

# Quest for Research Excellence On Computing, Mathematics and Statistics

**Editors**

Kor Liew Kee

Kamarul Ariffin Mansor

Asmahani Nayan

Shahida Farhan Zakaria

Zanariah Idrus

# **Quest for Research Excellence on Computing, Mathematics and Statistics**

## **Chapters in Book**

The 2<sup>nd</sup> International Conference on Computing, Mathematics  
and Statistics (iCMS2015)

Editors:

Kor Liew Lee  
Kamarul Ariffin Mansor  
Asmahani Nayan  
Shahida Farhan Zakaria  
Zanariah Idrus



**Quest for Research Excellence on Computing,  
Mathematics and Statistics**

**Chapters in Book**

The 2<sup>nd</sup> International Conference on Computing, Mathematics and Statistics  
(iCMS2015)

4-5 November 2015  
Langkawi Lagoon Resort  
Langkawi Island, Kedah  
Malaysia

Copyright © 2015 Universiti Teknologi MARA Cawangan Kedah

All rights reserved, except for educational purposes with no commercial interests. No part of this publication may be reproduced, copied, stored in any retrieval system or transmitted in any form or any means, electronic or mechanical including photocopying, recording or otherwise, without prior permission from the Rector, Universiti Teknologi MARA Cawangan Kedah, Kampus Merbok, 08400 Merbok, Kedah, Malaysia.

The views and opinions and technical recommendations expressed by the contributors are entirely their own and do not necessarily reflect the views of the editors, the Faculty or the University.

Publication by  
Faculty of Computer & Mathematical Sciences  
UiTM Kedah

ISBN 978-967-0314-26-6

# Content

*International Scientific Committee*

*Preface*

<b>CHAPTER 1 .....</b>	<b>1</b>
Towards Ameliorating the Problem of Packet Dropping in IDS using P System Model on GPU <i>Rufai Kazeem Idowu, Ravie Chandren M., and Zulaiha Ali Othman</i>	
<b>CHAPTER 2 .....</b>	<b>11</b>
Analyses of Software Testing Problems in Small and Medium Software Enterprises (SME's) and a Proposed Framework on Exploratory Testing <i>Murugan Thangiah and Shuib Basri</i>	
<b>CHAPTER 3 .....</b>	<b>25</b>
Senior Citizen and Online Form: Hybrid Guideline Form Design <i>Zanariah Idrus, Nor Hafizah Abdul Razak, and Noor Hasnita Abdul Talib</i>	
<b>CHAPTER 4 .....</b>	<b>35</b>
Research Paradigms in Computing Disciplines: A Review <i>Nor Hafizah Abdul Razak, Noor Hasnita Abdul Talib, and Jasmin Ilyani Ahmad</i>	
<b>CHAPTER 5 .....</b>	<b>41</b>
Dijkstra's Algorithm In Product Searching System (Prosearch) <i>Nur Hasni Nasrudin, Siti Hajar Nasaruddin, Syarifah Syafiqah Wafa Syed Abdul Halim and Rosida Ahmad Junid</i>	
<b>CHAPTER 6 .....</b>	<b>49</b>
Developing Waqf Land Computing: A Preliminary Study On The Used Of Web-based Applications And Spatial Database <i>Siti Nurbaya Ismail, Zanariah Idrus, Nor Hafizah Abdul Razak</i>	

<b>CHAPTER 7</b> .....	<b>59</b>
Implementation Of CORDIC Algorithm In Vectoring Mode <i>Anis Shahida Mokhtar, Abdullah bin Mohd Fadzullah</i>	
<b>CHAPTER 8</b> .....	<b>71</b>
A Description of Projective Contractions in the Orlicz-Kantorovich Lattice <i>Inomjon Ganiev and M. Azram</i>	
<b>CHAPTER 9</b> .....	<b>83</b>
The Geometry of the Accessible Sets of Vector Fields <i>A.Y.Narmanov, and I. Ganiev</i>	
<b>CHAPTER 10</b> .....	<b>89</b>
Existence Result of Third Order Functional Random Integro-Differential Inclusion <i>D. S. Palimkar</i>	
<b>CHAPTER 11</b> .....	<b>105</b>
Fourth Order Random Differential Equation <i>D. S. Palimkar and P.R. Shinde</i>	
<b>CHAPTER 12</b> .....	<b>115</b>
New Concept of $e$ - $I$ -open and $e$ - $I$ -Continuous Functions <i>W.F. Al-omeri, M.S. Md. Noorani, and A. AL-Omari</i>	
<b>CHAPTER 13</b> .....	<b>123</b>
Visualization of Constrained Data by Rational Cubic Ball Function <i>Wan Zafira Ezza Wan Zakaria, and JamaludinMd Ali</i>	
<b>CHAPTER 14</b> .....	<b>133</b>
Octupole Vibrations in Even–Even Isotopes of Dy <i>A.A. Okhunov, G.I. Turaeva, and M. Jahangir Alam</i>	
<b>CHAPTER 15</b> .....	<b>141</b>
Characterization of $p$ -Groups with a Maximal Irredundant 10-Covering <i>Rawdah Adawiyah Tarmizi and Hajar Sulaiman</i>	

<b>CHAPTER 16</b> .....	<b>149</b>
Sensitivity Index of HIV-1 model Parameters with Vertical transmission	
<i>Amiru Sule, Mamman Mamuda, Abdullahi Mohammed Baba, Jibril Lawal, and I.G. Usman</i>	
<b>CHAPTER 17</b> .....	<b>163</b>
Derivation of Four-Point Explicit Block Methods for Direct Solution of Initial Value Problems of Third Order Ordinary Differential Equations	
<i>Z. Omar, J. O. Kuboye, and Y.A. Abdullah</i>	
<b>CHAPTER 18</b> .....	<b>175</b>
Absolute Translativity of Generalized Nörlund Mean	
<i>Amjed Zraiqat</i>	
<b>CHAPTER 19</b> .....	<b>189</b>
Type I Error of the Modified Wilcoxon Signed Rank Test under Leptokurtic Distribution	
<i>Nor Aishah Ahad, Sharipah Soaad Syed Yahaya, Suhaida Abdullah, Lim Yai Fung and Zahayu Md Yusof</i>	
<b>CHAPTER 20</b> .....	<b>199</b>
The Combined EWMA-CUSUM Control Chart with Autocorrelation	
<i>Abbas Umar Farouk, and Ismail Bin Mohamad</i>	
<b>CHAPTER 21</b> .....	<b>213</b>
Estimating Philippine Dealing System Treasury (PDST) Reference Rate Yield Curves using a State-Space Representation of the Nelson-Siegel Model	
<i>Len Patrick Dominic M. Garces, and Ma. Eleanor R. Reserva</i>	
<b>CHAPTER 22</b> .....	<b>225</b>
A Structural Equation Model Analyzing the Relationship Model on Perception Students toward Mathematics	
<i>Siti Fairus Mokhtar</i>	

<b>CHAPTER 23</b> .....	<b>233</b>
Partial Least Squares Based Financial Distressed Classifying Model of Small Construction Firms	
<i>Amirah-Hazwani Abdul Rahim, Ida-Normaya M. Nasir, Abd-Razak Ahmad, and Nurazlina Abdul Rashid</i>	
<b>CHAPTER 24</b> .....	<b>245</b>
Logit Bankruptcy Model of Industrial Product Firms	
<i>Asmahani Nayan, Siti-Shuhada Ishak, and Abd-Razak Ahmad</i>	
<b>CHAPTER 25</b> .....	<b>255</b>
Data Mining in Predicting Firms Failure: A Comparative Study Using Artificial Neural Networks and Classification and Regression Tree	
<i>Norashikin Nasaruddin, Wan-Siti-Esah Che-Hussain, Asmahani Nayan, and Abd-Razak Ahmad</i>	
<b>CHAPTER 26</b> .....	<b>265</b>
Risks of Divorce: Comparison between Cox and Parametric Models	
<i>Sanizah Ahmad, Norin Rahayu Shamsuddin, Nur Niswah Naslina Azid @ Maarof, and Hasfariza Farizad</i>	
<b>CHAPTER 27</b> .....	<b>277</b>
Reliability and Construct Validity of DASS 21 using Malay Version: A Pilot Study	
<i>Kartini Kasim, Norin Rahayu Shamsuddin, Wan Zulkipli Wan Salleh, Kardina Kamaruddin, and Norazan Mohamed Ramli</i>	
<b>CHAPTER 28</b> .....	<b>285</b>
Outlier Detection in Time Series Model	
<i>Nurul Sima Mohamad Shariff, Nor Aishah Hamzah, and Karmila Hanim Kamil</i>	
<b>CHAPTER 29</b> .....	<b>297</b>
ROAD Algorithm for Control Charts	
<i>Gejza Dohnal</i>	

<b>CHAPTER 30 .....</b>	<b>311</b>
Learning Numerals for Down Syndrome by applying Cognitive Principles in 3D Walkthrough	
<i>Nor Intan Shafini Nasaruddin, Khairul Nurmazianna Ismail, and Aleena Puspita A.Halim</i>	
<b>CHAPTER 31 .....</b>	<b>329</b>
Predicting Currency Crisis: An Analysis on Early Warning System from Different Perspective	
<i>Nor Azuana Ramli</i>	
<b>CHAPTER 32 .....</b>	<b>341</b>
Using Analytic Hierarchy Process to Rank Takaful Companies based on Health Takaful Product	
<i>Noor Hafizah Zainal Aznam, Shahida Farhan Zakaria, and Wan Asma 'a Wan Abu Bakar</i>	
<b>CHAPTER 33 .....</b>	<b>349</b>
Service Discovery Mechanism for Service Continuity in Heterogeneous Network	
<i>Shaifizat Mansor, Nor Shahniza Kamal Basha, Siti Rafidah Muhamat Dawam, Noor Rasidah Ali, and Shamsul Jamel Elias</i>	
<b>CHAPTER 34 .....</b>	<b>361</b>
Ranking Islamic Corporate Social Responsibility Activities under Product Development Theme using Analytic Hierarchy Process	
<i>Shahida Farhan Zakaria, Wan-Asma ' Wan-Abu-Bakar, Roshima Said, Sharifah Nazura Syed-Noh, and Abd-Razak Ahmad</i>	
<b>CHAPTER 35 .....</b>	<b>369</b>
A Fuzzy Rule Base System For Mango Ripeness Classification	
<i>Ab Razak Mansor, Mahmud Othman, Noor Rasidah Ali , Khairul Adilah Ahmad, and Samsul Jamel Elias</i>	



**CHAPTER 36.....381**

**Technology Assistance for Kids with Learning Disabilities:  
Challenges and Opportunities**

*Suhailah Mohd Yusof, Noor Hasnita Abdul Talib, and Jasmin Ilyani  
Ahmad*

## CHAPTER 17

# Derivation of Four-Point Explicit Block Methods for Direct Solution of Initial Value Problems of Third Order Ordinary Differential Equations

Z. Omar, J. O. Kuboye, and Y.A. Abdullah

**Abstract.** Two block methods for solving third order ordinary differential equations directly are presented. These methods are derived using two different approaches; direct integration and collocation. Both methods are capable of approximating the numerical solutions at four points simultaneously. The advantages and drawbacks of each method are also discussed.

---

**Keywords:** Explicit Block method, Direct Integration, Collocation, Direct Solution, Initial Value Problems, Ordinary Differential Equations

---

Z. Omar (✉) • J. O. Kuboye  
School of Quantitative Sciences, College of Art and Sciences,  
Universiti Utara Malaysia.  
e-mail: zurni@uum.edu.my, Kubbysholly2007@yahoo.com

Y.A. Abdullah  
Hassan Usman Katsina Polytechnic, Katsina State, Nigeria  
e-mail: yahayaaliyyu@gmail.com

# 1 Introduction

We are interested in solving the following initial value problem of third order ordinary differential equation (ODE) directly

$$y''' = f(x, y, y', y''), y(a) = y_0, y'(a) = y_0', y''(a) = y_0'', a \leq x \leq b. \quad (1)$$

Several methods for solving higher ODEs directly have been proposed by researchers such as Awoyemi and Kayode [1], Gear [2-4], Hall and Suleiman [5], Kayode and Obarhua [6], Odekunle et. al. [7] and Suleiman [8-9]. These methods are capable of solving higher order initial value problems of ODEs without going through the process of reduction. Reducing higher order ODE to its equivalent system of first order equations is not only enlarging the system of equations, it may also jeopardize the performance of the methods particularly in term of computational time.

In this paper, two explicit block methods for solving (1) are derived using two different approaches namely direct integration and collocation. The details derivations of both methods are described in the following sections

## 2 Derivation of Four-Point Explicit Block Method Using Direct Integration

Let  $x_{n+j} = x_n + jh, j = 1,2,3,4$ . Integrating (1) once, twice and thrice and taking the limit from  $x_n$  to  $x_{n+j}$  gives

$$\begin{aligned} y''(x_{n+j}) - y''(x_n) &= \int_{x_n}^{x_{n+j}} f(x, y, y') dx \\ y'(x_{n+j}) - y'(x_n) - jhy''(x_n) &= \int_{x_n}^{x_{n+j}} \int_{x_n}^x f(x, y, y') dx dx = \int_{x_n}^{x_{n+j}} (x_{n+j} - x)f dx \\ y(x_{n+j}) - y(x_n) - jhy'(x_n) - \frac{(jh)^2}{2!} y''(x_n) &= \int_{x_n}^{x_{n+j}} \int_{x_n}^x \int_{x_n}^x f(x, y, y') dx dx dx = \int_{x_n}^{x_{n+j}} \frac{(x_{n+j} - x)^2}{2!} f dx \end{aligned} \quad (2)$$

Replacing  $f(x, y, y')$  with a polynomial  $P_{k,n}(x) = \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n$

which interpolates  $f(x, y, y')$  at the set of points  $(x_{n-i}, f_{n-i})$ ,  $i = 0, 1, 2, \dots, k-1$  gives the following results

$$\begin{aligned}
 y''_{n+j} - y''_n &= \int_{x_n}^{x_{n+j}} \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n dx. \\
 y'(x_{n+j}) - y'(x_n) - (jh)y''(x_n) &= \int_{x_n}^{x_{n+j}} (x_{n+j} - x) \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n dx. \\
 y(x_{n+j}) - y(x_n) - (jh)y'(x_n) - \frac{(jh)^2}{2!} y''(x_n) &= \int_{x_n}^{x_{n+j}} \frac{(x_{n+j} - x)^2}{2!} \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n dx.
 \end{aligned} \tag{3}$$

where  $s = \frac{x - x_n}{h}$ .

Changing the limit integration and substituting  $dx = hds$  in (3) yields

$$\begin{aligned}
 y''(x_{n+j}) - y''(x_n) &= \int_0^j \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n hds. \\
 y'(x_{n+j}) - y'(x_n) - (jh)y''(x_n) &= \int_0^j h(j-s) \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n hds. \\
 y(x_{n+j}) - y(x_n) - (jh)y'(x_n) - \frac{(jh)^2}{2!} y''(x_n) &= \int_0^j \frac{(h(j-s))^2}{2!} \sum_{m=0}^{k-1} (-1)^m \binom{-s}{m} \nabla^m f_n hds.
 \end{aligned} \tag{4}$$

which can be written as

$$\begin{aligned}
y''(x_{n+j}) &= y''(x_n) + h \sum_{m=0}^{k-1} \alpha_{j,m}^{(1)} \nabla^m f_n \\
y'(x_{n+j}) &= y'(x_n) + (jh)y''(x_n) + h^2 \sum_{m=0}^{k-1} \alpha_{j,m}^{(2)} \nabla^m f_n \\
y(x_{n+j}) &= y(x_n) + (jh)y'(x_n) + \frac{(jh)^2}{2!} y''(x_n) + h^3 \sum_{m=0}^{k-1} \alpha_{j,m}^{(3)} \nabla^m f_n
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
\alpha_{j,m}^{(1)} &= (-1)^m \int_0^j \binom{-s}{m} ds \\
\alpha_{j,m}^{(2)} &= (-1)^m \int_0^j (j-s) \binom{-s}{m} ds \\
\alpha_{j,m}^{(3)} &= (-1)^m \int_0^j \frac{(j-s)^2}{2!} \binom{-s}{m} ds
\end{aligned} \tag{6}$$

Let's  $G_j^{(1)}(t)$ ,  $G_j^{(2)}(t)$  and  $G_j^{(3)}(t)$  be the generating functions defined as follows

$$\begin{aligned}
G_j^{(1)}(t) &= \sum_{m=0}^{\infty} \alpha_{j,m}^{(1)} t^m = \sum_{m=0}^{\infty} (-t)^m \int_0^j \binom{-s}{m} ds = \int_0^j (1-t)^{-s} ds = \int_0^j e^{-s \log(1-t)} ds \\
G_j^{(2)}(t) &= \sum_{m=0}^{\infty} \alpha_{j,m}^{(2)} t^m = \sum_{m=0}^{\infty} (-t)^m \int_0^j (j-s) \binom{-s}{m} ds = \int_0^j (j-s)(1-t)^{-s} ds = \int_0^j (j-s) e^{-s \log(1-t)} ds \\
G_j^{(3)}(t) &= \sum_{m=0}^{\infty} \alpha_{j,m}^{(3)} t^m = \sum_{m=0}^{\infty} (-t)^m \int_0^j \frac{(j-s)^2}{2!} \binom{-s}{m} ds = \int_0^j \frac{(j-s)^2}{2!} (1-t)^{-s} ds = \int_0^j \frac{(j-s)^2}{2!} e^{-s \log(1-t)} ds
\end{aligned} \tag{7}$$

The constant step size formulations of implicit k-step method corresponding to (5) are

$$\begin{aligned}
y''_{n+j} &= y''_n + h \sum_{m=0}^{k-1} \beta_{k-1,m}^{(j,1)} f_{n-m} \\
y'_{n+j} &= y'_n + (jh)y''_n + h^2 \sum_{m=0}^{k-1} \beta_{k,m}^{(j,2)} \nabla^m f_{n-m} \\
y_{n+j} &= y_n + (jh)y'_n + \frac{(jh)^2}{2!} y''_n + h^3 \sum_{m=0}^{k-1} \beta_{k,m}^{(j,3)} \nabla^m f_{n-m}
\end{aligned} \tag{8}$$

respectively. The coefficients  $\beta_{k,m}^{(j,p)}$ ,  $m = 0, 1, \dots, k-1$  and  $p = 1, 2, 3$  are well known and defined by Shampine and Gordon (1975) as follows

$$\beta_{k,m}^{(j,p)} = (-1)^m \sum_{r=m}^k \binom{r}{m} \alpha_{j,m}^{(p)}$$

The values of  $\alpha_{j,m}^{(p)}$  are then substituted in (9) and subsequently in (8) respectively to get a four-point explicit block method. The most crucial part in deriving this method using direct integration approach is, therefore, to determine the values of  $\alpha_{j,m}^{(p)}$  which we are going to discuss as follows.

From (7), we have

$$\begin{aligned}
G_j^{(1)}(t) &= \sum_{m=0}^{\infty} \alpha_{j,m}^{(1)} t^m = \int_0^j e^{-s \log(1-t)} ds \\
G_j^{(2)}(t) &= \sum_{m=0}^{\infty} \alpha_{j,m}^{(2)} t^m = \int_0^j (j-s) e^{-s \log(1-t)} ds = \frac{j - G_j^{(1)}(t)}{\log(1-t)} \\
G_j^{(3)}(t) &= \sum_{m=0}^{\infty} \alpha_{j,m}^{(3)} t^m = \int_0^j \frac{(j-s)^2}{2!} e^{-s \log(1-t)} ds = \frac{j^2 - 2!G_j^{(2)}(t)}{2! \log(1-t)}
\end{aligned} \tag{10}$$

which implies

$$\begin{aligned}
\sum_{m=0}^{\infty} \alpha_{j,m}^{(1)} t^m &= \int_0^j e^{-s \log(1-t)} ds \\
\sum_{m=0}^{\infty} \alpha_{j,m}^{(2)} t^m &= \frac{j - \sum_{m=0}^{\infty} \alpha_{j,m}^{(1)} t^m}{\log(1-t)} \\
\sum_{m=0}^{\infty} \alpha_{j,m}^{(3)} t^m &= \frac{j^2 - 2! \sum_{m=0}^{\infty} \alpha_{j,m}^{(2)} t^m}{2! \log(1-t)}
\end{aligned}
\tag{11}$$

From Equation (11) it is observed that the integration coefficients depend very much on  $\sum_{m=0}^{\infty} \alpha_{j,m}^{(1)} t^m$ . Therefore, we will focus on determining the values of

$$\begin{aligned}
&\sum_{m=0}^{\infty} \alpha_{j,m}^{(1)} t^m \text{ for } j=1,2,3,4. \text{ When } j=1, \text{ we have} \\
\sum_{m=0}^{\infty} \alpha_{1,m}^{(1)} t^m &= \int_0^1 e^{-s \log(1-t)} ds = \frac{t}{-(1-t)\log(1-t)}
\end{aligned}$$

which leads to

$$\left( \sum_{m=0}^{\infty} \alpha_{1,m}^{(1)} t^m \right) \left( \frac{-\log(1-t)}{t} \right) = \frac{1}{1-t}
\tag{12}$$

Replacing  $\left( \frac{-\log(1-t)}{t} \right) = 1 + \frac{1}{2}t + \frac{1}{3}t^2 + \dots$  and  $\frac{1}{1-t} = 1 + t + t^2 + \dots$  in (12), equating coefficients  $t^m$  and on simplifying we obtain

$$\begin{aligned}
\alpha_{1,0}^{(1)} &= 1 \\
\alpha_{1,m+1}^{(1)} &= 1 - \sum_{r=0}^m \frac{\alpha_{1,r}^{(1)} t^m}{m+2-r} \\
\alpha_{1,0}^{(p)} &= \alpha_{1,1}^{(p-1)} \\
\alpha_{1,m+1}^{(p)} &= \alpha_{1,m+2}^{(p-1)} - \sum_{r=0}^m \frac{\alpha_{1,r}^{(p-1)}}{m+2-r} \quad \text{for } p = 2,3, \quad m = 0,1,\dots,k-1
\end{aligned} \tag{13}$$

Repeat the same procedure for  $j=2,3,4$ , we have

$$\begin{aligned}
\alpha_{j,0}^{(1)} &= j \\
\alpha_{j,m+1}^{(1)} &= \frac{(m+j+1)(m+j)(m+j-1)\dots(m+3)}{(j-1)!} - \sum_{r=0}^m \frac{\alpha_{j,r}^{(1)}}{m+2-r} \\
\alpha_{j,0}^{(p)} &= \alpha_{j,1}^{(p-1)} \\
\alpha_{j,m+1}^{(p)} &= \alpha_{j,m+2}^{(p-1)} - \sum_{r=0}^m \frac{\alpha_{j,r}^{(p-1)}}{m+2-r} \quad \text{for } j = 2,3,4, \quad p = 2,3, m = 0,1,\dots,k-1.
\end{aligned} \tag{14}$$

Substituting the values of integration coefficients obtained in (13) and (14) into (9), followed by (9) into (8) produces a four-point explicit block method.

### 3 Derivation of Four-Point Explicit Block Method Using Collocation Approach

Let power series of the form

$$y(x) = \sum_{i=0}^{k+3} a_i x^i \tag{15}$$

be an approximate solution to (1) where the number of points  $k = 4$ . Differentiating (15) thrice we have

$$y''' = \sum_{i=3}^{k+3} i(i-1)(i-2)a_i x^{i-3} \tag{16}$$



Equation (15) is interpolated at  $x = x_{n+j}, j = 0, 2, 3$  and equation (16) is collocated at  $x = x_{n+j}, j = 0(1)3$ . As a result, we have

$$\begin{aligned} \sum_{i=0}^{k+3} a_i x_{n+j}^i &= y_{n+j} \\ \sum_{i=2}^{k+3} i(i-1)(i-2) a_i x_{n+j}^i &= f_{n+j} \end{aligned} \quad (17)$$

In order to find the values of  $a$ 's in (17), Gaussian elimination method is employed. These values are then substituted into equation (15) and using transformation  $t = \frac{x - x_{n+3}}{h}$ , a continuous explicit scheme of the following form is obtained

$$y(t) = \alpha_0(t)y_n + \sum_{j=2}^{k-1} \alpha_j(t)y_{n+j} + h^3 \sum_{j=0}^{k-1} \beta_j(t)f_{n+j} \quad (18)$$

where

$$\begin{aligned} \alpha_0(t) &= \frac{t}{6} + \frac{t^2}{6} \\ \alpha_2(t) &= -\frac{3t}{2} - \frac{t^2}{2} \\ \alpha_3(t) &= \frac{4t}{3} + \frac{t^2}{3} \\ \beta_0(t) &= \frac{t}{120} + \frac{11t^2}{720} - \frac{t^4}{72} - \frac{t^5}{120} - \frac{t^6}{720} \\ \beta_1(t) &= \frac{t}{20} + \frac{t^2}{60} + \frac{t^4}{16} + \frac{t^5}{30} + \frac{t^6}{240} \\ \beta_2(t) &= \frac{3t}{8} + \frac{37t^2}{80} - \frac{t^4}{8} - \frac{t^5}{24} - \frac{t^6}{240} \\ \beta_3(t) &= \frac{t}{15} + \frac{31t^2}{180} + \frac{t^3}{6} + \frac{11t^4}{144} + \frac{t^5}{60} + \frac{t^6}{720} \end{aligned} \quad (19)$$

The first derivative of (19) yields

$$\begin{aligned}
\alpha'_0(t) &= \frac{1}{6} + \frac{t}{3} \\
\alpha'_2(t) &= -\frac{3}{2} - t \\
\alpha'_3(t) &= \frac{4}{3} + \frac{2}{3}t \\
\beta'_0(t) &= \frac{1}{120} + \frac{11t}{360} - \frac{t^3}{18} - \frac{t^4}{24} - \frac{t^5}{120} \\
\beta'_1(t) &= \frac{1}{20} + \frac{t}{30} + \frac{t^3}{4} + \frac{t^4}{6} + \frac{t^5}{40} \\
\beta'_2(t) &= \frac{3}{8} + \frac{37}{40}t - \frac{t^3}{2} - \frac{5t^4}{24} - \frac{t^5}{40} \\
\beta'_3(t) &= \frac{1}{15} + \frac{31}{90}t + \frac{t^2}{2} + \frac{11t^3}{36} + \frac{t^4}{12} + \frac{t^5}{120}
\end{aligned}
\tag{20}$$

and the second derivative of (19) produces

$$\begin{aligned}
\alpha''_0(t) &= \frac{1}{3} \\
\alpha''_2(t) &= -1 \\
\alpha''_3(t) &= \frac{2}{3} \\
\beta''_0(t) &= \frac{11}{360} - \frac{t^2}{6} - \frac{t^3}{6} - \frac{t^4}{24} \\
\beta''_1(t) &= \frac{1}{30} + \frac{3t^2}{4} + \frac{2t^3}{3} + \frac{t^4}{8} \\
\beta''_2(t) &= \frac{37}{40} - \frac{3}{2}t^2 - \frac{5}{6}t^3 - \frac{t^4}{8} \\
\beta''_3(t) &= \frac{31}{90} + t + \frac{11}{12}t^2 + \frac{t^3}{3} + \frac{t^4}{24}
\end{aligned}
\tag{21}$$

Evaluating (19) at non-interpolating points i.e. at  $t = -2, 1$  and evaluating (20) and (21) at all the grid points. i.e.  $t = -3, -2, -1, 0,$  and  $1$  produces the discrete schemes and its derivatives which are combined in a matrix form as follows

$$\begin{pmatrix} 1 & -1 & \frac{1}{3} & 0 \\ 0 & 2 & -\frac{8}{3} & 1 \\ 0 & -\frac{3}{2} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \end{pmatrix} \begin{pmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & -\frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} y_{n-3} \\ y_{n-2} \\ y_{n-1} \\ y_n \end{pmatrix} + h \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y'_{n-3} \\ y'_{n-2} \\ y'_{n-1} \\ y'_n \end{pmatrix} \\ + h^2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} y''_{n-3} \\ y''_{n-2} \\ y''_{n-1} \\ y''_n \end{pmatrix} + h^3 \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{2} & 0 \\ \frac{6}{75} & \frac{3}{36} & \frac{1}{2} & 0 \\ \frac{120}{393} & \frac{120}{72} & \frac{120}{11} & 0 \\ \frac{360}{360} & \frac{360}{360} & \frac{360}{360} & 0 \end{pmatrix} \begin{pmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \end{pmatrix} + h^3 \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{8}{120} \\ 0 & 0 & 0 & \frac{124}{360} \end{pmatrix} \begin{pmatrix} f_{n-3} \\ f_{n-2} \\ f_{n-1} \\ f_n \end{pmatrix}$$

Therefore, multiplying both  $y$  and  $f$  function by the inverse of the coefficients of  $y_{n+i}, i=0(1)4$ . This gives the explicit block method below

$$\begin{aligned} y_{n+1} &= y_n + hy'_n + \frac{1}{2}h^2y''_n + h^3\left(\frac{19}{180}f_n + \frac{7}{80}f_{n+1} - \frac{1}{30}f_{n+2} + \frac{1}{144}f_{n+3}\right). \\ y_{n+2} &= y_n + 2hy'_n + 2h^2y''_n + h^3\left(\frac{5}{9}f_n + \frac{14}{15}f_{n+1} - \frac{1}{5}f_{n+2} + \frac{2}{45}f_{n+3}\right). \\ y_{n+3} &= y_n + 3hy'_n + \frac{9}{2}h^2y''_n + h^3\left(\frac{27}{20}f_n + \frac{243}{80}f_{n+1} + \frac{9}{80}f_{n+3}\right). \\ y_{n+4} &= y_n + 4hy'_n + 8h^2y''_n + h^3\left(\frac{112}{45}f_n + \frac{32}{5}f_{n+1} + \frac{16}{15}f_{n+2} + \frac{32}{45}f_{n+3}\right) \end{aligned} \quad (22)$$

The corresponding first derivatives of (22) are

$$\begin{aligned} y'_{n+1} &= y'_n + hy''_n + h^2\left(\frac{97}{360}f_n + \frac{37}{60}f_{n+1} - \frac{11}{120}f_{n+2} + \frac{1}{45}f_{n+3}\right). \\ y'_{n+2} &= y'_n + 2hy''_n + h^2\left(\frac{28}{45}f_n + \frac{22}{15}f_{n+1} - \frac{2}{15}f_{n+2} + \frac{2}{45}f_{n+3}\right). \\ y'_{n+3} &= y'_n + 3hy''_n + h^2\left(\frac{39}{40}f_n + \frac{27}{10}f_{n+1} + \frac{27}{40}f_{n+2} + \frac{3}{20}f_{n+3}\right). \end{aligned}$$

$$y'_{n+4} = y'_n + 4hy'_n + h^2\left(\frac{56}{45}f_n + \frac{64}{15}f_{n+1} + \frac{16}{15}f_{n+2} + \frac{64}{45}f_{n+3}\right).$$

while the corresponding second derivatives of (22) are

$$y''_{n+1} = y''_n + h\left(\frac{3}{8}f_n + \frac{19}{24}f_{n+1} - \frac{5}{24}f_{n+2} + \frac{1}{24}f_{n+3}\right).$$

$$y''_{n+2} = y''_n + h\left(\frac{1}{3}f_n + \frac{4}{3}f_{n+1} + \frac{1}{3}f_{n+2}\right).$$

$$y''_{n+3} = y''_n + h\left(\frac{3}{8}f_n + \frac{9}{8}f_{n+1} + \frac{9}{8}f_{n+2} + \frac{3}{8}f_{n+3}\right).$$

$$y''_{n+4} = y''_n + h\left(0f_n + \frac{8}{3}f_{n+1} - \frac{4}{3}f_{n+2} + \frac{8}{3}f_{n+3}\right).$$

## 4 Discussion and Conclusion

Two numerical methods for the solution of general second order initial value problems using direct integration and collocation approaches have been proposed in this paper. Both methods have their strengths and weaknesses. In terms of simplicity, the derivation of block method using collocation method seems simpler than its counterpart. But this approach fails to generalize the formulation of unknowns  $a$ 's to any point since the order of differential equation that determines the number of interpolation points. In addition, the order of the approximated power series is also determined by the number of interpolation and collocation points. Although the derivation using direct integration method is more complicated, this approach is able to generalize the formulation of the integration coefficients for any back values used at any point.

## References

- [1]. D.O Awoyemi and S.J Kayode, "Maximal Order Multistep Derivative Collocation Method for Direct Solution of Fourth Order Initial Value Problems of Ordinary Differential Equations". *J. Nig. Math. Soc.*23, 53-64, 2004.
- [2]. C. W. Gear, "The Numerical Integration of Ordinary Differential Equations. *Math. Comp.*21:146-156, 1976

- [3]. C. W. Gear, "The Numerical Initial Value Problems in Ordinary Differential Equations". *New Jersey. Prentice Hall, Inc.* 1971
- [4]. C. W. Gear, "The Stability of Numerical Methods for Second –Order Ordinary Differential Equations". *SIAMJ. Numer. Anal.* *15(1): 118-187*, 1978.
- [5]. G. Hall, M. B. Suleiman, "Stability of Adams- Type Formulae for Second – Order Ordinary Differential Equations". *IMAJ. Numer. Anal.* *1: 427-428*, 1981.
- [6]. S. J. Kayode and F. O. Obarhua, " Continuous y-Function Hybrid Methods for Direct Solution of Differential Equations". *International Journal of Diff. Equations and Applications*. Volume 12(37-38), 2013.
- [7]. M. R. Odekunle, M. O.Egwurube, A. O. Adesanya, M. O. Udo, "Five-Step Block Predictor-Corrector Method for the Solution of Second Order Ordinary Differential Equations". *Applied Mathematics*, *5(1252-1266)*, 2014.
- [8]. M. B. Suleiman, " Generalised Multistep Adams and Backward Differential Methods for the Solution of Stiff and Non-stiff Ordinary Differential Equations. *PhD Thesis. University of Manchester*, 1979
- [9]. M. B. Suleiman, "Solving Higher Order Ordinary Differential Equations Directly by the Direct Integration Method. *Applied Mathematics and Computation* 33(3):197-219, 1989



ISBN 978-967-0314-25-6



9 789670 314256