion time ( μ ) = summation of time in critical path = 2 + 2 + 4 + 5 + 2 = 15

The project variance ( $\sigma^2$ ) = summation of variance in critical path = 28/9

b) x = 16

$$P(x < 16) = P(z < \frac{16-15}{\sqrt{\frac{28}{9}}})$$

$$P(x < 16) = P(z < 0.57)$$

$P(x < 16) = 1 - P(z > 0.57)$ since the value for > and < are the same and the standard normal table are in $P(Z \geq z)$. The z value is round off to 2 decimal places.

| z | Second decimal place of z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | $\boxed{.2843}$ | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |

$$P(x < 16) = 1 - 0.2843 = 0.7157$$

c)

i. less than 14 weeks

$$P(x < 16) = P(z < \frac{14-15}{\sqrt{\frac{28}{9}}})$$

$$P(x < 16) = P(z < -0.57)$$

$$P(x < 16) = P(z > 0.57) = 0.2843$$

ii. between 14 weeks to 16 weeks

$$P(14 < x < 16) = P(\frac{14-15}{\sqrt{\frac{28}{9}}} < z < \frac{16-15}{\sqrt{\frac{28}{9}}})$$

$$P(14 < x < 16) = P(-0.57 < z < 0.57)$$

$$P(14 < x < 16) = 1 - 2\,P(z > 0.57)$$

$$P(14 < x < 16) = 1 - 2(0.2843) = 0.4314$$

## GUIDED EXAMPLE 4

The expected completion time of a project XYZ is 36 days. If the probability that the project can be completed within 38 days is 0.7939, find the project variance.

**Solution:**

$P(z < 38) = P\left(z < \frac{38-36}{\sigma^2}\right) = 0.7939$

$P\left(z < \frac{38-36}{\sigma^2}\right) = 1 - \ 0.7939$ (since the larger value in the standard normal table is 0.5)

$P\left(z < \frac{38-36}{\sigma^2}\right) = 0.2061$ (find the closest value in the standard normal table if you cannot find the exact value : find the value with the smallest different)

| z | Second decimal place of z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | [illegible] | [illegible] | $\boxed{.2061}$ | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |

$\frac{38-36}{\sigma^2} = 0.82$

$\sigma^2 = 2.439$

$\sigma = 1.5617$

## HANDS-ON EXAMPLE 2

Baby Food Company was inspected by the Environmental and Protection Agency (EPA) and found to be in violation of a number of safety regulations. The EPA inspectors ordered the company to alter some existing machinery to make it safer, purchase some new machinery to replace older, dangerous machinery; and install water pollution control equipment. The agency gives **the factory only 30 weeks** to make the changes; if the changes were not made by then, the company would be fined RM350,000. The company determined the activities that would have to be completed and then estimated the activity times, as summarized in the following table.

| Activity | Immediate Predecessor | Time (weeks) | | |
|---|---|---|---|---|
| | | Optimistic(a) | Most likely(m) | Pessimistic(b) |
| A | - | 3 | 6 | 9 |
| B | - | 2 | 3 | 4 |
| C | - | 1 | 2 | 3 |
| D | C | 6 | 7 | 8 |
| E | B, D | 2 | 4 | 6 |
| F | A, E | 6 | 9 | 12 |
| G | A, E | 1 | 2 | 3 |
| H | F | 3 | 6 | 9 |
| I | B, D | 10 | 11 | 12 |

a) Draw the project network.
b) Find the critical path, the expected project duration, and the project standard deviation.
c) What is the probability that the company will be fined RM350,000?

**Solution:**

Ans: Critical Path : CDEFH:28, $\sigma = 1.633$, P(x>30) = 0.1112

## HANDS-ON EXAMPLE 3

Khalish Venture Sdn Bhd is planning to produce and market a new product. Activities, activity precedence relationships and activity times involve in the project are as follows:

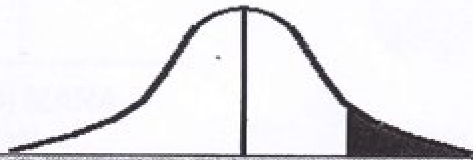
| Activity | Immediate Predecessors | Time Estimate (days) | | |
|---|---|---|---|---|
| | | Optimistic(a) | Most Likely(m) | Pessimistic(b) |
| A | - | 1 | 4 | 7 |
| B | A | 3 | 6 | 15 |
| C | - | 6 | 6 | 6 |
| D | - | 4 | 10 | 16 |
| E | B, C, D | 6 | 12 | 30 |
| F | D | 2 | 3 | 4 |
| G | D | 8 | 8 | 14 |
| H | E, F | 2 | 5 | 8 |
| I | E, F | 3 | 9 | 27 |
| J | G, I | 2 | 4 | 6 |
| K | H | 2 | 2 | 2 |

a) Construct a project network and then determine the critical path and the expected project completion time.
b) What is the maximum delay time without delaying the entire project for activity F and activity K?
c) Determine the probability that the project will be completed in 48 days.

**Solution:**

Ans: ABEIJ=40, Act F : S = 12, Act K : S=8, P(x<48) = 0.9049

## STANDARD NORMAL TABLE



**Normal Curve Areas :** ***P( Z ≥ z )***
***[Standard Normal Probability in right-hand tail]***

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Second decimal place of z | | | | | | | | | |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| 1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| 1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| 1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| 1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| 1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| 1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| 1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| 1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| 1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| 1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| 2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| 2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| 2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| 2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| 2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| 2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| 2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| 2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| 2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| 2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| 3.0 | .00135 | | | | | | | | | |
| 3.5 | .000 233 | | | | | | | | | |
| 4.0 | .000 031 7 | | | | | | | | | |
| 4.5 | .000 003 40 | | | | | | | | | |
| 5.0 | .000 000 287 | | | | | | | | | |

## 6.4 Critical Path Method (CPM)

- Project duration can be reduced by assigning more resources to project activities.
- Doing this however increases project cost.
- Decision is based on analysis of *trade-off* between time and cost.
- *Project crashing* is a method for **shortening project duration by reducing one or more critical activities to a time less than normal activity time.**
- Crashing achieved by devoting more resources to crashed activities.

**Several definitions:**

| | |
|---|---|
| **Normal time (NT)** | The average time to complete an activity under normal condition |
| **Normal cost (NC)** | An estimate on how much money it will take to complete an activity in its normal time |
| **Crash time (CT)** | The shortest possible time it will take to complete an activity if additional funds and resources were allocated |
| **Crash cost (CC)** | The price of completing the activity on a crash/deadline basis |

## 6.5 Project Crashing

**Steps of Project Crashing**

1. Find the normal critical path and identify the critical activities.

2. Compute the crash cost per week for all activities in the network using this formula:

   $$\text{Crash cost/time period} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

3. Compute the maximum time to be crash (total allowable crash time) for all activities = NT – CT

4. List all possible paths to be considered and determine the completion time for each path.

5. **Select the activity on the critical path with the smallest crash cost per unit time** and **crash this activity to the maximum extent possible** or to the point at which you desired deadline has been reached.
   - Each activity can be crash each time up to the extent that it will *remain* the normal critical path.
   - If there are multiple critical path, select the critical activity for all critical paths (with the smallest crash cost per unit time) to be crash.
   - **If there is no such kind of critical activity, select the activity from each critical path that has the smallest crash cost per unit time to be crash.**

6. Revise the network by adjusting the time and cost of the crash activity. Identify the critical path (can be more than one). If the completion time = required completion time, terminate the procedure. Otherwise return to step 4.

**GUIDED EXAMPLE 5**

The following table gives the activities and related data of a construction project to build a solar system in UiTM.

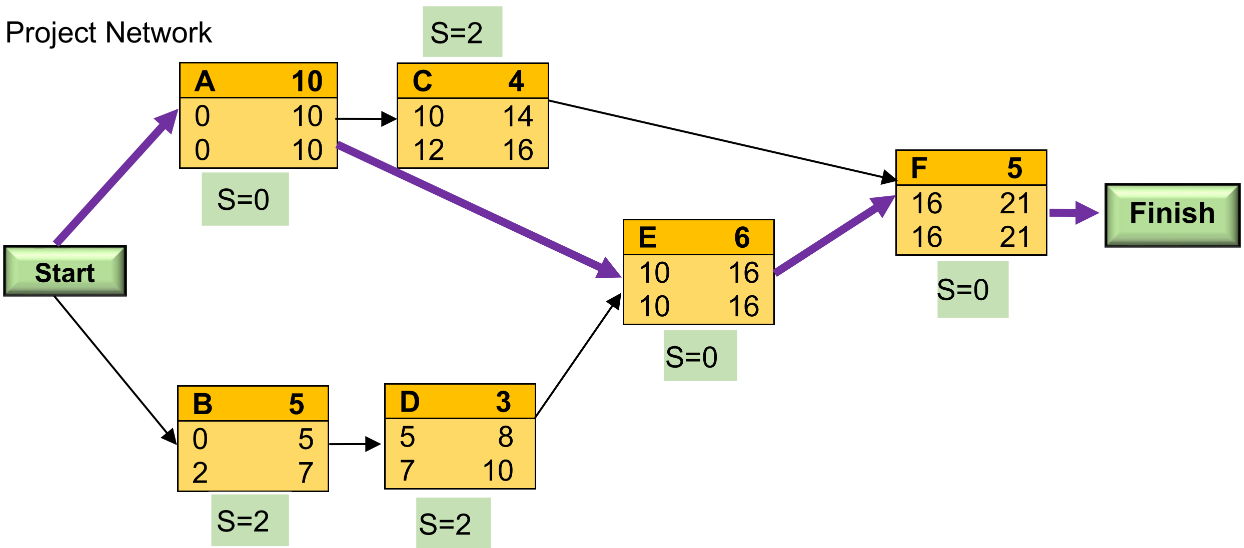
| Activity | Immediate predecessor | Time (weeks) | | Cost (RM) | |
|---|---|---|---|---|---|
| | | Normal | Crash | Normal | Crash |
| A | - | 10 | 8 | 1000 | 3400 |
| B | - | 5 | 4 | 1200 | 1400 |
| C | A | 4 | 2 | 500 | 710 |
| D | B | 3 | 2 | 600 | 920 |
| E | A,D | 6 | 3 | 500 | 800 |
| F | C,E | 5 | 4 | 1000 | 2000 |

a) Construct the project network for this problem.
b) Find the earliest start time, latest start time and slack for each activity.
c) Determine the critical path and the expected project completion time.
d) Crash the project to 18 weeks and find the minimum total cost.

**Solution:**

a)
b)

Project Network




c) Critical Path : AEF : 21 weeks

d)

| Activity | Time (weeks) | | Cost (RM) | | Crash cost – normal cost | Normal time – crash time | Crash cost /time period |
|---|---|---|---|---|---|---|---|
| | Normal | Crash | Normal | Crash | | | |
| **A** | **10** | **8** | **1000** | **3400** | **2400** | **2** | **1200** |
| B | 5 | 4 | 1200 | 1400 | 200 | 1 | 200 |
| C | 4 | 2 | 500 | 710 | 210 | 2 | 105 |
| D | 3 | 2 | 600 | 920 | 320 | 1 | 320 |
| **E** | **6** | **3** | **500** | **800** | **300** | **3** | **100** |
| **F** | **5** | **4** | **1000** | **2000** | **1000** | **1** | **1000** |
| **Total Normal Cost** | | | **4800** | | | | |

Choose E since it is has smallest amount. Crash for 3 days.

Crash Time : Project completion time – crash time project = 21 – 18 = 3 weeks

From the table above select the activity on the critical path with the smallest crash cost per unit time and crash this activity to the maximum extent possible.

Act E is choose since it has the smallest value among the critical path activity (A,E and F). Since crash time is 3 weeks and activity E can crash up to 3 weeks. Therefore we will only choose activity E.

Additional cost for activity E : RM 100 x 3 = RM 300
Total Normal Cost : RM 4800

Minimum Total cost = Total Normal Cost + Additional Cost for Crashing
= RM 4800 + RM 300
= RM 510

**HANDS-ON EXAMPLE 4**

Purewater Company is a construction and maintenance firm whose business is plumbing and piping systems. Following are activities and costs involved in a water supply pipe maintenance project. The company has 30 days to complete the project. For every one additional day the project is delayed, there will be a RM2500 penalty charge to the company.

| Activity | Immediate Predecessor | Normal Time (days) | Crash Time (days) | Normal Cost (RM) | Crash Cost (RM) |
|---|---|---|---|---|---|
| A | - | 12 | 10 | 3,000 | 5,000 |
| B | A | 8 | 6 | 2,000 | 3,500 |
| C | A | 4 | 3 | 4,000 | 7,000 |
| D | B,C | 4 | 1 | 1,000 | 2,500 |
| E | B,C | 12 | 9 | 50,000 | 71,000 |
| F | D | 4 | 1 | 1,500 | 3,000 |
| G | E,F | 4 | 3 | 15,000 | 22,000 |

a) Draw the project network.
b) Can the company complete the project on the required time? What will be the cost to complete the project under the normal time including the penalty charged (if any) to the company?
c) Crash the project to the required completion time and calculate the total cost.
d) Should the company complete the project under the normal time and pay the penalty charged for the delay or to crash the project to the required time to avoid the penalty?

**Solution:**

Ans: (a) ABEG=36, (b) No, RM 91, 500, c) Crash Act A : 2 days, Act B : 2 days and Act E : 2 days Total Cost(normal+crash)=RM 94 000, d) The company should complete project under normal time and pay the penalty

### HANDS-ON EXAMPLE 5

A student's project consists of eight activities with the following relevant information:

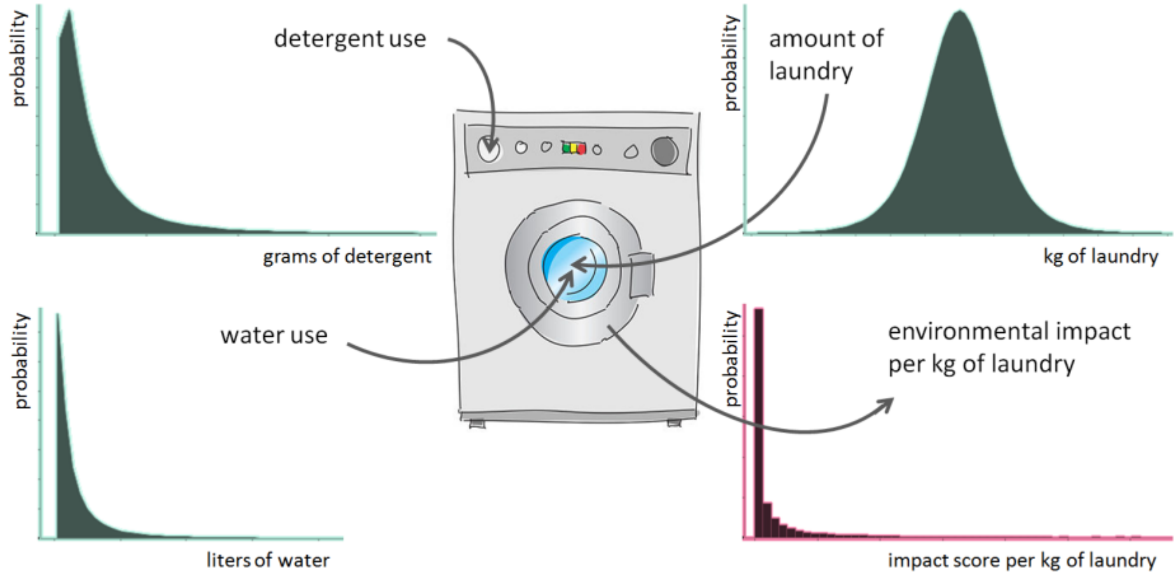
| Activity | Immediate Predecessor | Normal Time (days) | Crash Time (days) | Normal Cost (RM) | Crash Cost (RM) |
|---|---|---|---|---|---|
| A | - | 5 | 3 | 20000 | 30000 |
| B | - | 3 | 2 | 10000 | 20000 |
| C | A | 4 | 2 | 16000 | 24000 |
| D | A | 6 | 3 | 25000 | 43000 |
| E | B, D | 5 | 4 | 22000 | 30000 |
| F | B | 7 | 4 | 30000 | 48000 |
| G | C, E | 9 | 5 | 25000 | 45000 |
| H | F | 8 | 6 | 30000 | 44000 |

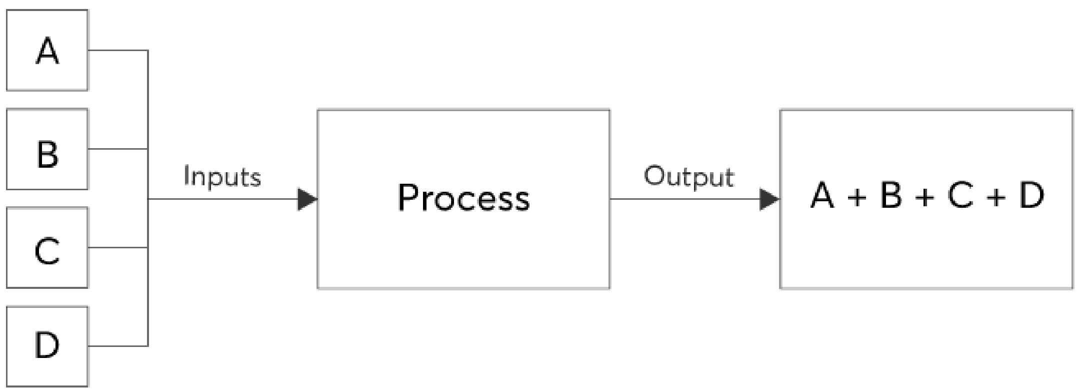
a) Construct the project network for this problem
b) Determine the critical path(s), the project completion time and the total cost of the project under normal time.
c) If the student wants to complete the project in 20 days, crash the project to meet the desired completion time at the least possible cost.
d) Calculate the total project cost to meet the 20 days completion time.

**Solution:**

Ans: b) ADEG = 25 days Total Cost = RM178,000 c) Crash Act A = 2 days and Act G = 3 days. Total Crash Cost = RM 25,000, d)Total project cost = RM 203,000

# CHAPTER 7: SIMULATION




**Learning Outcome (LO):**
At the end of this chapter, students will be able to

- Understand and able to solve simulation problem using Monte Carlo Simulation
- Solve any problem involving simulation

## 7.1 Introduction

- Simulation is one of the most widely used quantitative analysis tools.
- It is a method for learning about a real system by experimenting with a model that represents the system.
- To simulate means that to try to duplicate the features, appearance and characteristics of a real system.
- In this chapter we will discuss how to simulate a business or management system by building a mathematical model that comes as close as possible to representing the original system.
- Our mathematical model is then will be used to experiment and to estimate the effects of various action.
- The idea behind simulation is to imitate a real-world situation mathematically, then to study its properties and operating characteristics and finally to draw conclusions and make action decisions based on the results of simulation.
- In this way, the real-life system is not touched until the advantages and disadvantages of a decision change are first measured on the system's model.
- The simulation model has mathematical relationship on how to determine the output values for certain known inputs.
- A simulation model contains two types of input:
  a) Controllable input – can be controlled by the decision maker
  b) Uncontrollable input (probabilistic input) – not known and have to be generated (normally generated using random process)

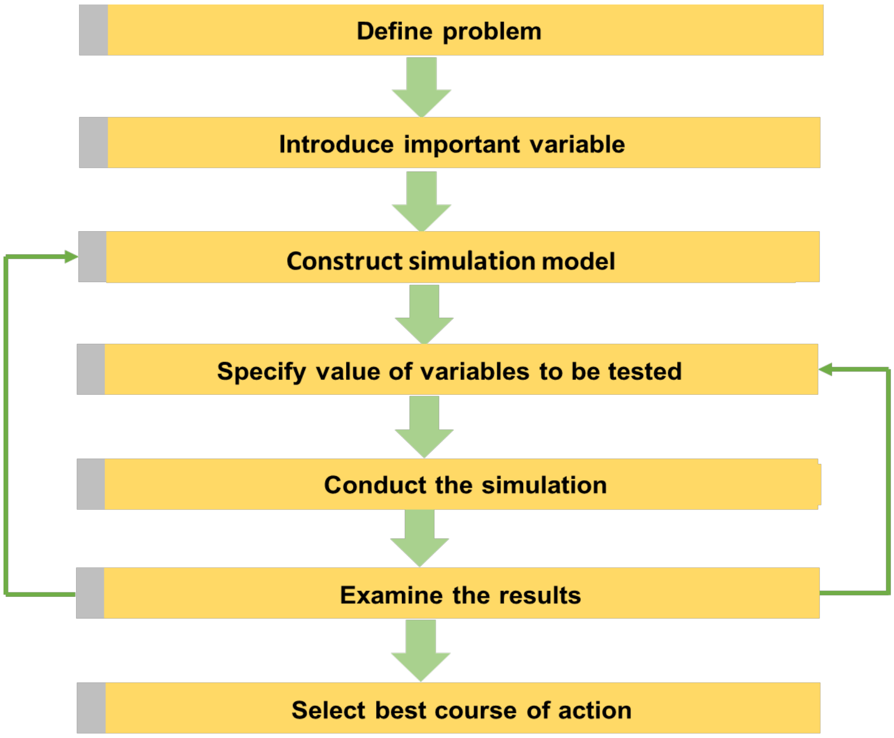
### Advantages of Simulation

1. It is relatively straightforward and flexible.
2. Software – make simulation model very easy to develop.
3. It can be used to analyze large and complex real world situations that cannot be solved by conventional quantitative analysis models.
4. Simulation allows 'what-if?' types of questions. This enables the decision maker evaluate / consider more than one alternative before choosing the best one.
5. Simulation allows us to study the interactive effect of several variables to determine which one is important.
6. The effects of changes in variables over many months / years can be obtained by computer simulation in a short time.

### Disadvantages of Simulation

1. Developing a good simulation model is often a long and complicated process especially for large problems.
2. Simulation does not give us the optimal solution. It is a trial and error approach that produces different solutions in repeated runs.
3. The user must generate all of the conditions and constraints for solutions that they want to examine.
4. The simulation must be done for many trials (100s or 1000s) to get reliable / usable results.

**Process of Simulation**




## 7.2 Monte Carlo Simulation

- When a system contains elements that exhibit chance (probability) in their behavior, the Monte Carlo method of simulation can be applied.
- The basic idea of Monte Carlo simulation is to generate values for the variables making up the model being studied.
- There are a lot of variables in real world systems that are probabilistic in nature such as:
  - Inventory demand on daily or weekly basis
  - Time between machine breakdowns
  - Time between arrivals
  - Service time
  - Time to complete project activities
  - Number of employees absent from work each day
- The basis of Monte Carlo simulation is experimentation on the chance (or probabilistic) elements through random sampling.

**Five steps in Monte Carlo Simulation**

**Step 1**

- Setting up a probability distribution for important variables.
- Probability distribution column are create by divided each demand frequency by the total demand.

**Step 2**

- Building a cumulative probability distribution for each variable in STEP 1.
- Cumulative probability is the sum of the number in the probability column added to the previous cumulative probability.

**Step 3**

- Establish interval of random numbers for each variable.
- Assign a set of numbers to represent each possible value or outcome and these are referred to as *random number interval.* Random number is a series of digits that have been selected by a totally random process.
- The interval selected to represent each possible daily demand is very closely related to the cumulative probability on its left. The interval will start with 01 and end with 00. The top end of each interval is always equal to the cumulative probability percentage.

**Step 4**

- Generating random numbers.
- For a very large and the process being studied involves thousands of simulation trials, computer programs are available to generate the random numbers needed.
- For simple process, we used this random number table:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 52 | 06 | 50 | 88 | 53 | 30 | 10 | 47 | 99 | 37 | 66 | 91 | 35 |
| 37 | 63 | 28 | 02 | 74 | 35 | 24 | 03 | 29 | 60 | 74 | 85 | 90 |
| 82 | 57 | 68 | 28 | 05 | 94 | 03 | 11 | 27 | 79 | 90 | 87 | 92 |
| 69 | 02 | 36 | 49 | 71 | 99 | 32 | 10 | 75 | 21 | 95 | 90 | 94 |
| 98 | 94 | 90 | 36 | 06 | 78 | 23 | 67 | 89 | 85 | 29 | 21 | 25 |
| 96 | 52 | 62 | 87 | 49 | 56 | 49 | 23 | 78 | 71 | 72 | 90 | 57 |
| 33 | 69 | 27 | 21 | 11 | 60 | 95 | 89 | 68 | 48 | 17 | 89 | 34 |
| 50 | 33 | 50 | 95 | 13 | 44 | 34 | 62 | 63 | 39 | 55 | 29 | 30 |
| 88 | 32 | 18 | 50 | 62 | 57 | 34 | 56 | 62 | 31 | 15 | 40 | 90 |
| 90 | 30 | 36 | 24 | 60 | 82 | 51 | 74 | 30 | 35 | 36 | 85 | 01 |
| 50 | 48 | 61 | 18 | 85 | 23 | 08 | 54 | 17 | 12 | 80 | 69 | 24 |
| 27 | 88 | 21 | 62 | 69 | 64 | 48 | 31 | 12 | 73 | 02 | 68 | 00 |
| 45 | 14 | 46 | 32 | 13 | 49 | 66 | 62 | 74 | 41 | 86 | 98 | 92 |

**Source:** Excerpted from A Million Random Digits with 100000 Normal Deviates, Rand (New York: The Free Press, 1995). Used by permission.

- We can either choose 1$^{st}$ row, 5$^{th}$ column or any other row and column as the random number.
- However, in this chapter random number will be provided in the question given.

**Step 5**

Simulating a series of trials.

## GUIDED EXAMPLE 1

Sulate Sdn Bhd sells all types of tires, but a popular radial tire accounts for a large portion of Sulate's overall sales. Recognizing that inventory costs can be quite significant with this product, the company wishes to determine a policy for managing this inventory. To see what the demand would look like over a period of time, the manager of the company wishes to simulate the daily demand for 10 days. Find the

a) expected daily demand (using the original probability distribution)
b) average daily demand (from simulation)

| Demand for Tire A | Frequency(days) |
|---|---|
| 0 | 10 |
| 1 | 20 |
| 2 | 40 |
| 3 | 60 |
| 4 | 40 |
| 5 | 30 |
| **Total** | **200** |

**Solution:**

**Step 1** **Step 2** **Step 3**

| Demand for Tire A | Probability | Cumulative Probability | Random Number Interval |
|---|---|---|---|
| 0 | 0.05 | 0.05 | 01 – 05 |
| 1 | 0.10 | 0.15 | 06 – 15 |
| 2 | 0.20 | 0.35 | 16 – 35 |
| 3 | 0.30 | 0.65 | 36 – 65 |
| 4 | 0.20 | 0.85 | 66 – 85 |
| 5 | 0.15 | 1.00 | 86 – 00 |

**Step 4** **1st column are chosen from Random Number table** **Step 5**

| Day | Random Number | Simulated Daily Demand |
|---|---|---|
| 1 | 52 | 3 |
| 2 | 37 | 3 |
| 3 | 82 | 4 |
| 4 | 69 | 4 |
| 5 | 98 | 5 |
| 6 | 96 | 5 |
| 7 | 33 | 2 |
| 8 | 50 | 3 |
| 9 | 88 | 5 |
| 10 | 90 | 5 |
| | **Total** | **39** |

**a) Expected daily demand (original probability table)**

$$E(X) = \sum xP(X = x)$$

$$= (0\times 0.05) + (1\times 0.10) + (2\times 0.20) + (3\times 0.30) + (4\times 0.20) + (5\times 0.15)$$

$$= 2.95 \text{ tires}$$

**b) Average daily demand (simulated)**

$$\frac{39}{10} = 3.9 \text{ tires}$$

→ If this simulation were repeated hundreds or thousands of times, it is much more likely that the average simulated demand would be nearly the same as the expected demand.

Simulation can be use in many application but in this book we will only show example of simulation for:

a) Inventory Control
b) Queuing Problem
c) Maintenance Policy.

**A. Simulation of Inventory Analysis**

- Simulation is useful when demand and lead time are probabilistic.

**GUIDED EXAMPLE 2**

Calvin an owner and general manager of Good Hardware. He wants to find a good, low cost inventory policy for one particular product that is the Ace model electric drill. He has decided to use simulation to help with this.

Two types of variables defined the controllable and uncontrollable inputs. The controllable inputs (decision variables) are the order quantity and the reorder point. The uncontrollable inputs are the fluctuating daily demand and the variable lead time.

Daily demand for the Ace model drill is relatively low but subject to some variability. Over the past 300 days, Simkin has observed the sales of the drill and the data is given in the following table:

| Demand for Ace drill | Frequency (days) |
|---|---|
| 0 | 15 |
| 1 | 30 |
| 2 | 60 |
| 3 | 120 |
| 4 | 45 |
| 5 | 30 |
| **Total** | **300** |

When he places an order to replenish his inventory of the drills, there is a delivery lag of one to three days.

Note that if the lead time is 1 day, the order will not arrive the next morning but rather at the beginning of the following morning. The numbers of days it took to receive the past 50 orders is given in the following table:

| Lead Time (days) | Frequency (orders) |
|---|---|
| 1 | 10 |
| 2 | 25 |
| 3 | 15 |
| **Total** | **50** |

Calvin wants to conduct a simulation analysis for an order quantity of 10 units with a reorder point of 5. That is, every time the on-hand inventory level at the end of the day is 5 or less. Assuming the beginning inventory is 10 units on day 1.

Calvin's store is open for business 200 days per year. He estimates that the cost of placing each order for Ace drills is RM10. The cost of holding a drill in stock is RM6 per drill per year, which can also be viewed as RM0.03 per drill per day (over a 200-day year). Finally, Calvin estimates that the cost of each shortage or lost sale is RM8.

Use the following random numbers:

06 63 57 94 52 69 32 30 48 88

a) Compute the probability distribution, cumulative probability and random number interval for both tables.
b) What is Calvin's daily inventory cost for the ordering policy of order quantity (Q) = 10 and reorder point (ROP) = 5?

*Tips : Total daily inventory cost = daily order cost + daily holding cost + daily stock out cost*

**Solution:**

**a)**

| Demand for Ace drill | Frequency (days) | Probability | Cum. Prob. | Rn. Nmb. Interval |
|---|---|---|---|---|
| 0 | 15 | 0.05 | 0.05 | 01 - 05 |
| 1 | 30 | 0.10 | 0.15 | 06 – 15 |
| 2 | 60 | 0.20 | 0.35 | 16 – 35 |
| 3 | 120 | 0.40 | 0.75 | 36 – 75 |
| 4 | 45 | 0.15 | 0.90 | 76 – 90 |
| 5 | 30 | 0.10 | 1.00 | 91 - 00 |
| **Total** | **300** | **1.00** | | |

| Lead time (days) | Frequency (orders) | Probability | Cum. Prob. | Rn. Nmb. Interval |
|---|---|---|---|---|
| 1 | 10 | 0.2 | 0.2 | 01 – 20 |
| 2 | 25 | 0.5 | 0.7 | 21 - 70 |
| 3 | 15 | 0.3 | 1.0 | 71 - 00 |
| **Total** | **50** | | | |

| Day | Units Received | Beginning Inventory | Rn | Demand | Ending Inventory | Lost sales | Order? (Y/N) | Rn2 | Lead Time |
|---|---|---|---|---|---|---|---|---|---|
| 1 | … | 10 | 06 | 1 | 9 | 0 | No | | |
| 2 | 0 | 9 | 63 | 3 | 6 | 0 | No | | |
| 3 | 0 | 6 | 57 | 3 | 3 | 0 | Yes | 02 | 1 |
| 4 | 0 | 3 | 94 | 5 | 0 | 2 | No | | |
| 5 | 10 | 10 | 52 | 3 | 7 | 0 | No | | |
| 6 | 0 | 7 | 69 | 3 | 4 | 0 | Yes | 33 | 2 |
| 7 | 0 | 4 | 32 | 2 | 2 | 0 | No | | |
| 8 | 0 | 2 | 30 | 2 | 0 | 0 | No | | |
| 9 | 10 | 10 | 48 | 3 | 7 | 0 | No | | |
| 10 | 0 | 7 | 88 | 4 | 3 | 0 | Yes | 14 | 1 |

**Notes:**

- The beginning inventory is 10 units. No units received on the 1st day. Random number is 06 with demand 1 unit (see table demand).
- Ending inventory = beginning inventory – demand
- Beginning inventory for the next day will take the value from ending inventory of previous day.
- At day 3, the ending inventory balance 3 units and it less than 5 units. So an order is placed.
- The random number 02 is generated to represent the first lead time and was drawn from table lead time. Therefore the lead time is 1 day.
- At day 4, no order received since the lead time is 1 day which means that the order will arrive at the beginning of the following working day. Thus it will arrive at day 5.
- At day 4, if on-hand inventory is insufficient to meet the day's demand, satisfy as much as possible and note the number of lost sales in lost sales column.

**b) Analyzing Calvin's Inventory Costs**

The average daily ending inventory (daily demand)

Average ending inventory = $\frac{41}{10} = 4.1$ units per day

The average lost sales and number of orders placed per day

Average lost sales = $\frac{2}{10} = 0.2$ units per day

Average number of orders placed = $\frac{3}{10} = 0.3$ order per day

Daily order cost = (cost of placing one order) x (number of orders placed per day)
= RM10 per order x 0.3 order per day
= RM3

Daily holding cost = (cost of holding one unit for one day) x (average ending inventory)
= RM0.03 per unit per day x 4.1 units per day
= RM0.12

Daily stock out cost = (cost per lost sale) x (average number of lost sales per day)
= RM8 per lost sale x 0.2 lost sales per day
= RM1.60

Total daily inventory cost = daily order cost + daily holding cost + daily stock out cost
= RM4.72

Annually (200 day working year) = RM944.

**B. Simulation of Queuing Problem**

- An important area of simulation application has been in the analysis of waiting line problems.

**GUIDED EXAMPLE 3**

Fully loaded barges arrive at night in New Orleans following their long trips down the Mississippi River from industrial Midwestern cities. The number of barges docking on any given night ranges from 0 to 5. The probability of 0, 1, 2, 3, 4 or 5 arrivals is as given in the following table:

| Number of arrivals | Probability |
|---|---|
| 0 | 0.13 |
| 1 | 0.17 |
| 2 | 0.15 |
| 3 | 0.25 |
| 4 | 0.20 |
| 5 | 0.10 |

The barges will be unloaded on a first come first serve basis and the unloading ranges from 1 to 5 as given.

| Daily unloading rate | Probability |
|---|---|
| 1 | 0.05 |
| 2 | 0.15 |
| 3 | 0.50 |
| 4 | 0.20 |
| 5 | 0.10 |

Find cumulative probability and random number interval for both tables.

| Number of arrivals | Probability | Cum. Prob. | Rn. No. Int |
|---|---|---|---|
| 0 | 0.13 | 0.13 | 01 – 13 |
| 1 | 0.17 | 0.30 | 14 – 30 |
| 2 | 0.15 | 0.45 | 31 – 45 |
| 3 | 0.25 | 0.70 | 46 – 70 |
| 4 | 0.20 | 0.90 | 71 – 90 |
| 5 | 0.10 | 1.00 | 91 – 00 |

| Daily unloading rate | Probability | Cum. Prob. | Rn. No. Int |
|---|---|---|---|
| 1<br>2<br>3<br>4<br>5 | 0.05<br>0.15<br>0.50<br>0.20<br>0.10 | 0.05<br>0.20<br>0.70<br>0.90<br>1.00 | 01 – 05<br>06 – 20<br>21 – 70<br>71 – 90<br>91 – 00 |

The barges that cannot be unloaded on the night of arrival will be unloaded on the following day. This involves extra cost of lost time.

The officer in charge of this process wants to simulate this system for 15 days.

**Random Numbers:**

Number of nightly arrivals: 52 06 50 88 53 30 10 47 99 37 66 91 35 32 00

Number unloaded: 37 63 28 02 74 35 24 03 29 60 74 85 90 73 59

**Solution :**

| Day | Num. delayed From previous day | Rn1 | Number of Nightly arrivals | Total to be unloaded | Rn2 | Daily unloading rate (no. unloaded) |
|---|---|---|---|---|---|---|
| 1 | - | 52 | 3 | 3 | 37 | 3 |
| 2 | 0 | 06 | 0 | 0 | 63 | 0 |
| 3 | 0 | 50 | 3 | 3 | 28 | 3 |
| 4 | 0 | 88 | 4 | 4 | 02 | 1 |
| 5 | 3 | 53 | 3 | 6 | 74 | 4 |
| 6 | 2 | 30 | 1 | 3 | 35 | 3 |
| 7 | 0 | 10 | 0 | 0 | 24 | 0 |
| 8 | 0 | 47 | 3 | 3 | 03 | 1 |
| 9 | 2 | 99 | 5 | 7 | 29 | 3 |
| 10 | 4 | 37 | 2 | 6 | 60 | 3 |
| 11 | 3 | 66 | 3 | 6 | 74 | 4 |
| 12 | 2 | 91 | 5 | 7 | 85 | 4 |
| 13 | 3 | 35 | 2 | 5 | 90 | 4 |
| 14 | 1 | 32 | 2 | 3 | 73 | 3 |
| 15 | 0 | 00 | 5 | 5 | 59 | 3 |
| | **Total delay: 20** | | **Total arrival: 41** | | | **Total unloading: 39** |

Notes :

- We begin with no delays from the previous day. The total to be unloaded is a summing of number delayed from previous day with number of nightly arrivals.
- Three barges could have been unloaded on day 2. But because there were no arrivals and no backlog existed, zero unloading took place.
- At day 14, 4 barges could have been unloaded, but since only 3 were in the queue, the number unloaded is recorded as 3.

**Analysis:**

**Average no. of barges delayed to the next day** = $\frac{20}{15} = 1.33$ barges delayed per day

**Average no. of nightly arrivals** = $\frac{41}{15} = 2.73$ arrivals

**Average no. of barges unloaded each day** = $\frac{39}{15} = 2.60$ unloadings

## C. Simulation Model for a Maintenance Policy

Simulation is a valuable technique for analyzing various maintenance policies before actually implementing them. A firm can decide whether to add more maintenance staff based on machine downtime costs and costs of additional labor. It can be simulate replacing parts that have not yet failed in exploring ways to prevent future breakdowns. Many companies used computerized simulation model to decide if and when to shutdown an entire plant for maintenance activities.

**GUIDED EXAMPLE 4**

Three Hills Power Company (THPC) provides electricity to a large metropolitan area through a series of almost 200 hydroelectric generators. Management recognizes that even well maintained generators will have periodic failures or breakdowns. The probability distribution for the time between generator breakdowns ($T_1$) is given in the table below:

| **$T_1$ (hours)** | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|
| **$P(T_1)$** | 0.05 | 0.06 | 0.16 | 0.33 | 0.21 | 0.19 |

Currently, it employs four highly skilled and highly paid (RM30 per hour) repairpersons. Each works every fourth 8-hour shift. In this way, there is a repairperson on duty 24 hours a day, seven days a week (one repairperson is on duty per shift to repair generators).

As expensive as the maintenance staff salaries are, breakdown expenses are even more costly. For each hour that one of its generators is down, THPC loses approximately RM75. This amount is the charge for reserve power that THPC must borrow from the neighbouring utility company.

The probability distribution for the time required to repair a generator (one repairperson is on duty per shift) is given below:

| **$T_2$ (hours)** | 1 | 2 | 3 |
|---|---|---|---|
| **$P(T_2)$** | 0.28 | 0.52 | 0.20 |

Stephanie Robbins is the THPC management analyst assigned to simulate maintenance costs.

Simulate this maintenance system over **15-generator breakdown** period and calculate the total maintenance cost (service maintenance cost + machine breakdown) incurred. Assume the first shift start on midnight 00.

Random numbers for time between breakdowns:

57 17 36 72 85 31 44 30 26 09 49 13 33 89 13

Random numbers for generator repair times:

07 60 77 49 76 95 51 16 14 85 59 85 40 42 52

Simulate time between generator breakdowns

| Time between the breakdowns | Probability | Cum. Prob. | Rn. No. Int |
|---|---|---|---|
| 0.5 | 0.05 | 0.05 | 01 – 05 |
| 1.0 | 0.06 | 0.11 | 06 – 11 |
| 1.5 | 0.16 | 0.27 | 12 – 27 |
| 2.0 | 0.33 | 0.60 | 28 – 60 |
| 2.5 | 0.21 | 0.81 | 61 – 81 |
| 3.0 | 0.19 | 1.00 | 82 – 00 |

Simulate generator repair times:

| Repair time Required | Probability | Cum. Prob. | Rn. No. Int |
|---|---|---|---|
| 1.0 | 0.28 | 0.28 | 01 – 28 |
| 2.0 | 0.52 | 0.80 | 29 – 80 |
| 3.0 | 0.20 | 1.00 | 81 – 00 |

**Solution**

| Breakdow n Number | Random No for Breakdowns | Time between the breakdowns | Time of breakdown | Time repairperson is free to begin this repair | Random no for repair time | Repair time required | Time repair ends | No of hour machine down |
|---|---|---|---|---|---|---|---|---|
| 1 | 57 | 2.0 | 02:00 | 02:00 | 07 | 1 | 03:00 | 1 |
| 2 | 17 | 1.5 | 03:30 | 03:30 | 60 | 2 | 05:30 | 2 |
| 3 | 36 | 2.0 | 05:30 | 05:30 | 77 | 2 | 07:30 | 2 |
| 4 | 72 | 2.5 | 08:00 | 8:00 | 49 | 2 | 10:00 | 2 |
| 5 | 85 | 3 | 11:00 | 11:00 | 76 | 2 | 13:00 | 2 |
| 6 | 31 | 2 | 13:00 | 13:00 | 95 | 3 | 16:00 | 3 |
| 7 | 44 | 2 | 15:00 | 16:00 | 51 | 2 | 18:00 | 3 |
| 8 | 30 | 2 | 17:00 | 18:00 | 16 | 1 | 19:00 | 2 |
| 9 | 26 | 1.5 | 18:30 | 19:00 | 14 | 1 | 20:00 | 1.5 |
| 10 | 09 | 1 | 19:30 | 20:00 | 85 | 3 | 23:00 | 3.5 |
| 11 | 49 | 2 | 21:30 | 23:00 | 59 | 2 | 01:00 | 3.5 |
| 12 | 13 | 1.5 | 23:00 | 01:00 | 85 | 3 | 04:00 | 5 |
| 13 | 33 | 2 | 01:00 | 04:00 | 40 | 2 | 06:00 | 5 |
| 14 | 89 | 3 | 04:00 | 06:00 | 42 | 2 | 08:00 | 4 |
| 15 | 13 | 1.5 | 05:30 | 08:00 | 52 | 2 | 10:00 | 4.5 |
| | | | | | | | **Total** | **44** |

**Cost Analysis of the Simulation:**

Service maintenance cost = 34 hours of worker service time x RM30 per hour
= RM1,020

Simulated Machine breakdown cost = 44 hours of breakdown x RM75 lost per hour of downtime
= RM3,300

Total maintenance cost = Service maintenance cost + Simulated Machine breakdown cost
= RM1,020 + RM3,300
= RM4,320

**HANDS-ON EXAMPLE 1**

Chan Electric runs a store for electric and electronics item. One of their hot sales are Charp rice cooker. Hana, the manager of store have collect data concerning the demand for the number of this rice cooker over the weeks. She estimates that the cost of placing an order is RM40, holding cost is RM2 per week and the stockout cost is RM10 per occurrence.

| Weekly Demand | Probability |
|---|---|
| 0 | 0.14 |
| 1 | 0.16 |
| 2 | 0.20 |
| 3 | 0.15 |
| 4 | 0.25 |
| 5 | 0.10 |

Chan Electric has a policy that is to order 9 rice cooker whenever its inventory reached 5 rice cooker or less at the end of the week. Currently the store has 7 rice cooker. The lead time for an order is given in the following table:

| Lead Time (weeks) | Probability |
|---|---|
| 1 | 0.40 |
| 2 | 0.50 |
| 3 | 0.10 |

a) Conduct a 12-week simulation. Use the following random numbers:

**Demand:** 37 97 09 42 50 81 33 00 16 77 61 10

**Lead Time:** 39 66 22 97

b) Compute the average ending inventory, average number of orders placed, and average inventory cost.

**Solution:**

Ans: b) Average Ending Inventory : 4.3
Average No of orders : 0.3
Average No of stockout : 0.08
Total Inventory Cost : RM 21.40

| Week | Unit Received | Beginning Inventory | RN | Demand | Ending Inventory | Lost Sales | Order? | RN | Lead Time |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |
| 6 | | | | | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |
| 10 | | | | | | | | | |
| 11 | | | | | | | | | |
| 12 | | | | | | | | | |

## HANDS-ON EXAMPLE 2

Kutan Medical Centre (KMC) currently has two doctors available to treat the patients on Saturday. KMC opens at time zero (t=0) and patient begins to arrive at some time later. The time between arrival (minutes), service time (minutes) and attention needed by a patient who comes to KMC are defined according to the following probability distributions:

| Time Between Arrival (minutes) | Probability |
|---|---|
| 5 | 0.16 |
| 10 | 0.23 |
| 15 | 0.29 |
| 20 | 0.32 |

| Patients to see Doctor | Probability |
|---|---|
| Doctor 1 | 0.40 |
| Doctor 2 | 0.60 |

| Doctor 1 | |
|---|---|
| Service Time (minutes) | Probability |
| 5 | 0.08 |
| 10 | 0.24 |
| 15 | 0.51 |
| 20 | 0.17 |

| Doctor 2 | |
|---|---|
| Service Time (minutes) | Probability |
| 10 | 0.22 |
| 15 | 0.31 |
| 20 | 0.25 |
| 25 | 0.22 |

The random numbers to be used are as follows:

Time between Arrival : 19 65 51 17 63 85 37
Patients to see doctor : 39 73 72 75 37 02 87
Service Time for doctor : 58 47 23 69 35 21 41

Assume that when the patients arrive for service, they will proceed to the available doctor. If both doctors are busy then they will form a queue and are served in order of arrival.

a) Simulate the arrival of 7 patients to the KMC using random number above

b) Compute the average time per patient and average time in queue

c) Based on the simulation, does it appear that this system provides adequate patient care? Give reason.

**Solution:**

Ans: b) Average time per patient = 14.3 min
Average time in queue = 0 min
c) Adequate, since average time in queue is 0.

| Patient | RN | Inter arrival time | Time Arrive | RN | Meet Doctor? | RN | Time Begin service | Service Time | Time Finish Service | Waiting time |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |

## HANDS-ON EXAMPLE 3

Bulan Sabit Merah Rescue Squad has gained reputation for many years for its fast and quality service. Data have been collected from previous experience. The following are probability distribution regarding receipt of number of emergency calls at night :

| Number of calls | Probability |
|---|---|
| 0 | 0.06 |
| 1 | 0.15 |
| 2 | 0.18 |
| 3 | 0.30 |
| 4 | 0.24 |
| 5 | 0.07 |

Bulan Sabit Merah categorizes each cell as either Minor, Normal or Major and has found the following distribution for these categories :

| Category | Probability |
|---|---|
| Minor | 0.28 |
| Normal | 0.62 |
| Major | 0.10 |

Bulan Sabit Merah sends a team comprising 3, 5 or 7 persons for Minor, Normal or Major calls respectively. Use random numbers given below :

Number of Calls : 47 31 05 76 18 59 35 16 72 60

Type of Calls : 08 56 37 71 92 74 17 13 50 41 27 55
93 10 32 72 99 65 33 07 42 88 22

a) Simulate the emergency call received by Bulan Sabit Merah for 10 nights
b) Determine the average number of calls of each category per night.
c) Determine the average number of call received per night
d) Determine the average crew size required per night.

**Solution:**

Ans:
b) Average Number of call for Minor = 0.7
Average Number of call for Normal = 1.3
Average Number of call for Minor = 0.3
c) Average Average Number of call for Minor = 2.3
d) Average Total crew size = 10.7

| Night | RN | No of Calls | RN | Category |
|---|---|---|---|---|
| | | | | |
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## HANDS-ON EXAMPLE 4

Senai International Airport primarily serves domestic air traffic. However, chartered planes from abroad may occasionally arrive with passengers bound for different Island located around Johor. There is only one immigration and one customs officers available at any time. Whenever an international plane arrives at the airport, the immigration and customs officers on duty will set up operations to process the passengers.

Incoming passengers must have their passports and visas checked by the immigration officer. The time required to check a passenger's passport and visa can be described by the following probability distribution :

| Time required to check a passenger's Passport & Visa (seconds) | Probability |
|---|---|
| 20 | 0.20 |
| 40 | 0.20 |
| 60 | 0.30 |
| 80 | 0.10 |

After having their passports and visa checked, the passengers proceed to the customs officer who will inspect their baggage. Passenger from a single waiting line and baggage inspection has the following probability distribution:

| Time required for baggage inspection (minutes) | Probability |
|---|---|
| 0 | 0.25 |
| 1 | 0.60 |
| 2 | 0.10 |
| 3 | 0.05 |

a) Suppose a chartered plane from abroad with 100 passengers land at Senai Airport. Simulate the immigration and customs clearance process for the first 10 passengers and determine how long it will take them to clear the process.

Use the following random numbers for the passport control:
93 63 26 16 21 26 70 55 72 89

And random numbers for baggage inspection:
13 08 60 13 68 40 40 27 23 64

b) what is the average length of time a customer has to wait before having his baggage inspected after clearing passport control?

**Solution:**

Ans: b) Average Waiting Time : 16 seconds

| Passenger | Passport and Visa Control | | | | Baggage Inspection | | | | Wait |
|---|---|---|---|---|---|---|---|---|---|
| | Begin time | RN | Time | End Time | Begin Time | RN | Time | End Time | |
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |
| 6 | | | | | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |
| 10 | | | | | | | | | |

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