Quest for Research Excellence On Computing, Mathematics and Statistics

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Faculty of Computer and Mathematical Sciences

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Chapters in Book

The 2nd International Conference on Computing, Mathematics and Statistics (iCMS2015)

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Content

CHAPTER 10
Existence Result of Third Order Functional Random Integro-Differential Inclusion

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Abstract. In this paper, we prove the existence result for third order random integro-differential inclusions under non-convex case of multi-valued function.

Keywords: random differential inclusion; random solution; caratheodory condition.

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1 Description of the Problem

Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measure space. Let r be the real line and let J = [0, T] be a closed and bounded interval in R. Consider the functional random integro-differential inclusion (in short FRIGDI),

$$\left.\begin{aligned} x'''(t,\omega) &\in F\left(t, x\big(\theta(t),\omega\big), \int_0^{\sigma(t)} k\Big(s,\tau,x\big(\eta(\tau),\omega\big),\omega\Big)d\tau, \omega\right) \; a.e.\; t \in J \\ x(0,\omega) &= q_0(\omega), x'(0,\omega) = q_1(\omega), x''(0,\omega) = q_2(\omega) \end{aligned}\right\} \tag{1.1}$$

for all $\omega \in \Omega$ where $q_0 : \Omega \to R, q_1 : \Omega \to R, q_2 : \Omega \to R$, $F : J \times R \times R \times \Omega \to \mathcal{P}_p(\mathrm{R})$, and functions $\theta, \sigma, \eta : J \to J$ are continuous.

By a random solution of the FRIGDI (1.1) on $J \times \Omega$ we mean a measurable function $x : \Omega \to AC(J, \mathrm{R})$ satisfying for each $\omega \in \Omega$, $x'''(t,\omega) = v(t,\omega)$ for some measurable $v : \Omega \to L^1(J, \mathrm{R})$ such that

$$v(t,\omega) \in F\left(t, x\big(\theta(t),\omega\big), \int_0^{\sigma(t)} k\Big(s,\tau,x\big(\eta(\tau),\omega\big),\omega\Big)d\tau, \omega\right) \; a.e. \; t \in J,$$

where $AC(J, \mathrm{R})$ is the space of absolutely continuous real-valued functions on J.

The FRIGDI (1.1) seems to be new and includes several known random differential inclusions already studied in the literature as special cases has been discussed in the literature for various aspects of the solutions. See Papapgeorgiou [5,6] and the reference therein. In this paper we prove the existence result for FIGDI (1.1) under non-convex case of multi-valued function involved in it.

2 Auxiliary Results

Let $F : J \times R \times R \times \Omega \to \mathcal{P}_p(\mathrm{R})$ be a multi-valued mapping. Then for any measurable function $x : \Omega \to C(J, \mathrm{R}),$ let

$$S_F(\omega)(x) = \Big\{ \mathbf{v} \in \mathcal{M}\ (\Omega, M(J,\mathrm{R}))\ \Big|$$

$$v(t,\omega) \in F\left(t, x\big(\theta(t),\omega\big), \int_0^{\sigma(t)} k\big(t,s,x\big(\eta(t),\omega\big),\omega\big), \omega \right)\ a.e.\ t \in J \Big\}. \quad (2.1)$$

and $S_F^1(\omega)(x) = \Big\{\ v \in \mathcal{M}\ \big(\Omega, L^1(J,\mathrm{R})\big)\Big|$

$$v(t,\omega) \in F\left(t, x\big(\theta(t),\omega\big), \int_0^{\sigma(t)} k\big(t,s,x\big(\eta(t),\omega\big),\omega\big), \omega \right)\ a.e.\ t \in J \Big\}. \quad (2.2)$$

This is our set of selection functions for F on $J \times R \times R \times \Omega$. When there is no confusion, we denote $S_F^1(\omega)(x) = S_F^1(\omega)(y),$ where $y(t,\omega) = x\big(\theta(t),\omega\big)$ for some continuous function $\theta : J \to J.$ The integral of the random multi-valued function F is defined as

$$\int_0^t F\left(s, x\big(\theta(s),\omega\big), \int_0^{\sigma(s)} k\big(s,\tau,x\big(\eta(\tau),\omega\big),\omega\big)d\tau, \omega \right) ds$$

$$= \left\{ \int_0^t v(s,\omega)ds : v \in S_F^1(\omega)(x) \right\}.$$

Further, if the integral,

$$\int_0^t F\left(s, x\big(\theta(s),\omega\big), \int_0^{\sigma(s)} k\big(s,\tau,x\big(\eta(\tau),\omega\big),\omega\big)d\tau, \omega \right) ds$$

exists for every measurable function $x:\Omega\to C(J,\mathrm{R}),$ then we say the multi-valued mapping *F* is Lebesgue integrable on *J*.
We need the following definition.

Definition 2.1. A multi-valued mapping $\beta: J\times R\times R\times\Omega\to \mathrm{P}_{cp}(\mathrm{R})$ is called strong random Caratheodory if for each $\omega\in\Omega$,

(i) $(t,\omega)\mapsto\beta(t,x,y,\omega)$ is jointly measurable for all $x,y\in R$, and
(ii) $(x,y)\mapsto\beta(t,x,y,\omega)$ is Hausdorff continuous almost everywhere for $t\in J$.

Again, a random Caratheodory multi-valued function β is called strong L^1-Caratheodory if

(iii) for each real number *r* > 0 there exists a measurable function $h_r:\Omega\to L^1(J,\mathrm{R})$ such that for each $\omega\in\Omega$,

$$\|\beta(t,x,y,\omega)\|_{\mathrm{P}}=\sup\{|u|:u\in F(t,x,y,\omega)\}\le h_r(t,\omega)$$
$$\text{a.e. } t\in J$$

for all $x,y\in R$ with $|x|\ \le\ r$ and $|y|\ \le\ r$.

Then we have the following lemmas, which are well-known in the literature.

Lemma 2.1.(Caratheodory theorem [5]) Let E be a Banach space. If $\beta: J\times E\to\mathrm{P}_{cp}(E)$ is strong Caratheodory, then the multi-valued mapping $(t,x)\mapsto F\left(t,x(t)\right)$ is jointly measurable for any measurable function x *on J.*

3 Existence Results

We will seek the random solutions of FRIGDI (1.1) in the function space $C(J,\mathrm{R})$ of continuous real-valued functions defined on *J*. Define a norm $\|\cdot\|$ in $C(J,\mathrm{R})$ by

$$\| x \| = \sup_{t \in J} |x(t)| \cdot$$
(3.1)

Clearly, $C(J, \mathbb{R})$ becomes a separable Banach space with respect to the above supremum norm.

Definition 3.1. A multi-valued random operator $Q : \Omega \times X \to \mathcal{P}_{cl}(X)$ is called right monotone increasing if for each $\omega \in \Omega$ we have that $S_Q(\omega)(x) \overset{i}{\leq} S_Q(\omega)(y)$ for all $x, y \in X$ for which $x \leq y$.

We quote the following fixed-point theorem for right monotone increasing random multi-valued mappings on ordered Banach spaces.

Theorem 3.1 (Dhage [1]) Let $(\Omega, \mathcal{A})$ be a measurable space and let $[a,b]$ be a random interval in a separable ordered Banach space X. If $Q : \Omega \times [a,b] \to \mathcal{P}_{cl}([a,b])$ is a compact, upper semi-continuous right monotone increasing multi-valued random operator and the cone K in X is normal, then $Q(\omega)$ has a random fixed point in [a, b].

Then I have the following lemmas which are well-known in the literature, hence quote it.

Lemma 3.1. (Lasota and Opial [8]) Let E be a Banach space. If dim (E) < ∞ and let F : J × E × $\Omega \to \mathcal{P}_{cp}(E)$ be a random L^1-Caratheodory, then $S_F^1(\omega)(x) \neq \emptyset$ for each $x \in E$.

Lemma 3.2 (Lasota and Opial [8]) Let E be a Banach space, F a Caratheodory multi-valued operator with $S_F^1(\omega) \neq \emptyset$ and let $\mathcal{L} : L^1(J, E) \to C(J, E)$ be a continuous linear mapping. Then the composite operator $\mathcal{L} \circ S_F^1(\omega) : C(J, E) \to \mathcal{P}_{bd,cl}(C(J, E))$ has closed graph on $C(J, E) \times C(J, E)$.

We consider the following set of hypotheses .

(A_0) The single-valued mappings $q_0 : \Omega \to R,\ q_1 : \Omega \to R,$ $q_2 : \Omega \to R$ are measurable.

(A_1) The single-valued mapping $k : \Omega \to C(J \times J \times R, \mathrm{R})$ is measurable, and there exists a measurable function $\alpha : \Omega \to L^1(J, \mathrm{R}^+)$ such that

$$\left| \int_0^{\sigma(t)} k\big(t,s,y,\omega\big)ds \right| \le \alpha(t,\omega)\,|\,y\,| \text{ for all } t,s \in J \text{ and } y \in R.$$

(A_2) The multi-valued mapping $(t,\omega) \mapsto F\big(t, x(t,\omega), y(t,\omega), \omega\big)$ is jointly measurable for all $x, y \in \mathrm{M}\ \big(\Omega, C(J,R)\big)$.

(A_3) $F(t,x,y,\omega)$ is closed subset of R for each $(t,\omega) \in J \times \Omega$ and $x, y \in R$.

(A_4) F is random L^1-Caratheodory.

(A_5)For each $\omega \in \Omega,$ the multi-valued mapping $x \mapsto S_F^1(\omega)(x)$ is right monotone increasing in $x \in C(J,R)$ almost everywhere for $t \in J.$

(A_6) FRIGDI (1.1) has a strict lower random solution *a* and a strict upper random solution *b* with $a \le b$ defined on $J \times \Omega$.

4 Main Existence Results

Theorem 2.4.2 Assume that the hypotheses (A_0) − (A_1) and (B_1) − (B_5) hold. Furthermore, if $\|\alpha(\omega)\| \leq 1$, then the FRIGDI (1.1) has a random solution in [a, b] defined on J × Ω.

Proof. Let $X = C(J, R)$. Define a random order interval [*a*, *b*] in *X* which is well defined in view of hypothesis (B_5). Now the FRIGDI (1.1) is equivalent to the random integral inclusion

$$x(t,\omega) \in q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2 + \int_0^t F\left(s, x(\eta(s),\omega), \int_0^{\sigma(t)} k\big(s,\tau,x(\eta(\tau),\omega),\omega\big)d\tau, \omega \right) ds,$$

$$t \in J \tag{3.2}$$

for all $\omega \in \Omega$. Define a multi-valued operator $Q : \Omega \times [a,b] \to \mathrm{P}_p(X)$ by

$$Q(\omega)x = \left\{ u \in \mathrm{M}(\Omega, X) \mid u(t,\omega) = q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2 + \int_0^t v(s,\omega)ds, \;\; v \in S_F^1(\omega)(x) \right\}$$

$$= \left(\mathrm{K} \circ S_F(\omega)\right)(x) \tag{3.2}$$

where $\mathrm{K} : \mathrm{M}\left(\Omega, L^1(J,R)\right) \to \mathrm{M}(\Omega, X)$ is a continuous operator defined by

$$\mathrm{K}v(t,\omega) = q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2 + \int_0^t v(s,\omega)ds. \tag{3.3}$$

Clearly, the operator $Q(\omega)$ is well defined in view of hypothesis (B_3). We shall show that $Q(\omega)$ satisfies all the conditions of Theorem 3.1.

Step I : First, we show that Q is closed valued multi-valued random operator on Ω × [a, b]. Observe that the operator $Q(\omega)$ is equivalent to the composition $\mathcal{K} \circ S_F^1(\omega)$ of two operators on $L^1(J,R)$, where $\mathcal{K} : \mathcal{M}\left(\Omega, L^1(J,R)\right) \to \mathcal{M}(\Omega, X)$ is the continuous operator defined by (2.4.5). To show $Q(\omega)$ has closed values, it then suffices to prove that the composition operator $\mathcal{K} \circ S_F^1(\omega)$ has closed values on [a, b]. Let $x \in [a,b]$ be arbitrary and let $\{ v_n \}$ be a sequence in $S_F^1(\omega)(x)$ converging to v in measure. Then, by the definition of $S_F^1(\omega)$,

$$v_n(t,\omega) \in F\left(t, x\left(\theta(t),\omega\right), \int_0^{\sigma(t)} k\left(s,\tau,x\left(\eta(\tau),\omega\right),\omega\right) d\tau, \omega \right)$$

a.e. for $t \in J$.

Since $F\left(t, x\left(\theta(t),\omega\right), \int_0^{\sigma(t)} k\left(s,\tau,x\left(\eta(\tau),\omega\right),\omega\right) d\tau, \omega \right)$ is closed,

$$v(t,\omega) \in F\left(t, x\left(\theta(t),\omega\right), \int_0^{\sigma(t)} k\left(s,\tau,x\left(\eta(\tau),\omega\right),\omega\right) d\tau, \omega \right) \text{ a.e. for } t \in J.$$

Hence, $v \in S_F^1(\omega)(x)$. As a result, $S_F^1(\omega)(x)$ is closed set in $L^1(J,R)$ for each $\omega \in \Omega$. From the continuity of $\mathcal{K}$, it follows that $\left(\mathcal{K} \circ S_F^1(\omega)(x)\right)$ is a closed set in X. Therefore, $Q(\omega)$ is a closed-valued multi-valued operator on [a, b] for each $\omega \in \Omega$.

Next, we show that $Q(\omega)$ is a multi-valued random operator on *X*. First, we show that the multi-valued map $(\omega, x) \mapsto S_F^1(\omega)(x)$ is measurable. Let $f \in \mathcal{M}\left(\Omega, L^1(J, R)\right)$ be arbitrary. Then we have

$$d\left(f, S_F^1(\omega)(x)\right) = \inf\left\{\left\| f(\omega) - h(\omega)\right\|_{L^1} : h \in S_F^1(\omega)(x)\right\}$$

$$= \inf\left\{\int_0^T \left| f(t,\omega) - h(t,\omega)\right| dt : h \in S_F(\omega)(x)\right\}$$

$$= \int_0^T d\left(f(t,\omega), F\left(t, x\left(\theta(t),\omega\right), \int_0^{\sigma(t)} k\left(t,s,x\left(\eta(s),\omega\right),\omega\right) ds, \omega\right)\right).$$

It can be shown as in the Step I of the proof of Theorem 2.2 that the mapping $(t,\omega) \mapsto \int_0^{\sigma(t)} k\left(t,s,x\left(\eta(s),\omega\right),\omega\right) ds$ is jointly measurable for all $x \in \mathcal{M}\left(\Omega, X\right)$. Again the mapping $z \mapsto d\left(z, F(t,x,y,\omega)\right)$ is continuous and hence, in view of hypothesis (B_2), the mapping

$$(t,x,\omega,f) \mapsto d\left(f(t,\omega), F\left(t, x(\theta(t),\omega), \int_0^{\sigma(t)} k\left(t,s,x\left(\eta(s),\omega\right),\omega\right) ds, \omega\right)\right)$$

is measurable from $J \times X \times \Omega \times L^1(J,R)$ in to R^+. Now the integral is the limit of the finite sum of measurable functions, and so, $d\left(f, S_F^1(\omega)(x)\right)$ is measurable. As a result, the multi-valued mapping $(\cdot,\cdot) \to S_{F(\cdot)}^1(\cdot)$ is jointly measurable.

Define the multi-valued map ϕ on $J \times X \times \Omega$ by

$$\phi(t,x,\omega)=\int_0^t F\left(s,x(\theta(s),\omega),\int_0^{\sigma(s)} k\left(s,\tau,x(\eta(\tau),\omega),\omega\right)d\tau,\omega\right)ds.$$

We shall show that $\phi(t,x,\omega)$ is continuous in *t* in the Hausdorff metric on R. Let { t_n } be a sequence in *J* converging to $t \in J.$ Then we have

$$d_H\left(\phi(t_n,x,\omega),\phi(t,x,\omega)\right)$$

$$=\int_J \left|\chi_{[0,\sigma(t_n)]}(s)-\chi_{[0,\sigma(t)]}(s)\right| h_r(s,\omega)ds$$

$$=\int_J \left|\chi_{[0,\sigma(t_n)]}(s)-\chi_{[0,\sigma(t)]}(s)\right| h_r(s,\omega)ds$$

$$\to 0 \quad as \quad n\to\infty.$$

$$=d_H\left(\int_0^{t_n} F\left(s,x(\theta(s),\omega),\int_0^{\sigma(s)} k\left(s,\tau,x(\eta(\tau),\omega),\omega\right)d\tau,\omega\right)ds,\right.$$

$$\left.\int_0^{t} F\left(s,x(\theta(s),\omega),\int_0^{\sigma(s)} k\left(s,\tau,x(\eta(\tau),\omega),\omega\right)d\tau,\omega\right)ds\right)$$

$$=\int_J \left|\chi_{[0,\sigma(t_n)]}(s)-\chi_{[0,\sigma(t)]}(s)\right| h_r(s,\omega)ds$$

$$=\int_J \left|\chi_{[0,\sigma(t_n)]}(s)-\chi_{[0,\sigma(t)]}(s)\right| h_r(s,\omega)ds$$

$$\to 0 \quad as \quad n\to\infty.$$

Thus the multi-valued map $t \mapsto \phi(t,x,\omega)$ is continuous and hence, by Lemma 2.2.2, the multi-valued mapping

$$(t,x,\omega)\mapsto\int_0^t F\left(s,x\big(\eta(s),\omega\big),\int_0^{\sigma(t)} k\big(t,s,x\big(\eta(s),\omega\big),\omega\big)ds,\omega\right)ds$$

is measurable. Again, since the sum of two measurable multi-valued functions is measurable, the map

$$(t,x,\omega)\mapsto q_0(\omega)+q_1(\omega)\omega+\frac{q_2}{2}(\omega)\omega^2+\int_0^t F\left(s,x\big(\eta(s),\omega\big),\int_0^{\sigma(t)} k\big(s,\tau,x\big(\eta(\tau),\omega\big),\omega\big)d\tau,\omega\right)ds$$

is measurable. Consequently, $Q(\omega)$ is a random multi-valued operator on [*a, b*].

Step II : Secondly, we show that $Q(\omega)$ is right monotone increasing and multi-valued random operator on [*a, b*] into itself for each $\omega\in\Omega$. Let $x,y\in[a,b]$ be such that $x\le y$. Since (B_4) holds, we have that $S_F^1(\omega)(x)\overset{i}{\le}S_F^1(\omega)(y)$. Hence $Q(\omega)(x)\overset{i}{\le}Q(\omega)(y)$. From ($H_4$) it follows that $a\le Q(\omega)a$ adn $Q(\omega)b\le b$ for all $\omega\in\Omega$. Now $Q(\omega)$ is right monotone increasing, so we have for each $\omega\in\Omega$,

$$a\le Q(\omega)a\overset{i}{\le}Q(\omega)x\overset{i}{\le}Q(\omega)b\le b$$

for all $x\in$[*a, b*]. Hence *Q* defines a right monotone increasing multi-valued random operator $Q:\Omega\times[a,b]\to\mathcal{P}_{cl}\big([a,b]\big)$.

Step III : Next, we show that $Q(\omega)$ is completely continuous for each $\omega\in\Omega$. First, we show that $Q(\omega)$ ([a, b]) is compact for each $\omega\in\Omega$. Let { $y_n(\omega)$ } be a sequence in $Q(\omega)$ ([a, b]) for some $\omega\in\Omega$. We will show that { $y_n(\omega)$ } has a cluster point. This is achieved by showing that { $y_n(\omega)$ } is uniformly bounded and equi-continuous sequence in *X*.

Case I : First, we show that { $y_n(\omega)$ } is uniformly bounded sequence. Since the cone *K* in X is normal, the random order interval [*a, b*] is norm-bounded. Hence there is a real number r > 0 such that $\|y_n(\omega)\|\le r$ for all $n\in\mathbb{N}$.

By the definition of { $y_n(\omega)$ }, we have a $v_n(\omega) \in S_F^1(\omega)(x)$ for some $x \in [a, b]$ such that

$$y_n(t,\omega) = q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2 + \int_0^t v_n(s,\omega)ds, \quad t \in J.$$

Therefore,

$$|y_n(t,\omega)| \le \left| q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2 \right| + \int_0^t |v_n(s,\omega)| ds$$

$$\le |q_0(\omega)| + |q_1(\omega)\omega| + \left|\frac{q_2}{2}(\omega)\omega^2\right| + \int_0^t \left\| F\left(s, x(\theta(s),\omega), \int_0^{\sigma(s)} k\left(s,\tau,x(\eta(\tau),\omega),\omega) d\tau,\omega\right) ds \right) \right\|_{\mathrm{P}} ds$$

$$\le |q_0(\omega)| + |q_1(\omega)\omega| + \left|\frac{q_2}{2}(\omega)\omega^2\right| + \|h_r(\omega)\|_{L^1}$$

for all $t \in J$, where $r = \|a(\omega)\| + \|b(\omega)\|$. Taking the supremum over t in the above inequality yields,

$$\|y_n(\omega)\| \le |q_0(\omega)| + |q_1(\omega)\omega| + \left|\frac{q_2}{2}(\omega)\omega^2\right| + \|h_r(\omega)\|_{L^1}$$

which shows that { $y_n(\omega)$ } is a uniformly bounded sequence in $Q(\omega)([a,b])$.

Next we show that { $y_n(\omega)$ } is an equi-continuous sequence in $Q(\omega)([a,b])$. Let $t,\tau \in J$. Then we have

$$|y_n(t,\omega) - y_n(\tau,\omega)| \le \left| \int_0^t v_n(s,\omega)ds - \int_0^\tau v_n(s,\omega)ds \right|$$

$$\le |p(t,\omega) - p(\tau,\omega)|,$$

where, $p(t,\omega)=\int_0^t h_r(s,\omega)ds$. From the above inequality, it follows that

$$\left|y_n(t,\omega)-y_n(\tau,\omega)\right|\to 0 \quad \text{as} \quad t\to\tau.$$

This shows that { $y_n(\omega)$ } is an equi-continuous sequence in $Q(\omega)$ ([a, b]). Now { $y_n(\omega)$ } is uniformly bounded and equi-continuous for each $\omega\in\Omega$, so it has a cluster point in view of Arzela-Ascoli theorem. As a result, $Q(\omega)$ is a compact multi-valued random operator on [*a, b*].

Case II : Next we show that $Q(\omega)$ is an upper semi-continuous multi-valued random operator on [a, b]. Let { $x_n(\omega)$ } be a sequence in [a, b] such that $x_n(\omega)\to x_*(\omega)$. Let { $y_n(\omega)$ } be a sequence such that $y_n(\omega)\in Q(\omega)x_n$ and $y_n(\omega)\to y_*(\omega)$. We shall show that $y_*(\omega)\in Q(\omega)x_*$. Since $y_n(\omega)\in Q(\omega)x_n$, there exists a $v_n(\omega)\in S_F^1(\omega)(x_n)$ such that

$$y_n(t,\omega)=q_0(\omega)+q_1(\omega)\omega+\frac{q_2}{2}(\omega)\omega^2+\int_0^t v_n(s,\omega)ds, \quad t\in J.$$

We must prove that there is a $v_*(\omega)\in S_F^1(\omega)(x_*)$ such that

$$y_*(t,\omega)=q_0(\omega)+q_1(\omega)\omega+\frac{q_2}{2}(\omega)\omega^2+\int_0^t v_*(s,\omega)ds, \quad t\in J.$$

Consider the continuous linear operator $\mathrm{L}:\mathrm{M}\left(\Omega,L^1(J,R)\right)\to\mathrm{M}(\Omega,X)$ defined by

$$\mathrm{L}v(t,\omega)=\int_0^t v(s,\omega)ds, \quad t\in J.$$

Now
$$\left\| \left(y_n(\omega) - (q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2) \right) - \left(y_*(\omega) - (q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2) \right) \right\| \to 0 \ \ as \ \ n \to \infty$$

From lemma 2.2, it follows that $\mathcal{L} \circ S_F^1(\omega)$ is a closed graph operator. Also from the definition of $\mathcal{L}$, we have $y_n(t,\omega) - (q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2) \in \left(\mathcal{L} \circ S_F^1(\omega)\right)(x_n)$
.

Since $y_n(\omega) \to y_*(\omega)$, there is a point $v_*(\omega) \in S_F^1(\omega)(x_*)$. such that

$$y_*(t,\omega) = q_0(\omega) + q_1(\omega)\omega + \frac{q_2}{2}(\omega)\omega^2 + \int_0^t v_*(s,\omega)ds, \quad t \in J$$
.

This shows that $Q(\omega)$ is a upper semi-continuous multi-valued random operator on [a, b]. Thus $Q(\omega)$ is an upper semi-continuous and compact and hence completely continuous multi-valued random operator on [a, b]. Now an application of Theorem 3.1 yields that $Q(\omega)$ has a random fixed point which further implies that the FRIGDI (2.1.1) has a random solution on J × Ω. This completes the proof.

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