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FREE CONVECTION FLOW OF BRINKMAN TYPE FLUID THROUGH AN COSINE OSCILLATING PLATE

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In this study, heat transfer on the unsteady flow of Brinkman type fluid model induced by free convection phenomenon is investigate. The flow is considered to be bounded by a vertical oscillating plate. The dimensional governing equations, as well as the initial and oscillating boundary conditions, are reduced to dimensionless forms using appropriate dimensionless variables. The Laplace transformation is employed on the resulting dimensionless equations to solve for the closed form solution of temperature and velocity fields. The graphical results illustrating the temperature and velocity profiles under the consequences of related parameters such as Prandtl number, Grashof number, Brinkman type fluid parameter, phase angle, and time are displayed.

***Keywords*:** Free convection, Brinkman type fluids, heat transfer, mathematical solutions, oscillating plate, Laplace transform

1. Introduction

In physics field, a fluid is defined as a substance that flows constantly when a shear stress or external force is applied to it. Fluids can also be divided into two categories: Newtonian and non-Newtonian fluids. A fluid where the shear rate is directly proportional to the shear stress when the shear rate and shear stress are at zero is called as Newtonian fluid. The properties of this type of fluid are mostly found in mineral oils, water, alcohol, air and ethanol. The study of a rotating and incompressible viscous fluid moving unsteadily with natural convection near an infinite vertical plate that adapting a time-dependent shear stress $f(t)$ to the fluid was investigated by Imran et al. (2015). Chaudhary and Jain (2007a) worked on the previous work considering a non-rotating unsteady viscous fluid flow generated by Newtonian heating past an infinite vertical plate. Suitable dimensionless variables have been used to reduce the dimensional governing equations into dimensionless form which then are computed by applying the method of Laplace transform. The same technique is used by Zulkiflee et al. (2019) to generate the exact solutions taking the fluid is considered to be unsteadily flow with a Newtonian heating effect within two parallel plates in a vertical position. In another study considering the same geometry as Zulkiflee et al. (2019), Narahari (2009) extended the study for a Couette flow of incompressible viscous fluid with thermal radiation effect.

Non-Newtonian fluids are distinguished from Newtonian fluids by the fact that they do not satisfy Newton's Law of Viscosity, which states that shear rate is not always proportional to shear stress. The study of non-Newtonian fluids has gotten a lot of attention because of its vast range of applications in industry, engineering, and geoscience. In 1949, this type of fluid model was employed for the first time (Brinkman, 1949). Afterwards, the flow of Brinkman type fluid model was further scrutinized by Hsu and Cheng (1985) taking the fluid is placed in a medium with porous effect and the flow is caused by free convection in a semi-infinite perpendicular smooth plate. Furthermore, numerous researchers have took their part in investigating the flow of Brinkman model including Gorla et al. (1999), Varma and Babu (1985) and Rajagopal (2007). Using the Brinkman model and the Fourier transformation, Fetecau et al. (2011) obtained the closed form solution for the incompressible flow of time-dependent viscous fluid. Later, Ali et al. (2012) applied the Laplace transformation to solve the problem of Brinkman model for a n exact solution. Furthermore, Ali et al. (2013) analyzed the heat and flow propagations with Newtonian heating effect for an incompressible MHD Brinkman type fluid that unsteadily flow through a porous material with free convection across a vertical flat plate. Most recently, Islam et al. (2018) adapted the same model of fluid to investigate the flow characteristics moving within side walls. Khan et al. (2018) examined the similar problem as Islam et al. (2018) by imposing magnetic field effect perpendicular to the flow.

The imposition of oscillating wall to generate the flow for a viscous is found in numerous engineering applications such as flows in vibrating media. Basically, the Stokes' second problem is defined for the fluid that have been restricted only by the moving wall. The study of a rotating system for unsteady incompressible viscous fluid flow have been done by Thornley (1968), Puri (1975), Pop and Soundalgekar (1975), Gupta and Gupta (1975), Deka et al. (1999) and several other researchers. Bhattacharya et al. (2011) have investigated the physical behavior of unsteady viscous incompressible flow across an oscillating vertical plate. The method of Laplace transform is applied to solve for a closed form solution with the consideration of double diffusion and first-order homogeneous chemical reaction. Furthermore, a study on mass and heat transmission of an electrically conducting fluid flow through a permeable medium due to an oscillating plate was carried out by Chaudhary et al. (2007b). The study of quite the same field was done by Ali et al. (2014) where the analytical solutions for a second-grade fluid with an unsteady flow have been attained by employing the Laplace transform technique. The flow has been deformed due an oscillating plate and the unbalance temperature in the fluid.

In this study, Brinkman type fluid which is one of the complex models suggested by H.C. Brinkman has been chosen as non-Newtonian fluid. This model is applicable for the fluid that flowing over a highly porous media and it has a special term of viscosity. Ali et al. (2014) employed the Laplace transform method to attain solution for a Brinkman type fluid problem where the flow is influenced by the oscillating plate. The current study is mainly concerned on the mathematical solutions for the behavior of fluid flow and heat transfer. The objective of this study is to carry out a free convection flow of Brinkman type fluid past an oscillating plate. The problem is governed by dimensional energy and momentum equations, which are then solved analytically utilizing the method of Laplace transform. Further analysis on the temperature and velocity distributions under the impacts of involved parameters are illustrated graphically coupled with a discussion.

2. Mathematical Formulation

Let us consider the unsteady convective flow of a Brinkman type fluid over a vertical infinite oscillating plate. The flow is being enclosed to $y' > 0$, where y' axis is considered as the normal direction to the plate. Initially, when time $t' = 0$, both fluid and plate are assumed to be at rest with uniform temperature T'_∞. Later, when time $t' = 0^+$, the plate starts to oscillate in its plane $U_0 \cos \omega t$ where the constant U_0 is the amplitude of the plate oscillations and ω is the oscillating frequency of the plate. At the same time, the plate temperature is raised to T_w' which is thereafter maintained constant. The velocity and temperature depend on space variable y' and time t'. Since the flow is considered to be in unidirectional and one-dimensional as well as adapting the Boussinesq's approximation, the momentum and energy equations are written as follow

$$\frac{\partial u'(y,t')}{\partial t'} + \beta^* u' = v\frac{\partial^2 u'(y,t')}{\partial y'^2} + g\beta(T' - T'_\infty) \tag{1}$$

$$\frac{\partial T'(y,t')}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'(y,t')}{\partial y'^2} \tag{2}$$

together with initial and boundary conditions:

$$\begin{array}{ll} u'(y',0) = 0 & T'(y',0) = T'_\infty\,, \\ u'(0,t') = U_0 \cos\omega t & T'(0,t') = T'_\omega\,, \\ u'(\infty,t') \to 0 & T'(\infty,t') \to T'_\infty \end{array} \tag{3}$$

where $u'(y',t')$ is the velocity in the y' direction, $\beta^* = \alpha/\rho$ where α is referred as a drag coefficient that is usually positive constant, v is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, $T'(y',t')$ is fluid temperature, k is the thermal conductivity, ρ is the density, and c_p is the heat of the fluid at constant pressure. Here, the suitable dimensionless variables are introduced (Ali et al., 2014)

$$y = \frac{U_0}{v}y'\,, \quad t = \frac{U_0^2}{v}t'\,, \quad u = \frac{u'}{U_0}, \quad \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}\,, \quad \omega = \frac{v}{U_0^2}\omega' \tag{4}$$

to convert the (1), (2) and (3) into dimensionless form which can be written as follows

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \beta_1 u + Gr\theta\,, \tag{5}$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}\,, \tag{6}$$

with associated dimensionless conditions

$$\begin{array}{ll} u(y,0) = 0 & \theta(y,0) = 0\,, \\ u(0,t) = \cos\omega t & \theta(0,t) = 1\,, \\ u(\infty,t) \to 0 & \theta(\infty,t) \to 0\,. \end{array} \tag{7}$$

where $\beta_1 = \frac{\beta^* v}{U_0^2}$, $Gr = \frac{v g\beta(T'_\omega - T'_\infty)}{U_o^3}$, $\Pr = \frac{v\rho c_p}{k}$ are the Brinkman fluid parameter, Grashof number, and Prandtl number.

3. Solution of the problem

Next, applying the Laplace transforms on (5) and (6) yields the dimensionless governing equations in the transformed (y,s) plane. After substituting initial and boundary conditions (7) and having some manipulations, the following solutions are obtained as

$$\bar{\theta}(y,s) = \frac{1}{s}e^{-y\sqrt{s\Pr}} \tag{8}$$

$$\begin{aligned} \bar{u}(y,s) = {} & \frac{1}{2(s-i\omega)}e^{-y\sqrt{s+\beta_1}} + \frac{1}{2(s+i\omega)}e^{-y\sqrt{s+\beta_1}} - \frac{\beta_1}{s}e^{-y\sqrt{s+\beta_1}} \\ & + \frac{\beta_3}{s+\beta_2}e^{-y\sqrt{s+\beta_1}} + \frac{\beta_3}{s}e^{-y\sqrt{s\Pr}} - \frac{\beta_3}{s+\beta_2}e^{-y\sqrt{s\Pr}}\,. \end{aligned} \tag{9}$$

Then, (8) and (9) are imposed with the inverse Laplace and transforms to following expression

$$\theta(y,t) = \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) \tag{10}$$

$$u(y,t)=u_1(y,t)+u_2(y,t)-u_3(y,t)+u_4(y,t)+u_5(y,t)-u_6(y,t) \tag{11}$$

where

$$\begin{aligned} u_1(y,t)=&\frac{1}{4}H(t)e^{i\omega t+y\sqrt{\beta_1+i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(\beta_1+i\omega)t}\right)\\ &+\frac{1}{4}H(t)e^{i\omega t-y\sqrt{\beta_1+i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(\beta_1+i\omega)t}\right), \end{aligned} \tag{12}$$

$$\begin{aligned} u_2(y,t)=&\frac{1}{4}H(t)e^{-i\omega t+y\sqrt{\beta_1-i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(\beta_1-i\omega)t}\right)\\ &+\frac{1}{4}H(t)e^{-i\omega t-y\sqrt{\beta_1-i\omega}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(\beta_1-i\omega)t}\right), \end{aligned} \tag{13}$$

$$u_3(y,t)=\frac{\beta_3}{2}e^{y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\beta_1 t}\right)+\frac{\beta_3}{2}e^{-y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{\beta_1 t}\right), \tag{14}$$

$$\begin{aligned} u_4(y,t)=&\frac{\beta_3}{2}e^{-\beta_2 t+y\sqrt{\beta_1-\beta_2}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(\beta_1-\beta_2)t}\right)\\ &+\frac{\beta_3}{2}e^{-\beta_2 t-\sqrt{\beta_1-\beta_2}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(\beta_1-\beta_2)t}\right), \end{aligned} \tag{15}$$

$$u_5(y,t)=\beta_3\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}\right), \tag{16}$$

$$\begin{aligned} u_6(y,t)=&\frac{\beta_3}{2}e^{-\beta_2 t+y(i\sqrt{\beta_2\Pr}}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}+i\sqrt{\beta_2 t}\right)\\ &+\frac{\beta_3}{2}e^{-\beta_2 t-y(i\sqrt{\beta_2\Pr})}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}-i\sqrt{\beta_2 t}\right). \end{aligned} \tag{17}$$

It should be noted that the aforementioned velocity solutions are only applicable for $\Pr \neq 1$. Therefore, when $\Pr = 1$, the solutions can be acquired by substituting $\Pr = 1$ into (8) and the same step as discussed above are followed. The obtained solutions are

$$\bar{u}(y,t)=u_1(y,t)+u_2(y,t)+u_7(y,t)+u_8(y,t) \tag{18}$$

where

$$u_7(y,t)=\frac{\beta_4}{2}e^{y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\beta_1 t}\right)+\frac{\beta_4}{2}e^{-y\sqrt{\beta_1}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{\beta_1 t}\right) \tag{19}$$

$$u_8(y,t)=\beta_4\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) \tag{20}$$

with $\beta_2 = \frac{Gr}{\beta_1}$, $\beta_3 = \frac{Gr}{\beta_1}$ and $\beta_4 = \frac{Gr}{\beta_1}$ are constant parameters.

4. Results and Discussions

In this section, the impacts of the embedded parameters such as Prandtl number Pr, Grashof number Gr, Brinkman fluid parameter β_1, phase angle ωt, and time t on the temperature and velocity distributions are displayed using graphs. The obtained solutions are analyzed numerically by using several values of parameters. All the plotted graphs are displayed to analyze the velocity and temperature profiles. Figure 1 to Figure 5 present the physical behavior of the velocity, whereas Figure 6 and Figure 7 illustrate the temperature profiles. The results obtained here satisfied all the initial and boundary conditions (7). Figure 1 shows the behavior of velocity towards time change. It has been observed that as the value of t increases, the velocity also increases. As the time increases, the buoyancy force effectively dominates the flow due to receiving energy from an external source and eventually causes the velocity to increase. Figure 2 displays the velocity distribution in response to the enlargement of *Gr*. It is clear that as the value of *Gr* increases, the velocity increases. Physically, *Gr* is represented as ratio between buoyancy force to viscous force. Hence, the buoyancy force takes precedence during the free convection process and cause an increment in *Gr*, which subsequently accelerating the velocity profile. The effect of Pr on velocity propagation is demonstrated in Figure 3. In this study, different physical values for Pr have been selected which are $\mathrm{Pr} = 1.50, \mathrm{Pr} = 5.00$ (light organic fluid), $\mathrm{Pr} = 6.20$ (water) and $\mathrm{Pr} = 7.2$ (sea water). As clearly displayed, the velocity decreases as Pr is increased. Prandtl number Pr is defined as the ratio of kinematic viscosity to thermal diffusivity. Thus, increasing Pr values imply to the increase of kinematic viscosity but reduce the thermal diffusivity of the fluid. As a result, increase in kinematic viscosity leads the decrement of fluid velocity. In Figure 4, the graph illustrates the effect of Brinkman fluid parameter, β_1 on the velocity profile. The findings show that increasing value of Brinkman fluid parameter has dropped the velocity field. Moreover, in Figure 5, it has shown the velocity decreases when ωt is increased. Clearly, this figure satisfies the boundary conditions (7) in view of boundary conditions (3), hence the accuracy of the obtained solutions is verified. Figure 6 shows that the temperature profile increases when value of t is ascended but decreases in responses to the increase values of Pr (Figure 7). The curve of the temperature profile declines more slowly as the value of t increases but decline more rapidly as the value of Pr increases.

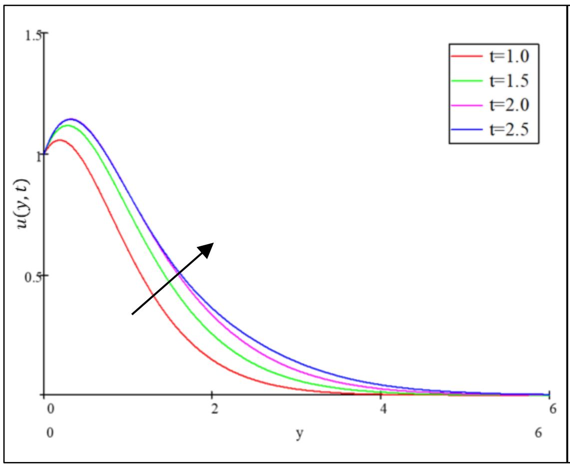

Figure 1: Velocity profiles for different values of t with $\beta_1 = 0.8, \mathrm{Pr} = 5.0,\ Gr = 5.0$ and $\omega = 0$

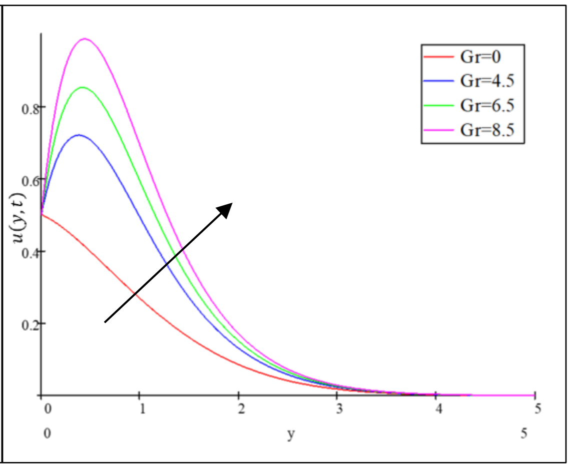

Figure 2: Velocity profiles for different values of Gr with $\beta_1 = 0.8,\ \mathrm{Pr} = 5.0,\ t = 1.0$ and $\omega = \frac{\pi}{3}$

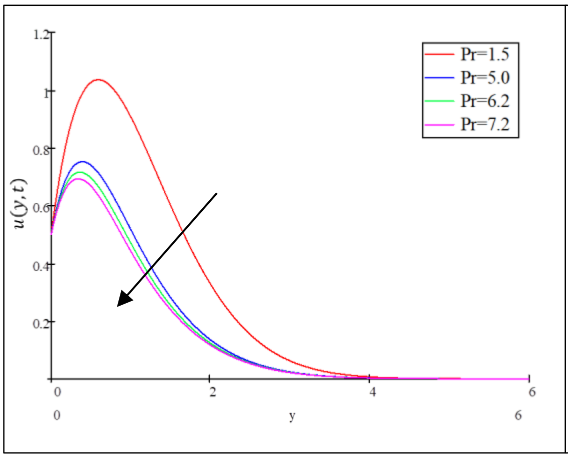

Figure 3: Velocity profiles for different values of Pr with $\beta_1 = 0.8,\ Gr = 5.0,\ t = 1.0$ and $\omega = \frac{\pi}{3}$

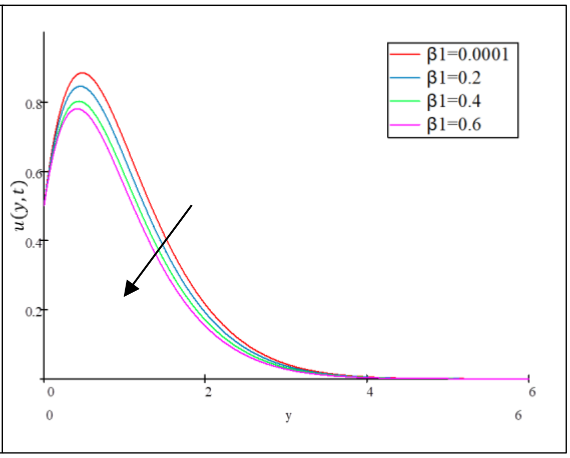

Figure 4: Velocity profiles for different values of β_1 with $\mathrm{Pr} = 5.0,\ Gr = 5.0,\ t = 1.0$ and $\omega = \frac{\pi}{3}$

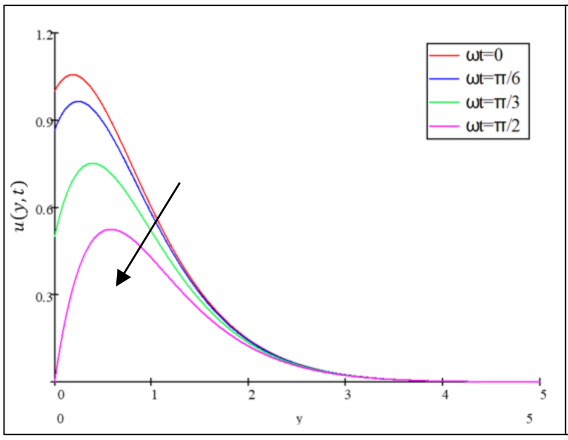

Figure 5: Velocity profiles for different values of ωt with $\beta_1 = 0.8, \mathrm{Pr} = 5.0,\ Gr = 5.0$ and $t = 1.0$

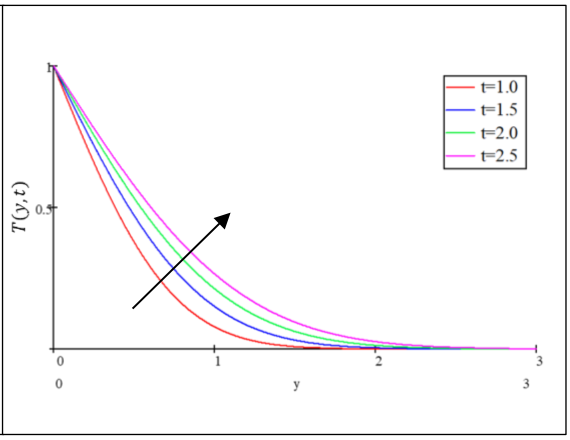

Figure 6: Temperature profiles for different values of t with $\mathrm{Pr} = 6.2$

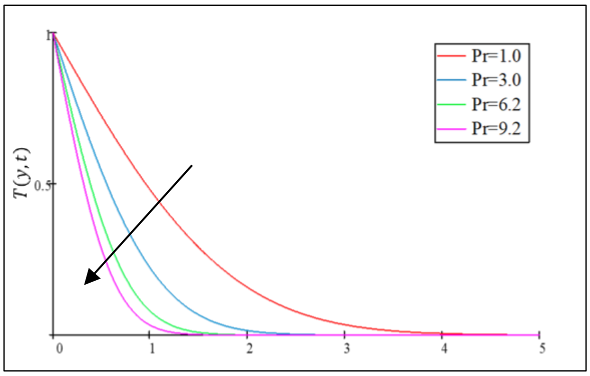

Figure 7: Temperature profiles for different values of Pr with $t = 1.0$

5. Conclusion

In this study, there are two governing dimensional equations which are momentum and energy equations. The governing equations are then reduced into dimensionless form by using suitable dimensionless variables. Here, the temperature and velocity solutions are acquired by solving the dimensionless governing equations using the Laplace transform method with the consideration of boundary and initial conditions. The resulting solutions are then plotted into several graphs by using MATHCAD software. By differentiate the values of various variables, the impact of the parameters towards the velocity and temperature distributions are observed. The significant findings in this study are the velocity amplifies with the increase of t and Gr but reduces with the increase of Pr, ωt and β_1. While temperature increases with t but it drops with ascending value of Pr.

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