Quest for Research Excellence On Computing, Mathematics and Statistics

> Editors Kor Liew Kee Kamarul Ariffin Mansor Asmahani Nayan Shahida Farhan Zakaria Zanariah Idrus



Faculty of Computer and Mathematical Sciences

Conceptor

# Quest for Research Excellence on Computing, Mathematics and Statistics

**Chapters in Book** 

The 2<sup>nd</sup> International Conference on Computing, Mathematics and Statistics (iCMS2015)

Editors:

Kor Liew Lee Kamarul Ariffin Mansor Asmahani Nayan Shahida Farhan Zakaria Zanariah Idrus



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# Content

## International Scientific Committee

Preface

CHAPTER 1	
CHAPTER 2	
<b>CHAPTER 3 </b>	
CHAPTER 4	
<b>CHAPTER 541</b> Dijkstra's Algorithm In Product Searching System (Prosearch) Nur Hasni Nasrudin, Siti Hajar Nasaruddin, Syarifah Syafiqah Wafa Syed Abdul Halim and Rosida Ahmad Junid	
CHAPTER 6	;

CHAPTER 7	
CHAPTER 8	
CHAPTER 9	
CHAPTER 10	
CHAPTER 11	
CHAPTER 12	
CHAPTER 13	
CHAPTER 14	
CHAPTER 15	

CHAPTER 16
CHAPTER 17
CHAPTER 18
CHAPTER 19
CHAPTER 20
CHAPTER 21213Estimating Philippine Dealing System Treasury (PDST)Reference Rate Yield Curves using a State-Space Representationof the Nelson-Siegel ModelLen Patrick Dominic M. Garces, and Ma. Eleanor R. Reserva
CHAPTER 22

CHAPTER 23
Partial Least Squares Based Financial Distressed Classifying Model of Small Construction Firms
Amirah-Hazwani Abdul Rahim, Ida-Normaya M. Nasir, Abd-Razak Ahmad, and Nurazlina Abdul Rashid
CHAPTER 24
CHAPTER 25
Data Mining in Predicting Firms Failure: A Comparative Study Using Artificial Neural Networks and Classification and
Regression Tree Norashikin Nasaruddin, Wan-Siti-Esah Che-Hussain, Asmahani Nayan, and Abd-Razak Ahmad
<b>CHAPTER 26265</b> Risks of Divorce: Comparison between Cox and Parametric Models
Sanizah Ahmad, Norin Rahayu Shamsuddin, Nur Niswah Naslina Azid @ Maarof, and Hasfariza Farizad
CHAPTER 27
Version: A Pilot Study Kartini Kasim, Norin Rahayu Shamsuddin, Wan Zulkipli Wan Salleh, Kardina Kamaruddin, and Norazan Mohamed Ramli
CHAPTER 28
Outlier Detection in Time Series Model Nurul Sima Mohamad Shariff, Nor Aishah Hamzah, and Karmila Hanim Kamil
CHAPTER 29

CHAPTER 30
CHAPTER 31
CHAPTER 32
CHAPTER 33
CHAPTER 34
CHAPTER 35

CHAPTER 36	381
Technology Assistance for Kids with Learning Disabilities:	
Challenges and Opportunities	

Challenges and Opportunities Suhailah Mohd Yusof, Noor Hasnita Abdul Talib, and Jasmin Ilyani Ahmad

## CHAPTER 9 The Geometry of the Accessible Sets of Vector Fields

A.Y.Narmanov, and I. Ganiev

**Abstract.** In this paper it is studied geometry of accessible sets of family of vector fields. It is proved that sets of 0- accessible sets are smooth submanifolds of an orbit. It is also proved that on the any manifold there is a system of vector fields consisting of three vector fields, such that each set of 0- accessible sets coincide with the manifold.

Keywords: manifold; vector field; orbit; 0- accessible sets

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#### 1 Introduction

The study of the geometry of the accessible set is one of the main objectives of quality control theory and is closely related to the geometry of the orbits of vector fields. Study of the structure of the accessible set and orbit systems of smooth vector fields are objectives of many mathematicians because of its importance in optimal control theory, dynamical systems, in geometry and in foliation theory [1-6].

#### 2 Main Results

Let M-smooth manifold of dimension n, V(M)- the set of all smooth vector fields defined on M. We denote by [X,Y] Lie bracket of vector fields  $X, Y \in V(M)$ . With respect to the Lie bracket of the set V(M) is an algebra Lee. Smoothness in this paper is smooth of class  $C^{\infty}$ .

Consider the set  $D \subset V(M)$ , through A(D) denote smallest Lie subalgebra containing the set D. Family D may contain a finite and an infinite number of smooth vector fields.

For a point  $x \in M$  denote by  $t \to X^{t}(x)$  the integral curve of the vector field X, passing through the point x at t = 0. The map  $t \to X^{t}(x)$  defined in a certain region  $I(x) \subset R$ , which generally depends on the field X, and from starting point x.

In the future, everywhere in the formulas of the form  $X^{t}(x)$ , we assume that  $t \in I(x)$ . If for all points  $x \in M$  domain I(x) of the curve  $t \to X^{t}(x)$  coincides with the real axis, then vector field X is called a complete vector field. In this case flow of X generates a dynamical system.

**Definition 1.** Orbit L(x) family D of vector fields, passing through the point x defined as the set of points y from M, for which there exist real numbers  $t_1, t_2, ..., t_k$  and vector fields  $X_1, X_2, ..., X_k$  from D (where k – is an arbitrary natural number) such that

$$y = X_k^{t_k} (X_{k-1}^{t_{k-1}} (... (X_1^{t_1} (x))...)).$$

#### Definition 2. Point

$$y = X_k^{t_k} \left( X_{k-1}^{t_{k-1}} (... (X_1^{t_1} (x))...) \right) \in L(x)$$

called T – accessible from the point  $x \in M$ , if  $\sum_{i} t_i = T$ .

We denote by  $A_x(T)$  the set of T-accessible points from the point  $x \in M$ .

In [Sussmann. H. ., 1973,] it is proved that every orbit of the family of vector fields with the topology Sussmann has a differential structure, with respect to which it is a smooth manifold smoothly immersed in M.

The following theorem shows that the set  $A_x(T)$  has a differential structure of a smooth manifold.

**Theorem 1.** Set  $A_x(T)$  for each  $x \in M$  for each T is immersed submanifold of orbit codimension one or zero.

**Theorem 2.** Let *M* - smooth connected manifold of dimension  $n \ge 2$ . There exist a system *D*, consisting of three vector fields, such that  $A_x(0) = M$  for each point  $x \in M$ .

In the event that the Euler characteristic of M is non-zero, there is a stronger statement.

**Theorem 3.** Let M - be a smooth compact connected manifold of dimension Euler characteristic which is different from zero. There is a system D on Mconsisting of two vector fields such that  $A_x(0) = M$  for each point  $x \in M$ .

The following example shows that the on the compact connected manifold M with zero Euler characteristic can also be a system consisting of two vector fields such that  $A_x(0) = M$  for each point  $x \in M$ .

Let the three-dimensional sphere  $S^3 \subset R^4$  is given by  $x^2 + y^2 + z^2 + w^2 = 1$ , where x, y, z, w- the Cartesian coordinates in  $R^4$ .

Let's consider a system on  $S^3$ , consisting of two on the vector fields:

$$X = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y} - \frac{\partial}{\partial z} + z\frac{\partial}{\partial z}, Y = -z\frac{\partial}{\partial x} + x\frac{\partial}{\partial z}$$

It is easy to verify that these fields are vector fields Killing, i.e. local diffeomorphisms  $x \to X^{t}(x)$ ,  $x \to Y^{t}(x)$  for every t are isometries sphere of  $S^{3}$ .

Lie bracket [X, Y] of vector fields X, Y is as follows:

$$[X,Y] = -w\frac{\partial}{\partial x} - z\frac{\partial}{\partial y} + y\frac{\partial}{\partial z} + x\frac{\partial}{\partial w}$$

At the point  $p(1,0,0,0) \in S^3$  the vectors X(p), Y(p), [X,Y](p) are linearly independent, subspace  $A_p(D) = \{X(p) : X \in A(D)\}$  is threedimensional. Therefore, the orbit is three-dimensional. Due to the fact that they are the Killing vector fields, the orbit is a closed set [Narmanov.A.Y.,Saitova S, 2014]. On the other hand, by virtue of the maximality of dimension of the orbit it is an open set. Consequently, the orbit is coincident with  $S^3$ .

Now consider the sets  $A_q(0)$  for  $q \in S^3$ . If the sets  $A_q$  are submanifolds of codi-mension one, owing to the fact that X, Y-are Killing vector fields, they generate two-dimensional Riemannian foliation on [3]. Recall that the foliation is called the Riemannian if every geodesic orthogonal to foliation at some point, it is orthogonal to foliation at all points [3]. How should follows from the results of [Thurston W, 1976], on the three-dimensional sphere doesn't exist two-dimensional Riemannian foliation. Consequently, the set  $A_q(0)$  is coincident with  $S^3$ .

In the following example sets  $A_q(0)$  are submanifolds of orbits L(p) codimension one. Let  $M = R^3$ , D - the family consists of following two Killing vector fields

$$X = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}, Y = \frac{\partial}{\partial z}.$$

In this case, for each point  $p(x, y, z) \in M$ , such that  $x^2 + y^2 > 0$  the orbit L(p) is a cylinder, and the set  $A_q(0)$  for each point  $q \in L(p)$  is a spiral line, tangent field vector field of which is  $Z = -y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y} - \frac{\partial}{\partial z}$ . For points of the axis *OZ* of orbit L(p) and sets  $A_q(0)$  coincides with the axis *OZ*.

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