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TABLE OF CONTENT

PART 1: MATHEMATICS

	Page
STATISTICAL ANALYSIS ON THE EFFECTIVENESS OF SHORT-TERM PROGRAMS DURING COVID-19 PANDEMIC: IN THE CASE OF PROGRAM BIJAK SIFIR 2020 <i>Nazihah Safie, Syerrina Zakaria, Siti Madhihah Abdul Malik, Nur Bains Ismail, Azwani Alias Ruwaidiah Idris</i>	1
RADIATIVE CASSON FLUID OVER A SLIPPERY VERTICAL RIGA PLATE WITH VISCOUS DISSIPATION AND BUOYANCY EFFECTS <i>Siti Khuzaimah Soid, Khadijah Abdul Hamid, Ma Nuramalina Nasero, NurNajah Nabila Abdul Aziz</i>	10
GAUSSIAN INTEGER SOLUTIONS OF THE DIOPHANTINE EQUATION $x^4 + y^4 = z^3$ FOR $x \neq y$ <i>Shahrina Ismail, Kamel Ariffin Mohd Atan and Diego Sejas Viscarra</i>	19
A SEMI ANALYTICAL ITERATIVE METHOD FOR SOLVING THE EMDEN-FOWLER EQUATIONS <i>Mat Salim Selamat, Mohd Najir Tokachil, Noor Aqila Burhanddin, Ika Suzieana Murad and Nur Farhana Razali</i>	28
ROTATING FLOW OF A NANOFLUID PAST A NONLINEARLY SHRINKING SURFACE WITH FLUID SUCTION <i>Siti Nur Alwani Salleh, Norfifah Bachok and Nor Athirah Mohd Zin</i>	36
MODELING THE EFFECTIVENESS OF TEACHING BASIC NUMBERS THROUGH MINI TENNIS TRAINING USING MARKOV CHAIN <i>Rahela Abdul Rahim, Rahizam Abdul Rahim and Syahrul Ridhwan Morazuk</i>	46
PERFORMANCE OF MORTALITY RATES USING DEEP LEARNING APPROACH <i>Mohamad Hasif Azim and Saiful Izzuan Hussain</i>	53
UNSTEADY MHD CASSON FLUID FLOW IN A VERTICAL CYLINDER WITH POROSITY AND SLIP VELOCITY EFFECTS <i>Wan Faezah Wan Azmi, Ahmad Qushairi Mohamad, Lim Yeou Jiann and Sharidan Shafie</i>	60
DISJUNCTIVE PROGRAMMING - TABU SEARCH FOR JOB SHOP SCHEDULING PROBLEM <i>S. Z. Nordin, K.L. Wong, H.S. Pheng, H. F. S. Saipol and N.A.A. Husain</i>	68
FUZZY AHP AND ITS APPLICATION TO SUSTAINABLE ENERGY PLANNING DECISION PROBLEM <i>Liana Najib and Lazim Abdullah</i>	78
A CONSISTENCY TEST OF FUZZY ANALYTIC HIERARCHY PROCESS <i>Liana Najib and Lazim Abdullah</i>	89
FREE CONVECTION FLOW OF BRINKMAN TYPE FLUID THROUGH AN OSCILLATING PLATE <i>Siti Noramirah Ibrahim, Ahmad Qushairi Mohamad, Lim Yeou Jiann, Sharidan Shafie and Muhammad Najib Zakaria</i>	98

RADIATION EFFECT ON MHD FERROFLUID FLOW WITH RAMPED WALL TEMPERATURE AND ARBITRARY WALL SHEAR STRESS	106
<i>Nor Athirah Mohd Zin, Aaiza Gul, Siti Nur Alwani Salleh, Imran Ullah, Sharena Mohamad Isa, Lim Yeou Jiann and Sharidan Shafie</i>	

PART 2: STATISTICS

A REVIEW ON INDIVIDUAL RESERVING FOR NON-LIFE INSURANCE	117
<i>Kelly Chuah Khai Shin and Ang Siew Ling</i>	
STATISTICAL LEARNING OF AIR PASSENGER TRAFFIC AT THE MURTALA MUHAMMED INTERNATIONAL AIRPORT, NIGERIA	123
<i>Christopher Godwin Udomboso and Gabriel Olugbenga Ojo</i>	
ANALYSIS ON SMOKING CESSATION RATE AMONG PATIENTS IN HOSPITAL SULTAN ISMAIL, JOHOR	137
<i>Siti Mariam Norrulashikin, Ruzaini Zulhusni Puslan, Nur Arina Bazilah Kamisan and Siti Rohani Mohd Nor</i>	
EFFECT OF PARAMETERS ON THE COST OF MEMORY TYPE CHART	146
<i>Sakthiseswari Ganasan, You Huay Woon and Zainol Mustafa</i>	
EVALUATION OF PREDICTORS FOR THE DEVELOPMENT AND PROGRESSION OF DIABETIC RETINOPATHY AMONG DIABETES MELLITUS TYPE 2 PATIENTS	152
<i>Syafawati Ab Saad, Maz Jamilah Masnan, Karniza Khalid and Safwati Ibrahim</i>	
REGIONAL FREQUENCY ANALYSIS OF EXTREME PRECIPITATION IN PENINSULAR MALAYSIA	160
<i>Iszuanie Syafidza Che Ilias, Wan Zawiah Wan Zin and Abdul Aziz Jemain</i>	
EXPONENTIAL MODEL FOR SIMULATION DATA VIA MULTIPLE IMPUTATION IN THE PRESENT OF PARTLY INTERVAL-CENSORED DATA	173
<i>Salman Umer and Faiz Elfaki</i>	
THE FUTURE OF MALAYSIA'S AGRICULTURE SECTOR BY 2030	181
<i>Thanusha Palmira Thangarajah and Suzilah Ismail</i>	
MODELLING MALAYSIAN GOLD PRICES USING BOX-JENKINS APPROACH	186
<i>Isnewati Ab Malek, Dewi Nur Farhani Radin Nor Azam, Dinie Syazwani Badrul Aidi and Nur Syafiqah Sharim</i>	
WATER DEMAND PREDICTION USING MACHINE LEARNING: A REVIEW	192
<i>Norashikin Nasaruddin, Shahida Farhan Zakaria, Afida Ahmad, Ahmad Zia Ul-Saufie and Norazian Mohamaed Noor</i>	
DETECTION OF DIFFERENTIAL ITEM FUNCTIONING FOR THE NINE-QUESTIONS DEPRESSION RATING SCALE FOR THAI NORTH DIALECT	201
<i>Suttipong Kawilapat, Benchlak Maneeton, Narong Maneeton, Sukon Prasitwattanaseree, Thoranin Kongsuk, Suwanna Arunpongpaisal, Jintana Leejongpermpool, Supattra Sukhawaha and Patrinee Traisathit</i>	

ACCELERATED FAILURE TIME (AFT) MODEL FOR SIMULATION PARTLY INTERVAL-CENSORED DATA	210
<i>Ibrahim El Feky and Faiz Elfaki</i>	
MODELING OF INFLUENCE FACTORS PERCENTAGE OF GOVERNMENTS' RICE RECIPIENT FAMILIES BASED ON THE BEST FOURIER SERIES ESTIMATOR	217
<i>Chaerobby Fakhri Fauzaan Purwoko, Ayuning Dwis Cahyasari, Netha Aliffia and M. Fariz Fadillah Mardianto</i>	
CLUSTERING OF DISTRICTS AND CITIES IN INDONESIA BASED ON POVERTY INDICATORS USING THE K-MEANS METHOD	225
<i>Khoirun Niswatin, Christopher Andreas, Putri Fardha Asa OktaviaHans and M. Fariz Fadilah Mardianto</i>	
ANALYSIS OF THE EFFECT OF HOAX NEWS DEVELOPMENT IN INDONESIA USING STRUCTURAL EQUATION MODELING-PARTIAL LEAST SQUARE	233
<i>Christopher Andreas, Sakinah Priandi, Antonio Nikolas Manuel Bonar Simamora and M. Fariz Fadillah Mardianto</i>	
A COMPARATIVE STUDY OF MOVING AVERAGE AND ARIMA MODEL IN FORECASTING GOLD PRICE	241
<i>Arif Luqman Bin Khairil Annuar, Hang See Pheng, Siti Rohani Binti Mohd Nor and Thoo Ai Chin</i>	
CONFIDENCE INTERVAL ESTIMATION USING BOOTSTRAPPING METHODS AND MAXIMUM LIKELIHOOD ESTIMATE	249
<i>Siti Fairus Mokhtar, Zahayu Md Yusof and Hasimah Sapiri</i>	
DISTANCE-BASED FEATURE SELECTION FOR LOW-LEVEL DATA FUSION OF SENSOR DATA	256
<i>M. J. Masnan, N. I. Maha3, A. Y. M. Shakaf, A. Zakaria, N. A. Rahim and N. Subari</i>	
BANKRUPTCY MODEL OF UK PUBLIC SALES AND MAINTENANCE MOTOR VEHICLES FIRMS	264
<i>Asmahani Nayan, Amirah Hazwani Abd Rahim, Siti Shuhada Ishak, Mohd Rijal Ilias and Abd Razak Ahmad</i>	
INVESTIGATING THE EFFECT OF DIFFERENT SAMPLING METHODS ON IMBALANCED DATASETS USING BANKRUPTCY PREDICTION MODEL	271
<i>Amirah Hazwani Abdul Rahim, Nurazlina Abdul Rashid, Abd-Razak Ahmad and Norin Rahayu Shamsuddin</i>	
INVESTMENT IN MALAYSIA: FORECASTING STOCK MARKET USING TIME SERIES ANALYSIS	278
<i>Nuzlinda Abdul Rahman, Chen Yi Kit, Kevin Pang, Fauhatuz Zahroh Shaik Abdullah and Nur Sofiah Izani</i>	

PART 3: COMPUTER SCIENCE & INFORMATION TECHNOLOGY

- ANALYSIS OF THE PASSENGERS' LOYALTY AND SATISFACTION OF AIRASIA PASSENGERS USING CLASSIFICATION** 291
Ee Jian Pei, Chong Pui Lin and Nabilah Filzah Mohd Radzuan
- HARMONY SEARCH HYPER-HEURISTIC WITH DIFFERENT PITCH ADJUSTMENT OPERATOR FOR SCHEDULING PROBLEMS** 299
Khairul Anwar, Mohammed A.Awadallah and Mohammed Azmi Al-Betar
- A 1D EYE TISSUE MODEL TO MIMIC RETINAL BLOOD PERFUSION DURING RETINAL IMAGING PHOTOPLETHYSMOGRAPHY (IPPG) ASSESSMENT: A DIFFUSION APPROXIMATION – FINITE ELEMENT METHOD (FEM) APPROACH** 307
Harnani Hassan, Sukreen Hana Herman, Zulfakri Mohamad, Sijung Hu and Vincent M. Dwyer
- INFORMATION SECURITY CULTURE: A QUALITATIVE APPROACH ON MANAGEMENT SUPPORT** 325
Qamarul Nazrin Harun, Mohamad Noorman Masrek, Muhamad Ismail Pahmi and Mohamad Mustaqim Junoh
- APPLY MACHINE LEARNING TO PREDICT CARDIOVASCULAR RISK IN RURAL CLINICS FROM MEXICO** 335
Misael Zambrano-de la Torre, Maximiliano Guzmán-Fernández, Claudia Sifuentes-Gallardo, Hamurabi Gamboa-Rosales, Huizilopoztli Luna-García, Ernesto Sandoval-García, Ramiro Esquivel-Felix and Héctor Durán-Muñoz
- ASSESSING THE RELATIONSHIP BETWEEN STUDENTS' LEARNING STYLES AND MATHEMATICS CRITICAL THINKING ABILITY IN A 'CLUSTER SCHOOL'** 343
Salimah Ahmad, Asyura Abd Nassir, Nor Habibah Tarmuji, Khairul Firhan Yusob and Nor Azizah Yacob
- STUDENTS' LEISURE WEEKEND ACTIVITIES DURING MOVEMENT CONTROL ORDER: UİTM PAHANG SHARING EXPERIENCE** 351
Syafıza Saila Samsudin, Noor Izyan Mohamad Adnan, Nik Muhammad Farhan Hakim Nik Badrul Alam, Siti Rosiah Mohamed and Nazihah Ismail
- DYNAMICS SIMULATION APPROACH IN MODEL DEVELOPMENT OF UNSOLD NEW RESIDENTIAL HOUSING IN JOHOR** 363
Lok Lee Wen and Hasimah Sapiri
- WORD PROBLEM SOLVING SKILLS AS DETERMINANT OF MATHEMATICS PERFORMANCE FOR NON-MATH MAJOR STUDENTS** 371
Shahida Farhan Zakaria, Norashikin Nasaruddin, Mas Aida Abd Rahim, Fazillah Bosli and Kor Liew Kee
- ANALYSIS REVIEW ON CHALLENGES AND SOLUTIONS TO COMPUTER PROGRAMMING TEACHING AND LEARNING** 378
Noor Hasnita Abdul Talib and Jasmin Ilyani Ahmad

PART 4: OTHERS

- ANALYSIS OF CLAIM RATIO, RISK-BASED CAPITAL AND VALUE-ADDED INTELLECTUAL CAPITAL: A COMPARISON BETWEEN FAMILY AND GENERAL TAKAFUL OPERATORS IN MALAYSIA** 387
Nur Amalina Syafiqa Kamaruddin, Norizarina Ishak, Siti Raihana Hamzah, Nurfadhlina Abdul Halim and Ahmad Fadhly Nurullah Rasade
- THE IMPACT OF GEOMAGNETIC STORMS ON THE OCCURRENCES OF EARTHQUAKES FROM 1994 TO 2017 USING THE GENERALIZED LINEAR MIXED MODELS** 396
N. A. Mohamed, N. H. Ismail, N. S. Majid and N. Ahmad
- BIBLIOMETRIC ANALYSIS ON BITCOIN 2015-2020** 405
Nurazlina Abdul Rashid, Fazillah Bosli, Amirah Hazwani Abdul Rahim, Kartini Kasim and Fathiyah Ahmad@Ahmad Jali
- GENDER DIFFERENCE IN EATING AND DIETARY HABITS AMONG UNIVERSITY STUDENTS** 413
Fazillah Bosli, Siti Fairus Mokhtar, Noor Hafizah Zainal Aznam, Juaini Jamaludin and Wan Siti Esah Che Hussain
- MATHEMATICS ANXIETY: A BIBLIOMETRIX ANALYSIS** 420
Kartini Kasim, Hamidah Muhd Irpan, Noorazilah Ibrahim, Nurazlina Abdul Rashid and Anis Mardiana Ahmad
- PREDICTION OF BIOCHEMICAL OXYGEN DEMAND IN MEXICAN SURFACE WATERS USING MACHINE LEARNING** 428
Maximiliano Guzmán-Fernández, Misael Zambrano-de la Torre, Claudia Sifuentes-Gallardo, Oscar Cruz-Dominguez, Carlos Bautista-Capetillo, Juan Badillo-de Loera, Efrén González Ramírez and Héctor Durán-Muñoz

FREE CONVECTION FLOW OF BRINKMAN TYPE FLUID THROUGH AN COSINE OSCILLATING PLATE

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In this study, heat transfer on the unsteady flow of Brinkman type fluid model induced by free convection phenomenon is investigate. The flow is considered to be bounded by a vertical oscillating plate. The dimensional governing equations, as well as the initial and oscillating boundary conditions, are reduced to dimensionless forms using appropriate dimensionless variables. The Laplace transformation is employed on the resulting dimensionless equations to solve for the closed form solution of temperature and velocity fields. The graphical results illustrating the temperature and velocity profiles under the consequences of related parameters such as Prandtl number, Grashof number, Brinkman type fluid parameter, phase angle, and time are displayed.

Keywords: Free convection, Brinkman type fluids, heat transfer, mathematical solutions, oscillating plate, Laplace transform

1. Introduction

In physics field, a fluid is defined as a substance that flows constantly when a shear stress or external force is applied to it. Fluids can also be divided into two categories: Newtonian and non-Newtonian fluids. A fluid where the shear rate is directly proportional to the shear stress when the shear rate and shear stress are at zero is called as Newtonian fluid. The properties of this type of fluid are mostly found in mineral oils, water, alcohol, air and ethanol. The study of a rotating and incompressible viscous fluid moving unsteadily with natural convection near an infinite vertical plate that adapting a time-dependent shear stress $f(t)$ to the fluid was investigated by Imran et al. (2015). Chaudhary and Jain (2007a) worked on the previous work considering a non-rotating unsteady viscous fluid flow generated by Newtonian heating past an infinite vertical plate. Suitable dimensionless variables have been used to reduce the dimensional governing equations into dimensionless form which then are computed by applying the method of Laplace transform. The same technique is used by Zulkiflee et al. (2019) to generate the exact solutions taking the fluid is considered to be unsteadily flow with a Newtonian heating effect within two parallel plates in a vertical position. In another study considering the same geometry as Zulkiflee et al. (2019), Narahari (2009) extended the study for a Couette flow of incompressible viscous fluid with thermal radiation effect.

Non-Newtonian fluids are distinguished from Newtonian fluids by the fact that they do not satisfy Newton's Law of Viscosity, which states that shear rate is not always proportional to shear stress. The study of non-Newtonian fluids has gotten a lot of attention because of its vast range of applications in industry, engineering, and geoscience. In 1949, this type of fluid model was employed for the first time (Brinkman, 1949). Afterwards, the flow of Brinkman type fluid model was further scrutinized by Hsu and Cheng (1985) taking the fluid is placed in a medium with porous effect and the flow is caused by free convection in a semi-infinite perpendicular smooth plate. Furthermore, numerous researchers have took their part in investigating the flow of Brinkman model including Gorla et al. (1999), Varma and Babu (1985) and Rajagopal (2007). Using the Brinkman model and the Fourier transformation, Fetecau et al. (2011) obtained the closed form solution for the incompressible flow of time-dependent viscous fluid. Later, Ali et al. (2012) applied the Laplace transformation to solve the problem of Brinkman model for a n exact solution. Furthermore, Ali et al. (2013) analyzed the heat and flow propagations with Newtonian heating effect for an

incompressible MHD Brinkman type fluid that unsteadily flow through a porous material with free convection across a vertical flat plate. Most recently, Islam et al. (2018) adapted the same model of fluid to investigate the flow characteristics moving within side walls. Khan et al. (2018) examined the similar problem as Islam et al. (2018) by imposing magnetic field effect perpendicular to the flow.

The imposition of oscillating wall to generate the flow for a viscous is found in numerous engineering applications such as flows in vibrating media. Basically, the Stokes' second problem is defined for the fluid that have been restricted only by the moving wall. The study of a rotating system for unsteady incompressible viscous fluid flow have been done by Thornley (1968), Puri (1975), Pop and Soundalgekar (1975), Gupta and Gupta (1975), Deka et al. (1999) and several other researchers. Bhattacharya et al. (2011) have investigated the physical behavior of unsteady viscous incompressible flow across an oscillating vertical plate. The method of Laplace transform is applied to solve for a closed form solution with the consideration of double diffusion and first-order homogeneous chemical reaction. Furthermore, a study on mass and heat transmission of an electrically conducting fluid flow through a permeable medium due to an oscillating plate was carried out by Chaudhary et al. (2007b). The study of quite the same field was done by Ali et al. (2014) where the analytical solutions for a second-grade fluid with an unsteady flow have been attained by employing the Laplace transform technique. The flow has been deformed due an oscillating plate and the unbalance temperature in the fluid.

In this study, Brinkman type fluid which is one of the complex models suggested by H.C. Brinkman has been chosen as non-Newtonian fluid. This model is applicable for the fluid that flowing over a highly porous media and it has a special term of viscosity. Ali et al. (2014) employed the Laplace transform method to attain solution for a Brinkman type fluid problem where the flow is influenced by the oscillating plate. The current study is mainly concerned on the mathematical solutions for the behavior of fluid flow and heat transfer. The objective of this study is to carry out a free convection flow of Brinkman type fluid past an oscillating plate. The problem is governed by dimensional energy and momentum equations, which are then solved analytically utilizing the method of Laplace transform. Further analysis on the temperature and velocity distributions under the impacts of involved parameters are illustrated graphically coupled with a discussion.

2. Mathematical Formulation

Let us consider the unsteady convective flow of a Brinkman type fluid over a vertical infinite oscillating plate. The flow is being enclosed to $y' > 0$, where y' axis is considered as the normal direction to the plate. Initially, when time $t' = 0$, both fluid and plate are assumed to be at rest with uniform temperature T'_{∞} . Later, when time $t' = 0^+$, the plate starts to oscillate in its plane $U_0 \cos \omega t$ where the constant U_0 is the amplitude of the plate oscillations and ω is the oscillating frequency of the plate. At the same time, the plate temperature is raised to T'_w which is thereafter maintained constant. The velocity and temperature depend on space variable y' and time t' . Since the flow is considered to be in unidirectional and one-dimensional as well as adapting the Boussinesq's approximation, the momentum and energy equations are written as follow

$$\frac{\partial u'(y, t')}{\partial t'} + \beta^* u' = \nu \frac{\partial^2 u'(y, t')}{\partial y'^2} + g\beta(T' - T'_{\infty}) \quad (1)$$

$$\frac{\partial T'(y, t')}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'(y, t')}{\partial y'^2} \quad (2)$$

together with initial and boundary conditions:

$$\begin{aligned}
u'(y', 0) &= 0 & T'(y', 0) &= T'_{\infty}, \\
u'(0, t') &= U_0 \cos \omega t & T'(0, t') &= T'_{\omega}, \\
u'(\infty, t') &\rightarrow 0 & T'(\infty, t') &\rightarrow T'_{\infty}
\end{aligned} \tag{3}$$

where $u'(y', t')$ is the velocity in the y' direction, $\beta^* = \alpha/\rho$ where α is referred as a drag coefficient that is usually positive constant, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, $T'(y', t')$ is fluid temperature, k is the thermal conductivity, ρ is the density, and c_p is the heat of the fluid at constant pressure. Here, the suitable dimensionless variables are introduced (Ali et al., 2014)

$$y = \frac{U_0}{\nu} y', \quad t = \frac{U_0^2}{\nu} t', \quad u = \frac{u'}{U_0}, \quad \theta = \frac{T' - T'_{\infty}}{T'_{\omega} - T'_{\infty}}, \quad \omega = \frac{\nu}{U_0^2} \omega' \tag{4}$$

to convert the (1), (2) and (3) into dimensionless form which can be written as follows

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \beta_1 u + Gr\theta, \tag{5}$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}, \tag{6}$$

with associated dimensionless conditions

$$\begin{aligned}
u(y, 0) &= 0 & \theta(y, 0) &= 0, \\
u(0, t) &= \cos \omega t & \theta(0, t) &= 1, \\
u(\infty, t) &\rightarrow 0 & \theta(\infty, t) &\rightarrow 0.
\end{aligned} \tag{7}$$

where $\beta_1 = \frac{\beta^* \nu}{U_0^2}$, $Gr = \frac{\nu g \beta (T'_{\omega} - T'_{\infty})}{U_0^3}$, $Pr = \frac{\nu \rho c_p}{k}$ are the Brinkman fluid parameter, Grashof number, and Prandtl number.

3. Solution of the problem

Next, applying the Laplace transforms on (5) and (6) yields the dimensionless governing equations in the transformed (y, s) plane. After substituting initial and boundary conditions (7) and having some manipulations, the following solutions are obtained as

$$\bar{\theta}(y, s) = \frac{1}{s} e^{-y\sqrt{sPr}} \tag{8}$$

$$\begin{aligned}
\bar{u}(y, s) &= \frac{1}{2(s - i\omega)} e^{-y\sqrt{s+\beta_1}} + \frac{1}{2(s + i\omega)} e^{-y\sqrt{s+\beta_1}} - \frac{\beta_1}{s} e^{-y\sqrt{s+\beta_1}} \\
&\quad + \frac{\beta_3}{s + \beta_2} e^{-y\sqrt{s+\beta_1}} + \frac{\beta_3}{s} e^{-y\sqrt{sPr}} - \frac{\beta_3}{s + \beta_2} e^{-y\sqrt{sPr}}.
\end{aligned} \tag{9}$$

Then, (8) and (9) are imposed with the inverse Laplace and transforms to following expression

$$\theta(y, t) = \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} \right) \tag{10}$$

$$u(y, t) = u_1(y, t) + u_2(y, t) - u_3(y, t) + u_4(y, t) + u_5(y, t) - u_6(y, t) \quad (11)$$

where

$$u_1(y, t) = \frac{1}{4}H(t)e^{i\omega t + y\sqrt{\beta_1 + i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(\beta_1 + i\omega)t}\right) + \frac{1}{4}H(t)e^{i\omega t - y\sqrt{\beta_1 + i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(\beta_1 + i\omega)t}\right), \quad (12)$$

$$u_2(y, t) = \frac{1}{4}H(t)e^{-i\omega t + y\sqrt{\beta_1 - i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(\beta_1 - i\omega)t}\right) + \frac{1}{4}H(t)e^{-i\omega t - y\sqrt{\beta_1 - i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(\beta_1 - i\omega)t}\right), \quad (13)$$

$$u_3(y, t) = \frac{\beta_3}{2}e^{y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1 t}\right) + \frac{\beta_3}{2}e^{-y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1 t}\right), \quad (14)$$

$$u_4(y, t) = \frac{\beta_3}{2}e^{-\beta_2 t + y\sqrt{\beta_1 - \beta_2}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(\beta_1 - \beta_2)t}\right) + \frac{\beta_3}{2}e^{-\beta_2 t - y\sqrt{\beta_1 - \beta_2}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(\beta_1 - \beta_2)t}\right), \quad (15)$$

$$u_5(y, t) = \beta_3 \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\operatorname{Pr}}{t}}\right), \quad (16)$$

$$u_6(y, t) = \frac{\beta_3}{2}e^{-\beta_2 t + y(i\sqrt{\beta_2 \operatorname{Pr}})}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\operatorname{Pr}}{t}} + i\sqrt{\beta_2 t}\right) + \frac{\beta_3}{2}e^{-\beta_2 t - y(i\sqrt{\beta_2 \operatorname{Pr}})}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\operatorname{Pr}}{t}} - i\sqrt{\beta_2 t}\right). \quad (17)$$

It should be noted that the aforementioned velocity solutions are only applicable for $\operatorname{Pr} \neq 1$. Therefore, when $\operatorname{Pr} = 1$, the solutions can be acquired by substituting $\operatorname{Pr} = 1$ into (8) and the same step as discussed above are followed. The obtained solutions are

$$\bar{u}(y, t) = u_1(y, t) + u_2(y, t) + u_7(y, t) + u_8(y, t) \quad (18)$$

where

$$u_7(y, t) = \frac{\beta_4}{2}e^{y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1 t}\right) + \frac{\beta_4}{2}e^{-y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1 t}\right) \quad (19)$$

$$u_8(y, t) = \beta_4 \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) \quad (20)$$

with $\beta_2 = \frac{Gr}{\beta_1}$, $\beta_3 = \frac{Gr}{\beta_1}$ and $\beta_4 = \frac{Gr}{\beta_1}$ are constant parameters.

4. Results and Discussions

In this section, the impacts of the embedded parameters such as Prandtl number Pr , Grashof number Gr , Brinkman fluid parameter β_1 , phase angle ωt , and time t on the temperature and velocity distributions are displayed using graphs. The obtained solutions are analyzed numerically by using several values of parameters. All the plotted graphs are displayed to analyze the velocity and temperature profiles. Figure 1 to Figure 5 present the physical behavior of the velocity, whereas Figure 6 and Figure 7 illustrate the temperature profiles. The results obtained here satisfied all the initial and boundary conditions (7). Figure 1 shows the behavior of velocity towards time change. It has been observed that as the value of t increases, the velocity also increases. As the time increases, the buoyancy force effectively dominates the flow due to receiving energy from an external source and eventually causes the velocity to increase. Figure 2 displays the velocity distribution in response to the enlargement of Gr . It is clear that as the value of Gr increases, the velocity increases. Physically, Gr is represented as ratio between buoyancy force to viscous force. Hence, the buoyancy force takes precedence during the free convection process and cause an increment in Gr , which subsequently accelerating the velocity profile. The effect of Pr on velocity propagation is demonstrated in Figure 3. In this study, different physical values for Pr have been selected which are $Pr = 1.50$, $Pr = 5.00$ (light organic fluid), $Pr = 6.20$ (water) and $Pr = 7.2$ (sea water). As clearly displayed, the velocity decreases as Pr is increased. Prandtl number Pr is defined as the ratio of kinematic viscosity to thermal diffusivity. Thus, increasing Pr values imply to the increase of kinematic viscosity but reduce the thermal diffusivity of the fluid. As a result, increase in kinematic viscosity leads the decrement of fluid velocity. In Figure 4, the graph illustrates the effect of Brinkman fluid parameter, β_1 on the velocity profile. The findings show that increasing value of Brinkman fluid parameter has dropped the velocity field. Moreover, in Figure 5, it has shown the velocity decreases when ωt is increased. Clearly, this figure satisfies the boundary conditions (7) in view of boundary conditions (3), hence the accuracy of the obtained solutions is verified. Figure 6 shows that the temperature profile increases when value of t is ascended but decreases in responses to the increase values of Pr (Figure 7). The curve of the temperature profile declines more slowly as the value of t increases but decline more rapidly as the value of Pr increases.

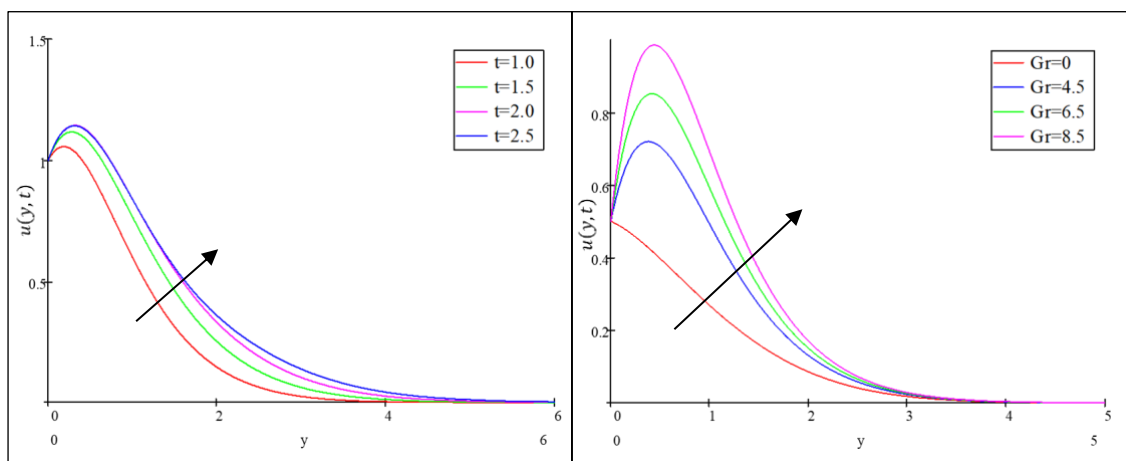


Figure 1: Velocity profiles for different values of t with $\beta_1 = 0.8$, $Pr = 5.0$, $Gr = 5.0$ and $\omega = 0$

Figure 2: Velocity profiles for different values of Gr with $\beta_1 = 0.8$, $Pr = 5.0$, $t = 1.0$ and $\omega = \frac{\pi}{3}$

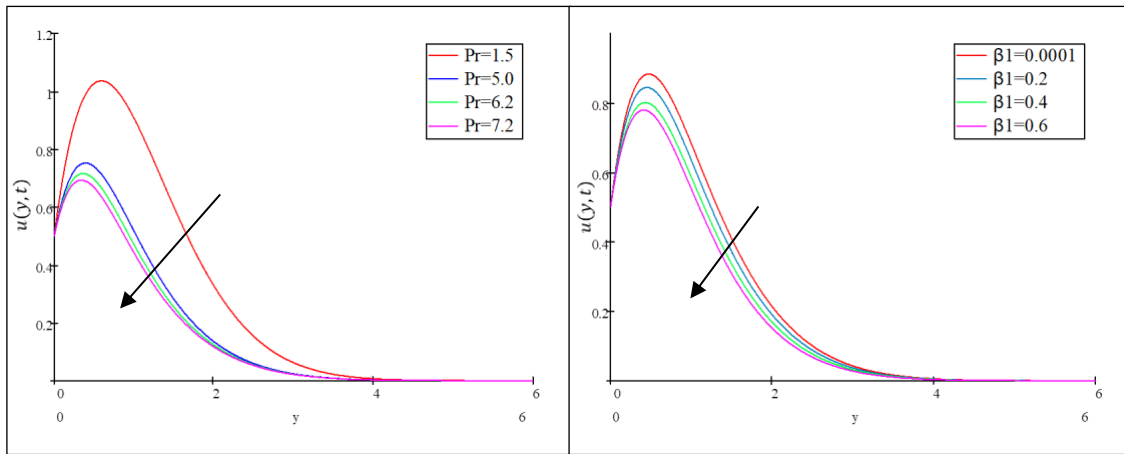


Figure 3: Velocity profiles for different values of Pr with $\beta_1 = 0.8$, $Gr = 5.0$, $t = 1.0$ and $\omega = \frac{\pi}{3}$

Figure 4: Velocity profiles for different values of β_1 with $Pr = 5.0$, $Gr = 5.0$, $t = 1.0$ and $\omega = \frac{\pi}{3}$

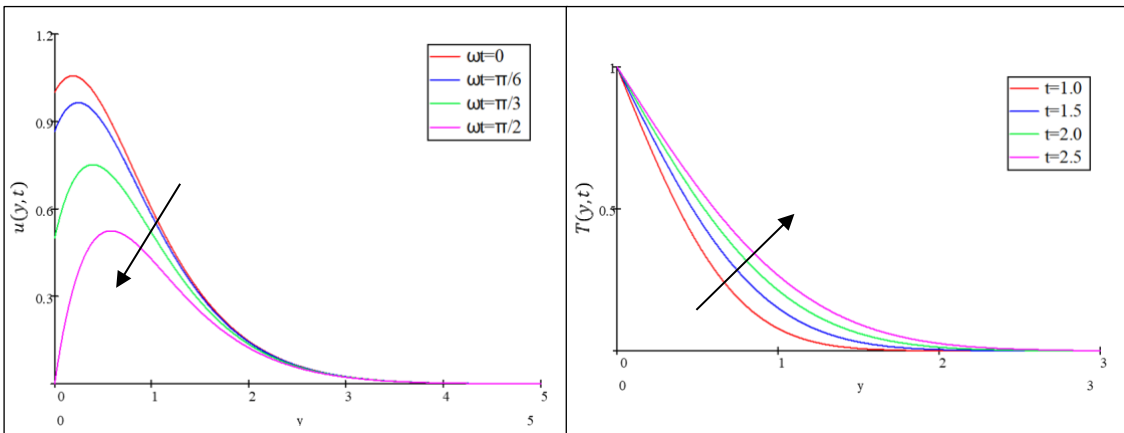


Figure 5: Velocity profiles for different values of ωt with $\beta_1 = 0.8$, $Pr = 5.0$, $Gr = 5.0$ and $t = 1.0$

Figure 6: Temperature profiles for different values of t with $Pr = 6.2$

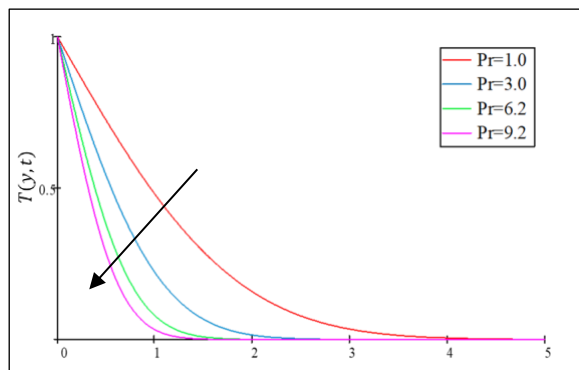


Figure 7: Temperature profiles for different values of Pr with $t = 1.0$

5. Conclusion

In this study, there are two governing dimensional equations which are momentum and energy equations. The governing equations are then reduced into dimensionless form by using suitable dimensionless variables. Here, the temperature and velocity solutions are acquired by solving the dimensionless governing equations using the Laplace transform method with the consideration of boundary and initial conditions. The resulting solutions are then plotted into several graphs by using MATHCAD software. By differentiate the values of various variables, the impact of the parameters towards the velocity and temperature distributions are observed. The significant findings in this study are the velocity amplifies with the increase of t and Gr but reduces with the increase of Pr , ωt and β_1 . While temperature increases with t but it drops with ascending value of Pr .

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