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EFFECT OF PARAMETERS ON THE COST OF MEMORY TYPE CHART

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A memory type control chart is a scheme that utilizes previous information for constructing the chart. An exponentially weighted moving average (EWMA) control chart is an example of a memory type chart. The EWMA control chart is well-known and widely employed by practitioners for monitoring small and moderate process mean shifts. In light of this, the cost of implementing an EWMA control chart needs to be determined. This study aims to investigate the effect of input parameters on the cost of implementing an EWMA control chart. Here, the economic model was used to compute the cost parameter. The 14 input parameters were identified and the analysis was carried out based on the one-parameter-at-a-time basis. When the input parameters change based on a predetermined percentage, the cost is also affected. According to the results, 10 input parameters with major effect and 4 input parameters with almost no effect on the cost parameter. However, the effect of each input parameter on the cost is different. This analysis is crucial to observe and determine the input parameters that have a significant impact on the cost of the EMWA control chart. Hence, practitioners can obtain an overview of the influence of the input parameters on the cost of implementing the EWMA control chart.

***Keywords*:** Cost, Economic model, Sensitivity analysis

1. Introduction

Statistical Process Control (SPC) is a collection of analytical decision making tools that are effective to achieve process reliability and enhance capability by reducing variability. A control chart is among the most efficient tools in SPC (Montgomery, 2019). The first control chart was pioneered by Dr. Walter A. Shewhart to monitor and determine whether a process is in a desired state for producing high quality goods (Abtew et al., 2018).

Since then, the control chart remains a valuable tool in SPC. This is because it can continuously monitor a process and ultimately improve its capability. Meanwhile, it is simple to use, yet achieves a significant impact on the enhancement of quality (You et al., 2020). The awareness and knowledge of using control charting techniques to monitor a process and deliver high-quality production has grown exponentially (Şengöz, 2018). Hence, control charts are used extensively in the manufacturing and service sectors.

The Shewhart chart has been known for its ease of construction and implementation. Nevertheless, it is insensitive towards small and moderate process shifts. Consequently, extensive research has been conducted to develop new control charts to improve the sensitivity of the Shewhart chart, such as the memory type control chart (Abbas, 2018). A memory type control chart is constructed based on past and current information. Here, an example of the memory type control chart is the exponentially weighted moving average (EWMA) chart.

The assessment of a control chart is necessary to reveal its effectiveness. This is vital, as it influences the decision to use the control chart. In practice, the performance of a control chart can be evaluated (Chong et al., 2019). The common criteria to measure the performance of a control chart are average run length (ARL) and standard deviation of the run length (SDRL). Nonetheless, the expected cost to implement the control chart cannot be identified. The economic designs of the memory type control chart have been studied by Hariba and Tukaram (2016) and Serel (2009), to name a few. The effect of input parameters on the implementation cost of the EWMA chart is not yet available in the literature.

In view of this, it is essential to investigate the impact of the input parameters on the expected cost of implementing the EWMA control chart. This work presents a brief description of the EWMA control chart. The economic model of the EWMA chart is also elaborated. Next, the sensitivity analysis of the EWMA control chart is conducted to identify the input parameters that influence the cost of the EWMA chart.

2. The EWMA Chart

The EWMA chart was developed by Roberts (1959), and is known as a memory type chart because it takes into account both previous and current data in establishing the control structure. In light of this, the EWMA chart is suitable to be used to detect small process mean shifts (Ramesh and Vasu, 2019).

The statistic of the EWMA chart at sampling period, *i*, is computed as follows:

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}, \tag{1}$$

where λ is a smoothing constant and $0 < \lambda \le 1$. When a z_i falls beyond the control limit, an out-of-control situation is said to have occurred. Note that the EWMA chart is converted to a Shewhart chart when λ becomes 1 (Patel and Divecha, 2011).

3. The Economic Model of the EWMA Chart

The economic model of the EWMA chart, obtains optimal charting parameters that minimizes cost (Chung, 1991). Here, the economic model of the EWMA chart involves finding the expected cost of the EWMA chart. The implementing cost of the EWMA control chart is calculated based on the cost function.

When an assignable cause occurs, the process is said to become out-of-control, the process mean becomes $\mu_0 + \delta\sigma$. It is assumed that a process initiates in a state of in-control, and the time for an assignable cause to occur is exponentially distributed with mean $1/\theta$. The expected cost per hour is obtained from dividing the expected cost-per-cycle by the expected cycle length. The cycle is composed of an in-control phase, followed by an out-of-control phase.

The expected cost per unit time in hours is denoted as *C*, and can be defined as follows:

$$C = \frac{\dfrac{C_0}{\theta} + C_1 B + \dfrac{b+cn}{h}\left(\dfrac{1}{\theta} + B\right) + \dfrac{sY}{\mathrm{ARL}_0} + W}{\dfrac{1}{\theta} + \dfrac{(1-\varphi_1)sT_0}{\mathrm{ARL}_0} + EH}, \tag{2}$$

where

$$B = (\mathrm{ARL}_1 - 0.5)h + F,$$
$$F = ne + \gamma_1 T_1 + \gamma_2 T_2,$$
$$s = \frac{1}{\lambda h} - 0.5,$$
$$EH = (\mathrm{ARL}_1 - 0.5)h + G,$$

and

$$G = ne + T_1 + T_2.$$

The notations in **(2)** are defined as follows:

C_0	Expected quality cost per unit time, while the process is in-control
C_1	Expected quality cost per unit time, while the process is out-of-control
θ	Process failure rate
b	Fixed cost per sample
c	Cost per unit sampled
n	Sample size
e	Expected time to sample and interpret one unit
h	Sampling interval
s	Expected number of samples taken before an assignable cause occurs
Y	Cost of false alarm
ARL_0	Average run length when the process is in-control
ARL_1	Average run length when the process is out-of-control
W	Cost of finding and fixing an assignable cause
γ_1	= 1 if production continues during search = 0 if production stops during search
γ_2	= 1 if production continues during repair = 0 if production stops during repair
T_0	Expected time to search for a false alarm
T_1	Expected time to find the assignable cause
T_2	Expected time to repair the process

4. The Sensitivity Analysis of the EWMA Chart

Sensitivity analysis is crucial to identify the input parameter that has an impact on the cost of implementing the EWMA chart. Meanwhile, sensitivity analysis of input parameters of the EWMA control chart helps to determine the input parameters that are critical. Here, 14 input parameters $\left(\theta,\delta,C_0,C_1,Y,W,b,c,e,T_0,T_1,T_2,\gamma_1,\gamma_2\right)$ have been identified. Sensitivity analysis can be performed on a one factor at a time basis, where one of the input parameters is modified each time, while the other 13 input parameters remain unchanged. For example, the value for θ is 0.01, 0.02 and 0.04 each time, with the remainder of the input parameters remaining unchanged. In view of this, 40 different input parameter combinations $\left(\theta,\delta,C_0,C_1,Y,W,b,c,e,T_0,T_1,T_2,\gamma_1,\gamma_2\right)$ were taken into account and are displayed in Table 1.

Table 2 demonstrates the corresponding cost values for the EWMA control chart. Here, the input parameters are $\theta = 0.02$, $\delta = 0.86$, $C_0 = 114.24$, $C_1 = 949.2$, $Y = 977.4$, $W = 977.4$, $b = 0$, $c = 4.22$, $e = 0.083$, $T_0 = 0.083$, $T_1 = 0.083$, $T_2 = 0.75$, $\gamma_1 = 1$, $\gamma_2 = 0$ and the corresponding optimal cost for the EWMA chart is 222.37. According to Table 2, the implementation cost of the EWMA control chart increases when the expected quality cost per unit time while the process is in-control, C_0, increased, and vice versa. For instance, an increase of C_0 from 114.24 to 228.48 as in number 9 in Table 1 results in an increase in the expected cost from 222.37 to 327.97. The same situation occurs to the expected quality cost per unit time while the process is out-of-control, C_1, and cost per unit sampled, c. Nevertheless, C_0 has a greater effect on expected cost than C_1. This can be seen when C_0 increased, expected cost increased by 47.5%, while when C_1 increased, the expected cost increased by 21%. Meanwhile, the input parameters Y and W have a minor effect on the expected cost. It can be noticed that the increase in the value of Y and W increased the expected cost by 3% and 8.1%, respectively. When fixed cost per sample, b, increased to 5 and 10, it was found that the expected cost for both cases increased by 7.1% and 12.4%, respectively.

According to Table 2, the expected cost remains the same, regardless of the increase or decrease of the input parameter expected time to search a false alarm, T_0. Nonetheless, the expected time to sample and interpret one unit, e, and the expected time to determine the assignable cause, T_1, have a small impact on the expected cost. It was observed that when the e and T_1 increase to 0.166, the expected cost increases by 1.0% and 0.5%, respectively. In contrast, the expected cost decreases by 1.4% when the expected time to repair the process, T_2, increases from 0.75 to 1.50. The process failure rate, θ, can be seen to have a significant effect on the optimal cost. This can be observed when θ decreased by 50% (i.e. from 0.02 to 0.01), the cost decreases from 222.37 to 187.91. However, when θ rises to 0.04, the cost rises to 273.66. The increase in cost occurs due to an increase in the process failure rate.

When the process shift, δ, increases by 50% from 0.86 to 1.72, the cost is reduced by 16.6%. As the size of process shift decreases, the cost increased by 25.3% from 222.37 to 278.56. A large process shift is easier to be detected than small process shift, hence, the cost is lower in the large process shift compared to the small process shift. The production status during search, γ_1, and production status during repair, γ_2, show that when production stops during search and repair, the optimal cost decreased from 222.37 to 220.67. The optimal cost increases by 6.1% when the production continues during search and repair. The same phenomenon occurs when the production stops during search and production continues during repair, i.e. increased by 5.4%. As discussed, 13 out of 14 input parameters have an effect on the output parameter, i.e. cost. However, the size of the effect of each input parameter on the cost is different.

Table 1: Different combinations of the input parameters.

No.	θ	δ	C_0($)	C_1($)	Y($)	W($)	b($)	c($)	e	T_0	T_1	T_2	γ_1	γ_2
1	**0.01**	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
2	**0.02**	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
3	**0.04**	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
4	0.02	**0.43**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
5	0.02	**0.86**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
6	0.02	**1.72**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
7	0.02	0.86	**57.12**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
8	0.02	0.86	**114.24**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
9	0.02	0.86	**228.48**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
10	0.02	0.86	114.24	**474.6**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
11	0.02	0.86	114.24	**949.2**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
12	0.02	0.86	114.24	**1898.4**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
13	0.02	0.86	114.24	949.2	**488.7**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
14	0.02	0.86	114.24	949.2	**977.4**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
15	0.02	0.86	114.24	949.2	**1954.8**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
16	0.02	0.86	114.24	949.2	977.4	**488.7**	0	4.22	0.083	0.083	0.083	0.75	1	0
17	0.02	0.86	114.24	949.2	977.4	**977.4**	0	4.22	0.083	0.083	0.083	0.75	1	0
18	0.02	0.86	114.24	949.2	977.4	**1954.8**	0	4.22	0.083	0.083	0.083	0.75	1	0
19	0.02	0.86	114.24	949.2	977.4	977.4	**0**	4.22	0.083	0.083	0.083	0.75	1	0
20	0.02	0.86	114.24	949.2	977.4	977.4	**5**	4.22	0.083	0.083	0.083	0.75	1	0
21	0.02	0.86	114.24	949.2	977.4	977.4	**10**	4.22	0.083	0.083	0.083	0.75	1	0
22	0.02	0.86	114.24	949.2	977.4	977.4	0	**2.11**	0.083	0.083	0.083	0.75	1	0
23	0.02	0.86	114.24	949.2	977.4	977.4	0	**4.22**	0.083	0.083	0.083	0.75	1	0
24	0.02	0.86	114.24	949.2	977.4	977.4	0	**8.44**	0.083	0.083	0.083	0.75	1	0
25	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	**0.0415**	0.083	0.083	0.75	1	0
26	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	**0.083**	0.083	0.083	0.75	1	0
27	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	**0.166**	0.083	0.083	0.75	1	0
28	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.0415**	0.083	0.75	1	0
29	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.083**	0.083	0.75	1	0
30	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.166**	0.083	0.75	1	0
31	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.0415**	0.75	1	0
32	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.083**	0.75	1	0
33	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.166**	0.75	1	0
34	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**0.375**	1	0
35	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**0.75**	1	0
36	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**1.50**	1	0
37	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**0**	**0**
38	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**1**	**0**
39	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**0**	**1**
40	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**1**	**1**

Table 2: Optimal cost of the EWMA control chart.

No.	Cost	Percentage difference from optimal cost, %
1	187.91	-15.5
2	222.37	0.0
3	273.66	23.1
4	278.56	25.3
5	222.37	0.0
6	185.48	-16.6
7	194.60	-12.5
8	222.37	0.0
9	327.97	47.5
10	188.49	-15.2
11	222.37	0.0
12	269.11	21.0
13	215.28	-3.2
14	222.37	0.0
15	229.03	3.0
16	213.31	-4.1
17	222.37	0.0
18	240.47	8.1
19	222.37	0.0
20	238.22	7.1
21	250.04	12.4
22	202.44	-9.0
23	222.37	0.0
24	246.67	10.9
25	221.20	-0.5
26	222.37	0.0
27	224.69	1.0
28	222.37	0.0
29	222.37	0.0
30	222.37	0.0
31	221.78	-0.3
32	222.37	0.0
33	223.53	0.5
34	223.92	0.7
35	222.37	0.0
36	219.32	-1.4
37	220.67	-0.8
38	222.37	0.0
39	234.29	5.4
40	236.00	6.1

5. Conclusion

The EWMA control chart has been known for its quick detection of small and moderate process mean shifts. In this study, sensitivity analysis was conducted to determine the effect of the input parameters on the cost of implementing the EWMA control chart. The sensitivity analysis was carried out by increasing and decreasing the values of the input parameters by 50%. All input parameters, namely, θ, δ, C_0, C_1, Y, W, b, c, e, T_0, T_1, T_2, γ_1 and γ_2 were subjected to sensitivity analysis. The findings of the study indicate that such a variation affects the implementation cost of the EWMA control chart. The input parameters that have a major effect on the cost are θ, δ, C_0, C_1, Y, W, b, c, γ_1 and γ_2. The expected quality cost per unit time while the process is in-control, C_0, has the most impact on the cost when C_0 is increased by 50%. The cost of implementing the EWMA control chart is increased by 47.5% when C_0 is increased by 50%. In contrast, input parameters e, T_0, T_1 and T_2 have almost no effect on the implementation cost of the EWMA control chart. As this study is based on the sensitivity analysis of the EWMA control chart, future research work can be extended by comparing the effect of the input parameters on the implementation cost for several control charts.

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