Quest for Research Excellence On Computing, Mathematics and Statistics

Editors

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Faculty of Computer and Mathematical Sciences

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Chapters in Book

The 2nd International Conference on Computing, Mathematics and Statistics (iCMS2015)

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Content

CHAPTER 20
The Combined EWMA-CUSUM Control Chart with Autocorrelation

Abbas Umar Farouk, and Ismail Bin Mohamad

Abstract. It is well known that statistical control charts such as Shewhart, CUSUM and EWMA charts have found widespread application in improving the quality for manufacturing and service processes. Control charts process monitoring have traditionally been designed and evaluated under assumption that observations on the process output at different times are independent. However, autocorrelation may be present in many processes, and may have a strong impact on the properties of control charts. Due to high sophisticated computer programs and soft ware's that are used in nowadays production processes the output of the production processes used to be very high and large in quantities which give rises to serial correlation among the output, hence the need to check for the effect of the autocorrelation in the production processes. The combined EWMA and CUSUM control chart is an improved technique proposed to check the excess of the autocorrelation in the production processes so that the effect can be minimized. A source code has been developed. A result of 10000 simulations was recorded for the ARL values. The performance of the proposed chart was evaluated in terms of average run length (ARL), the standard deviation of the run length as well as the percentile of the run length. Comparison of these charts with some existing control charts designed for monitoring small and large shifts revealed that the proposed chart is more sensitive and offer better shift detection than the others considered in this study.

Keywords: autocorrelation; CUSUM; EWMA; average run length; standard deviation;percentiles

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1 Introduction

Statistical process control (SPC) is a collection of powerful tools that are useful in maintaining and improving process performance through the reduction of variability [15]. This is done by collecting, organizing, analyzing and interpreting data so that the process can be maintained at its present level or improved to a higher level of quality. SPC is a strategy that can be applied to any process to reduce variation and it contains tools like histograms, check sheets, Pareto charts, cause and effect diagrams, defect concentration diagrams, scatter diagrams and control charts. This collection is formally known as the SPC tool kit. The control chart is the most important device that helps to determine if a process is in-control.

The design of control charts is based on first identifying the distribution of the process characteristics followed by monitoring the stability of its parameters. In general, a control chart is a trend chart with three additional lines: the centre line (CL) the upper control limit (*UCL*) and the Lower control limit (*LCL*). These limits are chosen such that almost all of the data will lie between these limits as long as the process remains statistically in-control. In this paper, we study the control chart for the location parameter. There are three major categories of charts to monitor processes, namely Shewhart-type control charts, cumulative sum (CUSUM) charts and exponentially weighted moving average (EWMA) charts. The Shewhart-type control charts are quite good in detecting large shifts, whereas CUSUM and EWMA charts are effective for smaller shifts in the parameters of interest (generally location and spread parameters).

When the presence of autocorrelation is detected in a process there is a question of which of the best method to use in monitoring the process? As an alternative to standard control charts which plot the original observations, several authors (see, for example, [1] and [6] have proposed control charts based on plotting the residuals from the forecasts of a time series model fitted to the original observations. Control charts based on residuals seem to work best when the level of correlation is high. When the level of correlation is low or moderate, forecasting is more difficult and residual charts are not particularly effective at detecting process changes. For evaluations of residual charts see, for example, [3], [11], [9], and [4]. In general, it appears that the best method for monitoring autocorrelated observations will depend on the specific circumstances of the problem such as the model assumed and the objectives of the monitoring. For many problems "optimal" methods for monitoring the process have not been developed. Thus it seems that there will be many applications for which it will be desirable from both practical and statistical considerations to apply a standard control chart to the original autocorrelated observations. This is particularly true for applications in which the level of autocorrelation is not high (see [10]. Then for these applications it will be necessary to determine the effect of the autocorrelation and how to adjust the parameters of the chart to account for the autocorrelation.

The objective of this paper is to examine the properties of exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts when the observations from the process are autocorrelated according to a relatively simple model, namely "the combine EWMA-CUSUM (CECUSUM) control chart" . In this model observations are modeled using a first order autoregressive (AR(1)) model with an additional random error which can correspond to sampling or measurement error. This model allows relatively accurate numerical techniques to be used to evaluate properties of the control charts. In particular, the methodology is developed for using the Markov chain method for Markov processes to evaluate the average run length for the EWMA and CUSUM charts when the process is in control and when there has been a step shift in the overall mean of the process. Numerical results are presented for levels of autocorrelation that range from low to moderately high. These results show that autocorrelation in the form of the assumed model has a strong effect on the ARL of these charts even when the correct variance is used to determine the control limits. Tables of in-control and out-of-control ARL values are also given for use in designing the control charts for application to processes which can be adequately modeled by the model used in this study. Our paper is structured as follows in section 2, we review the classical EWMA control chart and CUSUM control chart, in section 3 explains the autocorrelation techniques and how to design CECUSUM chart with autocorrelation. The run length (RL) distributions of the proposed charts are given in section 4. In section 5, we compared some of the existing charts with the proposed scheme. Finally, we summarize and conclude our findings in section 6.

2 A Model for Autocorrelated Process Observations

The model to be used for autocorrelated observations from a process will be described as follows: even though the model has been used severally by some authors, which is assumed to have advantages of accounting for the autocorrelation between samples that are close together in time, for variability in the process mean over time.

Let the samples of size $n \geq 1$ and X_n represent the ith observation from the process at time t, t=1,2,…

Assumed X_n can be written as

$$X_n = \mu_t + \varepsilon_n \tag{1}$$

Where
ε_t is a random error and μ_t is a random variable which in many application, can be considered to be the mean of the proposed time t.

The random error ε_n is assumed to be normally distributed with mean zero and variance σ_t^2 and independent of all other random errors. Hence, to detect the change in the overall mean $\xi = E(\mu_i)$. If μ_1, μ_2,.. is an AR(!) process, then μ_t can be written in terms of μ_{t-1} as

$$\mu_t = \xi(1-\phi) + \phi\mu_{t-1} + \alpha_t \tag{2}$$

Where ϕ is the correlation between μ_t and μ_{t-1} and α_t is a random noise which is normally distributed with mean zero and variance σ_n^2 and is independent of all random errors and of the random noise at any other time. It will be assumed that $|\phi| < 1$ so that the AR(1) process is stationary

The mean of an individual observation X_n is $E(X_n) = E(\mu_t) = \zeta$, and the variance, say σ_x^2, is

$$\sigma_x^2 = \sigma_\mu^2 + \sigma_\varepsilon^2 = \frac{\sigma_\alpha^2}{1-\phi^2} + \sigma_\varepsilon^2 \tag{3}$$

Where $n > 1$ most control charts for monitoring the mean use the sample mean to form a control statistic.

Let $\bar{X}_t = \sum_{t=1}^{n} X_n/n$ be the sample mean for the sample at time t, then

$$\bar{X}_t = \mu_t + \bar{\varepsilon}_t \tag{4}$$

Where $\bar{\varepsilon}_t = \sum_{t=1}^{n} \varepsilon_n/n$. Then it is easy to show that the variance of $\bar{X}_t$ say $\sigma_{\bar{x}}^2$ is

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{1-\phi^2} + \frac{\sigma_\varepsilon^2}{n} \tag{5}$$

And the covariance of two adjacent sample mean is

$$cov(\bar{X}_{t-1}, \bar{X}_t) = \phi\sigma_\mu^2 \tag{6}$$

The parameter ψ is defined by

$$\psi = \frac{\sigma_\mu^2}{\sigma_{\bar{x}}^2} \tag{7}$$

The correlation between $\bar{X}_{t-1}$ and $\bar{X}_t$ is simply $\phi\psi$, also note that when $\psi > 0$, $\sigma_{\bar{x}}^2$ does not converge to zero as $n \to \infty$.

The model given by (1) and (2) is a possible model for processes in which the variability of observations has both short and long term components.

2.1 The CUSUM Control Chart

CUSUM charts were introduced by [7] to effectively address small parameter shifts, Shewhart-type control chart effectively detect large shifts, whereas the smaller shifts CUSUM and EWMA procedures were of importance. The CUSUM procedures work by accumulating the deviations up and down from a target value (in general the mean of the in-control situation) for which the notation C^+ and C^- are used respectively.

Initially, C^+ and C^- are set to zero.

The CUSUM statistic is defined as

$$C^+ = \max[0, X_i - (\mu_0 + k) + C_{i-1}^+]$$
$$C^+ = \max[0, -X_i - (\mu_0 - k) + C_{i-1}^-] \tag{8}$$

where X_i Denote the ith observations, μ_0 is the target value (mean) and k is known as the reference value which is chosen about half of the shift (in standard unit) to detect shift; $k = \delta/2$, where δ-equals the shift in standard units. Once the values of

2.2 Classical EWMA Control Chart

Consider the EWMA control chart proposed by Robert, the EWMA statistic is defined as

$$Z_j = \lambda X_j + (-\lambda)Z_{j-1} \tag{9}$$

Where Z_{j-1} is the past information and $\boldsymbol{\lambda}$ is a smoothing constant 0<$\boldsymbol{\lambda}$<1. The starting value of Z_j denoted by Z_0 is set equal to the target mean μ_0 when the process is under control. As the X_{ij} are independent and identically normally distributed, the variance of Z_j is given by

$$var(Z_j) = \left(\frac{\sigma^2}{n}\right)\left(\frac{\lambda}{2-\lambda}\right)\{1-(1-\lambda)^{2j}\} \tag{10}$$

As j gets larger, the term (1-(1-$\lambda)^{2j}$) approaches unity, and hence the asymptotic value for the variances of EWMA statistic is given to be

$$var(Z_j) = \left(\frac{\sigma^2}{n}\right)(\frac{\lambda}{2-\lambda}) \tag{11}$$

Suppose that the chart parameters are known or estimated from preliminary in-control process data, thus, the combine EWMA-CUSUM control chart signals when either the statistic Z_j falls outside the CUSUM or the statistics Z_i falls outside the EWMA control chart limits

$$LCL = \mu_0 - L\frac{\sigma}{\sqrt{n}}\left(\sqrt{\frac{\lambda}{2-\lambda}}\right) \text{ And } UCL = \mu_0 + L\frac{\sigma}{\sqrt{n}}\left(\sqrt{\frac{\lambda}{2-\lambda}}\right) \tag{12}$$

Where LCL and UCL are the lower and upper control limit, respectively, both the CUSUM and EWMA control limits are usually cantered on the target value, centre line CL = μ_0.

The values of the design parameters K and L are chosen based on the choice of the smoothing constant λ to satisfy the required process control needs.

To measure how quickly a control chart respond to shifts in a process, run length (RL) properties are studied, of which average run length (ARL) and the standard deviation of RL (SDRL) distribution are often used ARL and SDRL which represent the average number and standard deviation of samples plotted within the control limits before in out-of-control notice is signaled.

3 The Proposed Scheme for the CECUSUM Chart

To improve the performance of a CECUSUM chart, so that it is more sensitive to both small and large shifts in the process mean with autocorrelation, we proposed using a CECUSUM control chart to monitor the process with autocorrelated data. The scheme has in archived in increasing the sensitivity of EWMA and CUSUM control charts. For example, [12], where they proposed a combined Shewhart. (EWMA) control chart for monitoring mean output using rank set sampling (RSS) instead of ordinary simple random sampling and the references therein.

The ARL of the CECUSUM chart may be computed numerically using an integral equation approximation to Markov chain, or by a Monte Carlo simulation. We follow a Monte Carlo simulation approach via an algorithm developed in R package.

4 Measuring the Performance of the Proposed Scheme

To evaluate the performance of the proposed scheme, we used Monte Carlo simulation and assumed that an in-control process follows a normal distribution with mean zero and variance one (without loss of generality). For each shift in mean δ, with different combinations of K, L and λ values of 0.1, 0.5 and 0.9 for better comparison with some of the existing schemes, we set in-control ARL_0 values for the control chart at 200, 500 and 1000. The computed ARL values are displayed in Tables 1-3.

The behavior of the RL distribution of the proposed CECUSUM control charts have been computed, the standard deviation and percentile points. Some parameters were used for ARLs. The results are displayed on Tables 4-6. Also provides estimates for the 10^{th}, 50^{th} and 90^{th} percentiles points of the RL distribution when the process is in-control at ARL_0=500, which were shown on Tables 7-9 with AR =0.1, 0.5 and 0.9, K=2.72.

Table 5. ARL values for the CECUSUM scheme with h=0.5, ARL_0=200 with AR=0.1.

δ	**λ**=0.1 K=21.3	**λ**=0.25 K=13.29	**λ**=0.5 K=8.42	**λ**=0.75 K=5.48
0.00	199.8505	117.2025	200.0675	199.478
0.25	58.7075	56.0777	50.7829	60.3214
.5	21.155	22.0145	20.4231	20.5621
0.75	10.6885	10.98	10.7681	10.7678
1.00	6.305	6.54	6.7891	7.6811
1.5	4.3785	4.5895	4.3678	3.7621
2.00	3.4385	3.4255	3.4341	3.4261

Table 6. ARL values for the CECUSUM scheme with h=0.5, ARL_0=500 with AR=0.5.

δ	**λ**=0.1 K=21.3	**λ**=0.25 K=13.29	**λ**=0.5 K=8.42	**λ**=0.75 K=5.48
0.00	499.3981	500.6781	500.6812	500.7681
0.25	79.5631	80.7621	99.7634	100.6813
0.5	30.4213	39.6787	28.7671	30.5671
0.75	20.6781	16.7651	14.8691	14.5167
1.00	16.4381	10.6781	10.4321	8.6781
1.5	10.3678	7.3671	5.3211	5.6781
2.00	9.7641	5.3421	5.0101	3.1678

Table 7. ARL values for the CECUSUM scheme with h=0.5, ARL_0=1000 with AR=0.9.

δ	**λ**=0.1 K=21.3	**λ**=0.25 K=13.29	**λ**=0.5 K=8.42	**λ**=0.75 K=5.48
0.00	1000.7071	999.6891	1000.6281	1001.8176
0.25	876.3481	738.1413	814.7681	740.7681
0.5	581.7814	431.6781	634.7641	674.6814
0.75	381.6781	281.8761	324.7816	245.6781
1.00	132.7814	120.7861	110.8476	95.6321
1.5	80.9671	75.2478	84.7681	32.6781
2.00	5.6781	10.7892	24.7681	10.7678

The standard errors are provided in Tables 1-3 and also calculated and found to be less than 1%. More so, the ARL results of the classical CUSUM and classical EWMA, using our simulation algorithm and found almost similar results as by [13] and [14] respectively, ensuring the validity of the simulation algorithm used.

The findings of the combine EWMA=CUSUM with autocorrelation for monitoring the mean of a normally distributed process yields the results:

i. The combine EWMA-CUSUM with autocorrelation improves the performance in quick shift detection.
ii. The detection of small shifts in the process, gives better performance of the proposed scheme than with smaller values of **λ** (smoothing constant).
iii. The proposed scheme's ARL unbiased, for fixed values of ARL_0, the ARL_1 decreases with a decrease in the values of **λ**.

Table 4. SDRL values for the CECUSUM with autocorrelation AR=0.1, ARL=500, K=2.67.

δ	λ=0.05 L=2.841	λ=0.1 L=2.963	λ=0.25 L=3.078	λ=0.5 L=3.172
0	500.6782	498.7621	499.7681	500.7660
0.25	25.3679	40.0563	86.9611	100.8111
0.5	10.4342	7.8913	13.4678	29.6781
0.75	4.7632	7.8913	3.4781	8.6000
1.0	1.6781	3.3431	2.2671	3.5341
1.5	1.3621	2.1031	1.0026	1.1678
2.0	1.0621	0.6381	0.5362	0.5881
3.0	0.1342	0.1214	0.1478	0.1311

Table 5. SDRL values for the CECUSUM with autocorrelation AR=0.5, ARL=500, K=2.67.

δ	λ=0.05 L=2.841	λ=0.1 L=2.963	λ=0.25 L=3.078	λ=0.5 L=3.172
0	500.2496	499.7672	498.7621	500.3224
0.25	20.5231	31.7681	55.4271	110.0421
0.5	4.7052	5.4871	9.6781	20.7512
0.75	2.9421	2.6781	3.5251	5.3681
1.0	1.8724	1.6531	1.4781	2.4987
1.5	0.7456	1.1211	0.9921	0.8621
2.0	0.5776	0.5321	0.3001	0.4381
3.0	0.1342	0.1627	0.1113	0.1821

Table 6. SDRL values for the CECUSUM with autocorrelation AR=0.9, ARL=500, K=2.71.

δ	λ=0.05 L=2.841	λ=0.1 L=2.963	λ=0.25 L=3.078	λ=0.5 L=3.172
0	497.2490	500.4170	499.3416	499.7621
0.25	20.5081	27.4781	50.7481	110.0341
0.5	3.4001	5.2671	9.4583	19.7431
0.75	2.4201	2.5781	3.4251	5.3671
1.0	1.4210	1.5171	1.7621	2.4310
1.5	0.8932	0.5629	0.9322	0.7314
2.0	0.5030	0.3421	0.5321	0.4314
3.0	0.0070	0.2710	0.0017	0.1035

Table 7. Percentile Points for CECUSUM with autocorrelation AR=0.1, ARL=500, K=2.72.

δ	Pi	0.0	0.25	0.5	0.75	1.0	1.5	2.0	3.0
0.05	P_{10}	53	13	6	4	2	1	1	1
	P_{50}	271	29	18	6	4	2	1	1
	P_{90}	1020	55	18	10	6	4	2	1
0.1	P_{10}	46	10	6	4	2	1	1	1
	P_{50}	320	30	10	5	4	1	2	1
	P_{90}	1080	75	19	8	6	3	1	1
0.25	P_{10}	50	10	4	2	1	1	1	1
	P_{50}	245	45	10	4	3	1	1	1
	P_{90}	1050	150	25	9	5	2	1	1
0.5	P_{10}	49	12	3	2	1	1	1	1
	P_{50}	335	72	15	4	3	1	1	1
	P_{90}	1120	250	45	15	1	2	1	1

Table 8. Percentile Points for CECUSUM with autocorrelation AR=0.5, ARL=500, K=2.72.

δ	Pi	0.0	0.25	0.5	0.75	1.0	1.5	2.0	3.0
0.05	P_{10}	50	12	5	4	3	1	1	1
	P_{50}	250	25	10	5	3	1	1	1
	P_{90}	1015	50	14	10	5	3	1	1
0.1	P_{10}	43	10	5	3	1	1	1	1
	P_{50}	300	25	8	3	3	1	1	1
	P_{90}	1000	70	18	5	5	2	1	1
0.25	P_{10}	49	8	3	1	3	1	1	1
	P_{50}	240	42	9	3	2	1	1	1
	P_{90}	1000	145	20	8	4	1	1	1
0.5	P_{10}	45	2	1	1	1	1	1	1
	P_{50}	330	14	3	2	1	1	1	1
	P_{90}	1115	43	14	5	2	1	1	1

Table 9. Percentile Points for CECUSUM with autocorrelation AR=0.9, ARL=500, K=2.72.

δ	Pi	0.0	0.25	0.5	0.75	1.0	1.5	2.0	3.0
0.05	P_{10}	49	10	4	3	1	1	1	1
	P_{50}	250	25	10	5	3	2	1	1
	P_{90}	1015	50	16	8	5	3	1	1
0.1	P_{10}	45	8	5	3	1	1	1	1
	P_{50}	300	25	8	4	3	3	1	1
	P_{90}	1060	70	17	7	5	5	2	1
0.25	P_{10}	45	8	3	1	1	1	1	1
	P_{50}	240	40	8	3	2	1	1	1
	P_{90}	1000	100	27	8	4	1	1	1
0.5	P_{10}	45	10	2	2	1	1	1	1
	P_{50}	330	70	10	3	2	1	1	1
	P_{90}	1010	245	40	10	5	1	1	1

Based on the results presented on Tables 4-9, the main findings are summarized below:

i. The proposed methods are quick at detecting small moderate and large shifts in the process mean.

ii. It is noted that when the process is in control the SDRL values for all the methods are almost the same with the ARL values.

iii. The percentile points of the RL distribution shows that the proposed control charts have RL distributions that are positively skewed.

5 Comparative Studies

In this section, we compared the ARL performance of the proposed control charts with some of the existing control charts used for quick detection of shifts in the process mean when the distribution is normal. The classical EWAM and CUSUM were used for the comparison. The results of the comparison were shown in Tables 10-12 below.

5.1 Comparison with Classical CUSUM and EWMA

Table 10. ARL values for the classical CUSUM, EWMA and CECUSUM with ARL=200, AR=0.1.

	CUSUM	EWMA		CECUSUM	
		λ=0.1	λ=0.4	λ=0.1	λ=0.4
δ	K=3.00	L=2.75	L=2.86	L=2.84	L=3.28
0	200	200.41	200.09	199.23	200.47
0.5	90.25	110.46	80.34	110.42	85.76
1.00	35.47	90.37	57.68	83.67	50.47
2.00	10.52	45.72	20.67	10.57	10.37
3.00	5.32	4.47	4.35	3.96	3.56

Table 11. ARL values for the classical CUSUM, EWMA and CECUSUM with ARL=200, AR=0.5.

	CUSUM	EWMA		CECUSUM	
		λ=0.1	λ=0.4	λ=0.1	λ=0.4
δ	K=3.00	L=2.75	L=2.86	L=2.84	L=3.28
0	200	200.41	200.09	199.23	200.47
0.5	90.25	110.46	80.34	110.42	85.76
1.00	35.47	90.37	57.68	83.67	50.47
2.00	10.52	45.72	20.67	10.57	10.37
3.00	5.32	4.47	4.35	3.96	3.56

Table 12. ARL values for the Classical CUSUM, EWMA and CECUSUM with ARL=200, AR=0.9.

	CUSUM	EWMA		CECUSUM	
		λ=0.1	λ=0.4	λ=0.1	λ=0.4
δ	K=3.00	L=2.75	L=2.86	L=2.84	L=3.28
0	200	200.41	200.09	199.23	200.47
0.5	90.25	110.46	80.34	110.42	85.76
1.00	35.47	90.37	57.68	83.67	50.47
2.00	10.52	45.72	20.67	10.57	10.37
3.00	5.32	4.47	4.35	3.96	3.56

6 Summary and Conclusion

Statistical process control (SPC) is a combination of many useful statistics that help to distinguish the variations in a process. Out of these techniques the control chart is the most important and commonly used tool. For small shifts, CUSUM charts and EWMA charts are considered most effective. We have proposed a scheme namely, the combine EWMA-CUSUM control chart with autocorrelation, this is due to the effect of autocorrelation to the process mean. The proposed mean can detect shift quickly as it occurs. By investigating the performance of the proposed scheme and by comparing it with the existing schemes, such as the classical EWMA and CUSUM, we found that the proposed scheme has the ability to perform better for small and moderate shifts. We therefore recommend the use of the propose scheme to be used whenever the data set is suspected to be autocorrelated for quick detection of shifts at different levels of autocorrelation for prompt attention.

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