Quest for Research Excellence On Computing, Mathematics and Statistics

Editors
Kor Liew Kee
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Faculty of Computer and Mathematical Sciences

Quest for Research Excellence on Computing, Mathematics and Statistics

Chapters in Book

The 2nd International Conference on Computing, Mathematics and Statistics (iCMS2015)

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CHAPTER 18
Absolute Translativity of Generalized Nörlund Mean

Amjed Zraiqat

Abstract. In this paper, two results involving left and right translativity for absolute generalized Nörlund mean $|(N,p,q)|$ are established. Two interesting non-trivial examples to show that $|(N,p,q)|$ may have only one-side translative being constructed, one non-trivial example to show that $|(N,p,q)|$ may be translative is given, and one non-trivial example to show that $|(N,p,q)|$ is non-translative from the left nor do from the right is constructed.

Keywords: Nörlund mean; translativity, one-side translative; and sequence-to-sequence transformation.

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1 Introduction

Let $A = (a_{n,k})$ be a sequence-to-sequence transformation, then A is said to be regular, if whenever $\{S_n\}$ has a bounded variation, it follows that $\{t_n\}$ has a bounded variation and does not alter the sum.
Let $p = \{p_n\}, q = \{q_n\}$ be two given sequences of real numbers. Let $q_n \neq 0 \ (\text{all } n \geq 0)$. The generalized Nörlund method (N,p,q) which was first considered by D. Borwin [5] is defined as the sequence-to-sequence transformation

$$t_n = \sum_{k=0}^{n} a_{n,k} S_k \ , \qquad \textbf{(1)}$$

where

$$a_{n,k} = \frac{1}{R_n} p_{n-k} q_k \ ,$$

$$R_n = p_n q_0 + p_{n-1} q_1 + \ldots + p_0 q_n \neq 0 \ ; \ \ (\text{all } n \geq 0).$$

It follows from Toeplitz's Theorem (Hardy 1949; Theorem (2)) that (N, p, q) is regular if, and only if

$$\sum_{k=0}^{n} |a_{n,k}| = O(1) \ , \qquad (2)$$

and

$$a_{n,k} \to 0 \ \text{ as } n \to \infty \ \text{ for each fixed k.} \qquad (3)$$

The following important result due to Mears [10] is required for our purposes:

<u>Theorem</u> (1.1) (Mears [10])
A is absolutely regular if, and only if

$$\sum_{k=0}^{\infty} a_{n,k} \ \text{ converges for every n,} \qquad (4)$$

$$a_{n,k} \to 0 \text{ as n} \to \infty \text{ for each fixed k,} \tag{5}$$

$$\sum_{k=0}^{\infty} a_{n,k} \to 1 \text{ as n} \to \infty \text{,} \tag{6}$$

and

$$\sum_{n=0}^{\infty} \left| \sum_{\mu=K}^{\infty} a_{n,\mu} - \sum_{\mu=k}^{\infty} a_{n+1,\mu} \right| = O(1) \text{.} \tag{7}$$

We define the sequence of constants $\{c_n\}$ formally by means of the identity

$$\left(\sum_{n=0}^{\infty} r_n z^n \right)^{-1} = \sum_{n-0}^{\infty} c_n z^n \text{ ; } c_{-n} = 0 \text{.} \tag{8}$$

A is called Translative <u>to</u> <u>the</u> <u>left</u> (written) $A \in T_L$ if the limitability of $\{S_j\}_{j=0}^{\infty}$ implies the limitability of $\{S_{j-1}\}_{j=0}^{\infty}$ with $S_{-1} = 0$.

$A \in T_R$ if the converse holds. $A \in T$ if, and only if $A \in T_L$ and $A \in T_R$.
On translativity of summability methods much work has been done already see [1]-[4], [6], [8] and [9].

2 Object of The Paper

The object of this paper is to obtain results for translativity of $|(N,p,q)|$ and to give two interesting examples to show that $|(N,p,q)|$ may belong to T_L or T_R without the other, and to give a non-trivial example to show that $|(N,p,q)|$ may be translative. Finally, an example to show that $|(N,p,q)| \notin T_L \cup T_R$ is given. These results will be included in sections (4) and (5).

3 Results

In this section we state and prove our two main results:

<u>**Theorem (3.1)**</u> Let $q_n \neq 0$; all $n \geq 0$, and let (N,p,q) be regular, then $|(N,p,q)| \in T_L$ if, and only if

$$M_{n,v} \to 0 \text{ as } n \to \infty \text{ , for each fixed v,} \tag{9}$$

and

$$\sum_{n=\mu-1}^{\infty}\left|\Delta\frac{p_{n+1}}{R_{n+1}}.\sum_{v=0}^{\mu-1}R_v\sum_{k=v}^{n+1}\frac{q_{k+1}}{q_k}c_{k-v}\Delta\frac{p_{n-k}}{R_{n+1}}\right| = O(1), \tag{10}$$

where

$$M_{n,v} = \frac{R_v}{R_{n+1}}\sum_{k=v}^{n}\frac{q_{k+1}}{q_k}p_{n-k}c_{k-v} \quad , \quad 0 \le v \le n, \tag{11}$$

$$=0 \quad \text{otherwise.} \tag{12}$$

Further, if $\{c_n\}$ is bounded and $\{q_n\}$ is strictly increasing, then condition (10) alone is necessary and sufficient.

<u>Theorem (3.2)</u> Let $q_n \neq 0;$ all $n \ge 0$, and let (N,p,q) be regular, then $|(N,p,q)| \in T_R$ if, and only if

$$H_{n,v} \to 0 \text{ as } n \to \infty \text{ , for each fixed v,} \tag{13}$$

and

$$\sum_{n=\mu-1}^{\infty}\left|.\sum_{v=0}^{\mu-1}R_{v+1}\sum_{k=v}^{n+1}\frac{q_k}{q_{k+1}}c_{k-v}\Delta\frac{p_{n-k}}{R_{n+1}}\right| = O(1), \tag{14}$$

where

$$H_{n,v} = \frac{R_{v+1}}{R_n}\sum_{k=v}^{n}\frac{q_k}{q_{k+1}}p_{n-k}c_{k-v} \quad , \quad 0 \le v \le n, \tag{15}$$

$$=0 \quad \text{otherwise.} \tag{16}$$

Further, if $\{c_n\}$ is bounded and $\{q_n\}$ is strictly increasing, then (14) alone is necessary and sufficient.

Proof of Theorem (3.1)

Let $\{t_n\}$ and $\{\bar{t}_n\}$ be respectively the (N,p,q) transform of $\{S_n\}$ and $\{S_{n-1}\}$, then

$$t_n = \frac{1}{R_n}\sum_{k=0}^{n} p_{n-k}q_k S_k, \tag{17}$$

so

$$\bar{t}_{n+1} = \frac{1}{R_{n+1}}\sum_{k=0}^{n} p_{n-k}q_{k+1}S_k . \tag{18}$$

Using the inversion formula in (17), we have

$$S_n = \frac{1}{q_n}\sum_{k=0}^{n} c_{n-k}R_k t_k, \tag{19}$$

where c_n is given as in (8).

Using (19), it follows from (18) that

$$\bar{t}_{n+1} = \sum_{v=0}^{\infty} M_{n,v}t_v, \tag{20}$$

where $M_{n,v}$ is given by (11) and (12).

Hence $|(N,p,q)| \in T_L$ if, and only if the transformation given by (20) is absolutely regular. Observe that when $S_n = 1$ $(\text{all } n \geq 0)$, (17) and (19) imply that

$$q_n = \sum_{k=0}^{n} c_{n-k}R_k . \tag{21}$$

Using (11), (12) and (21), one can easily seen that

$$\sum_{v=0}^{\infty} M_{n,v} = 1 - \frac{p_{n+1}}{R_{n+1}}. \tag{22}$$

Therefore, (4) and (6) follow from the regularity of (N,p,q) together with (22). Using (22) and (12), we see that (7) will be satisfied if, and only if (10) is satisfied. Hence, it follows from Mear's Theorem that $|(N,p,q)| \in T_L$ if, and only if (9) and (10) are satisfied. Next, if $\{c_n\}$ is bounded and $\{q_n\}$ is strictly increasing then

$$\frac{c_n}{q_n} \to 0 \text{ as } n \to \infty. \tag{23}$$

Using (11), we have

$$\left|M_{n,v}\right| \le \left|\frac{R_v}{R_{n+1}}\right| \left(\sum_{k=0}^{N} \left|p_{n-k} q_{k+1}\right| \left|\frac{c_{k-v}}{q_k}\right| + \sum_{k=N+1}^{n} \left|p_{n-k} q_{k+1}\right| \left|\frac{c_{k-v}}{q_k}\right| \right). \qquad (24)$$

Using (23) and the regularity of (N, p, q), (9) follows from (24). This completes the proof.

<u>Proof of Theorem (3.2)</u>

Using (18) to obtain S_n in terms of t_n, and then substitute this in (17), we have

$$t_n = \sum_{v=0}^{\infty} H_{n,v} \bar{t}_{v+1}\,, \qquad (25)$$

where $H_{n,v}$ is given by (15) and (16).

Now, $\left|(N, p, q)\right| \in T_R$ if, and only if the transformation given by (25) is absolutely regular. For this, we use (15) to get

$$\sum_{v=0}^{n} H_{n,v} = \sum_{v=0}^{n} \frac{R_{v+1}}{R_n} \sum_{k=v}^{n} \frac{q_k}{q_{k+1}} p_{n-k} c_{k-v}$$

$$= \frac{1}{R_n} \sum_{k=0}^{n} q_k p_{n-k} \frac{1}{q_{k+1}} \sum_{v=0}^{k} R_{v+1} c_{k-v}\,, \qquad (26)$$

which by (24) reduces to

$$\sum_{v=0}^{n} H_{n,v} = \frac{1}{R_n} \sum_{k=0}^{n} q_k p_{n-k} \frac{1}{q_{k+1}} \left(q_{k+1} - c_{k+1}\right) \qquad (27)$$

$$= 1 - \frac{1}{R_n} \sum_{k=0}^{n} \frac{q_k}{q_{k+1}} p_{n-k} c_{k+1} \qquad (28)$$

$$= 1 - \left(H_{n,-1} - \frac{q_{-1} p_{n+1}}{q_0 R_n} \right) \qquad (29)$$

$$= \mathbf{1}. \qquad (30)$$

So that (4) and (6) are satisfied. Hence it follows from Mear's Theorem that $\left|(N, p, q)\right| \in T_R$ if, and only if (13) and

$$\sum_{n=0}^{\infty}\left|\sum_{v=\mu}^{\infty} H_{n,v} - \sum_{v=\mu}^{\infty} H_{n+1}\right| = O(1), \tag{31}$$

are satisfied. Using (30), it can be easily shown that (31) reduces to (14). This completes the proof of the first part. Next, using the same technique used in the proof of the second part of Theorem(3.1) , the second part follows at once.

4 Examples

In this section, we will give three examples to show in each of the first two examples that $|(N, p, q)|$ can be only one-side translative. In the third example we will give a non-trivial case to show that $|(N, p, q)|$ may be translative.

EXAMPLE (4.1)

Let

$$p_n = 1 \ ; \ \ (\mathrm{n} \geq 0), \tag{32}$$

$$q_0 = 1 \ , \tag{33}$$

and

$$q_n = w! \ , \ \ (\mathrm{w}-1)! \leq \mathrm{n} < \mathrm{w}! \ ; \ \ \mathrm{w} = 2,3,..., \tag{34}$$

then $|(N, p, q)| \in T_R$, but $|(N, p, q)| \notin T_L$.

Proof Using(32), we have

$$c_0 = 1 \ , c_1 = -1 \text{ and } \mathrm{c_n} = 0 \ ; \ \ \mathrm{n} \geq 2 \,. \tag{35}$$

Using (32) and (35), it follows from (15) that

$$H_{n,v} = \frac{R_{v+1}}{R_n}\left(\frac{q_v}{q_{v+1}} - \frac{q_{v+1}}{q_{v+2}}\right), \ 0 \leq \mathrm{v} \leq \mathrm{n}\,. \tag{36}$$

observe that

$$R_{w!} = 1 + q_1 + q_2 + ... + q_{(w-1)!} + ... + q_{w!-1} + q_{w!}$$

$$=1+\sum_{k=2}^{w}\left(k!-(k-1)!\right)k!+(w+1)!$$

$$\sim (w!)^2, \qquad (37)$$

and using (36), (13) follows immediately.

Next, observe that q>0 ; $(all\ \mathrm{n} \geq 0)$ and using (32)-(35), the left hand side of (14) will be reduces to

$$\sum_{n=\mu-1}^{\infty}\left|\left(\frac{1}{R_{n-1}}-\frac{1}{R_{n-2}}\right)\sum_{v=0}^{\mu-1}R_{v+1}\left(\frac{q_v}{q_{v+1}}-\frac{q_{v+1}}{q_{v+2}}\right)\right|$$

$$=\sum_{n=\mu-1}^{\infty}\left|\left(\frac{1}{q_1}+R_\mu-R_\mu\frac{q_\mu}{q_{\mu+1}}-q_\mu\right)\left(\frac{1}{R_{n+1}}-\frac{1}{R_{n+2}}\right)\right|$$

$$=\left|\frac{1}{q_1R_\mu}+1-\frac{q_\mu}{q_{\mu+1}}-\frac{q_\mu}{R_\mu}\right|R_\mu\left|\sum_{n=\mu-1}^{\infty}\left(\frac{1}{R_{n+1}}-\frac{1}{R_{n+2}}\right)\right|$$

$$=O(1). \qquad (38)$$

Hence, (14) is satisfied and theorem (3.2) implies that $|(N,p,q)|\in T_R$. Next, to show that $|(N,p,q)|\notin T_L$, it is enough to show that (10) is not satisfied. Using (32)-(35), we have that the left hand side of (10) is equivalent to:

$$\sum_{n=\mu-1}^{\infty}\left|\left(\frac{1}{R_{n+1}}-\frac{1}{R_{n+2}}\right)\left(1+\sum_{v=0}^{\mu-1}R_v\Delta\frac{q_{v+1}}{q_v}\right)\right|$$

$$=\left(1+R_\mu-R_{\mu-1}\frac{q_{\mu+1}}{q_\mu}\right)\sum_{n=\mu-1}^{\infty}\Delta\frac{1}{R_{n+1}} \qquad (39)$$

$$\sim\frac{R_{\mu-1}q_{\mu+1}}{R_\mu q_\mu}. \qquad (40)$$

Taking $\mu=w!-1$, and using (34), it follows from (40) that

$$\frac{R_{\mu-1}q_{\mu+1}}{R_\mu q_\mu} = \frac{q_{w!}R_{w!-2}}{q_{w!-1}R_{w!-1?}} \sim \frac{(w+1)!}{w!} \cdot \frac{(w!-2)^2}{(w!-1)^2} \sim w+1 \neq O(1)\,, \tag{41}$$

so that (10) is not satisfied and consequently $|(N,p,q)| \notin T_L$. This completes the proof.

EXAMPLE (4.2)

Write $E_w = 2^{w-1}\mathrm{w}!$, and let

$$p_n = 1 \quad (\text{all } \mathrm{n} \geq 0)\,, \tag{42}$$

$$q_0 = 1\,, \tag{43}$$

and

$$q_n = \frac{1}{w!}\;;\quad \mathrm{W_{w-1}} \leq \mathrm{n} < E_w\;;\; \mathrm{w} = 2,3,\ldots, \tag{44}$$

then $|(N,p,q)| \in T_L$, but $|(N,p,q)| \notin T_R$.

Proof

Observe that

$$q_n > 0;\; (\text{all } \mathrm{n} \geq 0) \tag{45}$$

and

$$\sum_{\mathrm{n=E_{w-1}}}^{\mathrm{E_w}} \mathrm{q_n} = \frac{\mathrm{E_w} - E_{w-1}}{w!} \sim 2^{w-1}, \tag{46}$$

we have

$$R_{E_w} \sim 2^w\,. \tag{47}$$

Using (39), (42)-(47), we see that the quantity on the right hand side of (40) is bounded, so that (10) is satisfied. Also (9) is clearly satisfied. Hence, by Theorem (3.1), $|(N,p,q)| \in T_L$. Next, using (44), we have

$$\frac{q_\mu}{q_{\mu+1}} \sim \mu + 1 \neq O(1)\,. \tag{48}$$

Using (48), it follows from (38) that the left hand side of (14) is unbounded. Hence by Theorem (3.2), $|(N,p,q)| \notin T_R$.

EXAMPLE (4.3)
Let

$$p_0 = 1,\ p_1 = -1 \text{ and } \mathrm{p_n} = 0;\ (\mathrm{n} \geq 2), \tag{49}$$

and let

$$q_n = (n+1)!\ ;\ (\mathrm{n} \geq 0), \tag{50}$$

then $|(N, p, q)| \in T$.

Proof
Using (49) and (50), we have

$$\mathrm{c_n} = 1\ ;\ (\mathrm{n} \geq 0) \tag{51}$$

and

$$\mathrm{R_0} = 1,\ \mathrm{R_n} = \mathrm{q_n} - \mathrm{q_{n-1}}\ \mathrm{n!n};\ (\mathrm{n} \geq 1). \tag{52}$$

Using (50) and (51), it follows from Theorems (3.1) and (4.2) that $|(N, p, q)| \in T$ if, and only if, (10) and (14) are satisfied. Using (49)-(52), the left hand side of (10) will reduces to:

$$\sum_{\mathrm{n}=\mu-1}^{\infty} \sum_{\mathrm{v}=0}^{\mu-1} (\mathrm{q_v} - \mathrm{q_{v-1}}) \left(-\frac{\mathrm{q_{n+2}}}{\mathrm{q_{n+1}R_{n+2}}} + \frac{\mathrm{q_{n+1}}}{\mathrm{q_n}} \left(\frac{1}{\mathrm{R_{n+1}}} + \frac{1}{\mathrm{R_{n+2}}} \right) - \frac{\mathrm{q_n}}{\mathrm{q_{n-1}R_{n+1}}} \right) \tag{53}$$

$$= \mathrm{q_{\mu-1}} \sum_{\mathrm{n}=\mu-1}^{\infty} \left(\frac{1}{\mathrm{R_{n+1}}} - \frac{1}{\mathrm{R_{n+2}}} \right) = \mathrm{O}(1)\,. \tag{54}$$

So that (10) is satisfied and $|(N, p, q)| \in T_L$. Next, using (51) and (52), the left hand side of (14) will be reduces to:

$$\sum_{n=\mu-1}^{\infty} \sum_{v=0}^{\mu-1} (q_{v+1} - q_v) \left(-\frac{q_{n+1}}{q_{n+2}R_{n+2}} + \frac{q_n q_{n-1}}{q_{n+1}R_{n+1}} + \frac{1}{R_{n+2}} - \frac{q_{n-1}}{q_n R_{n+1}} \right) \tag{55}$$

$$= (q_\mu - 1) \sum_{n=\mu-1}^{\infty} \left(\frac{1}{(n+1)(n+2)R_{n+1}} - \frac{1}{(n+2)(n+3)R_{n+2}} \right) \tag{56}$$

$$= O(1), \tag{57}$$

so that (14) is satisfied and the proof is completed.

EXAMPLE (4.4)
Let

$$p_0 = 1,\ p_1 = -1 \text{ and } \mathrm{p_n} = 0;\ (\mathrm{n} \geq 2), \qquad (58)$$

and define $\{\mathrm{q_n}\}$ by

$$q_n = \begin{Bmatrix} n^3 & \text{if n is odd} \\ 1 & \text{if n is even} \end{Bmatrix}, \qquad (59)$$

then $|(N, p, q)| \notin T_L \cup T_R$.

Proof

Using (58) and (59), and the fact that

$$R_n = p_n q_0 + p_{n-1} q_1 + \ldots + p_1 q_{n-1} + p_0 q_n \ ,$$

we have

$$R_0 = 1 \ ,$$

and

$$R_n = q_n - q_{n-1} = \begin{Bmatrix} n^3 - 1 \ \text{if n is odd} \\ 2 - 3\mathrm{n} + 3\mathrm{n}^2 - n^3 \ \text{if n is even} \end{Bmatrix}, \qquad (61)$$

and these imply (2) and (3), so that (N,p,q) is regular. Also using (58), we have

$$c_n = 1\ ;\ (\mathrm{n} \geq 0). \qquad (62)$$

Using (58), and (59)-(62), and taking n to be given, we have from (11) that for every fixed v,

$$M_{n,v} = \frac{R_v}{R_{n+1}} \left[\frac{q_{n+1}}{q_n} - \frac{q_n}{q_{n-1}} \right] \qquad (63)$$

$$= \frac{R_v}{2 - 3(n+1) + 3(n+1)^2 - (n+1)^3} \left[\frac{1}{n^3} - \frac{n^3}{1} \right] \ \text{if n is odd} \qquad (64)$$

$$\rightarrow R_v \neq 0 \quad (65)$$

$$= \frac{R_v}{(n+1)^3 - 1}\left[\frac{(n+1)^3 - 1}{1} - \frac{1}{(n+1)^3 - 1}\right] \quad \text{if n is even} \qquad (66)$$

$$\to R_v \neq 0. \qquad (67)$$

Hence (9) does not satisfied, and Theorem (3.1) implies that $|(\mathrm{N}, \mathrm{p}, \mathrm{q})| \notin \mathrm{T_L}$. Finally,using (58) and (59)-(62), and let n be odd, we have from (15) that for ever fixed v,

$$\mathrm{H_{n,v}} = \frac{\mathrm{R_{v+1}}}{\mathrm{n}^3 - 1}\left[\frac{\mathrm{n}^3}{1} - \frac{1}{\mathrm{n}^3}\right], \qquad (68)$$

$$\to \mathrm{R_{v+1}} \neq 0, \qquad (69)$$

So that (13) is not satisfied, and Theorem (3.2) implies that $|(N, p, q)| \notin T_R$. This completes the proof.

References

1. AL-Madi, A.K. "On Translativity of the product of Riesz summability Methods" Indian Jour. Pure Appl. Maths. 11(11) (1980) pp. 1444-1457.
2. AL-Madi, A.K. "On Translativity of the product of Nörlund summability Methods" Indian Maths. Soc. 44(1980) pp. 83-90.
3. AL-Madi, A.K. "On Translativity of Absolute Rogosinski – Bernstein summability methods" Internet J. Math & Math. Soc v11 No. 3(1988) 517-522.
4. Amjed Abo-Izreik and Azmi k. almadi "On Translativity of Absolute Rogosinski – Bernstein summability methods of Different Order" Vesti BSV (Republic of Belarus) Beernus 6 ty.cep.1.2006 No1.

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