Properties of Hantavirus Infection Model

Norhuda Mohammed
Asyura Abd Nassir
Nazirah Ramli

ABSTRACT

Hantavirus is a group of viruses that can cause Hantavirus Pulmonary Syndrome (HPS) and Hemorrhagic Fever with Renal Syndrome (HFRS). Human contract this virus through their rodent hosts. Since the discovery of Hantavirus, there is still no vaccination for the diseases caused by the viruses. This situation has inspired researchers to explore the ecology and epidemiology of Hantaviruses in order to come out with a better solution in preventing this infection. In this paper, we investigate the mathematical model of Hantavirus infection known as Abramson-Kenkre (AK) model. The result shows that by changing the values of certain data such as the transmission rate responsible for infection, the birth rate and the natural death rate, different results can be obtained.

Keywords: *Hantavirus infection, Abramson-Kenkre (AK), disease*

Introduction

Numerous studies have shown a strong correlation between generalist rodent species and infectious diseases carried by them such as Hantavirus Pulmonary Syndrome (HPS). HPS is caused by a group of viruses that were first isolated in the laboratory from striped field mice captured near Korea's Hantaan River called as Hantavirus. Their potential for causing severe human illness makes these viruses an important public health concern.

The Hantavirus infection is carried by mice that move from location to location, and is transmitted to other mice through fight. The mice do not die or are impaired, from contraction of the virus. Vertical transmission (infected mother transfer the disease to offspring), appears to be negligible or nonexist. Humans get the virus from the mice but have no feedback effects on the mice in the infection process (Kenkre, 2005).

In this research, we investigate the mathematical model in modelling and analyzing Hantavirus infection developed by Abramson and Kenkre (2002) or known as AK model. The AK model analyzes spatio-temporal patterns in the spread of Hantavirus infection based on differential equation system. Figure 1 shows the result of analysis by AK model.

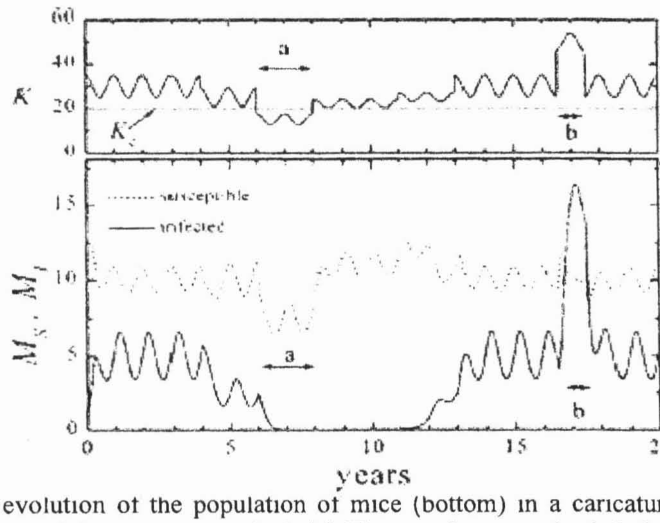

Figure1: Temporal evolution of the population of mice (bottom) in a caricature time-dependent carrying capacity (top). Two special events are marked: (a) The carrying capacity is below the *Kc* threshold (shown as a horizontal line). (b) An extraordinary one-year event of greater carrying capacity. Model parameters are $a = 0.1$, $b = 1$, and $c = 0.5$. (Abramson & Kenkre, 2002)

AK model is able to reproduce several observed features in Hantavirus epidemics such as refugia which is foci of infection in the landscape. It also has qualitative and semi-quantitative success in explaining observations such as spatio-temporal patterns in the epidemics. Besides, AK model can successfully explain several field observations as environmentally controlled phase transitions, and thus provide an analytical support to biological hypotheses such as the trophic cascade discussed in Yates et. al, 2002(Abramson, 2007).

However, based on the AK result (as shown in Figure 1), there is no relation between *K* (carrying capacity) and *Ms* (susceptible mouse) or *Mi* (infected mouse). In this paper we investigate in detail the relationship between *K* and *Ms* or *Mi* due to the changes of data. This relation is important as the changes in *K*, will affect the value of susceptible and infected mice. Thus, by changing the values of *K*, the relation between them can be obtained.

Preliminary

The AK model for mouse population with two classes represented by *Ms* for susceptible mouse and *Mi* for infected mouse is shown by Equation (1):

$$\frac{dMs}{dt} = bM - cMs - \frac{MsM}{K} - aMsMi \qquad (1)$$
$$\frac{dMi}{dt} = -cMi - \frac{MiM}{K} + aMsMi$$

where *a*, *b*, *c* and *K* are constant with
M is the total population,
a represent the rate of transformation from susceptible to infected, due to fight when the population is over crowded
b signify the births of mice, given that all mice are born susceptible,
c characterizes the death rate for natural reason such that the rodent not infected by the virus carried by them,
K is carrying capacity that is the resource used by mice for the survival such as food and shelter and,
– ((*MsM or MiM)/K*) show the limitation process of population growth.

The range of the deer mice comprises a diverse landscape with a variety of habitats. The inhomogeneous way in which this spatial extent affects local populations can be included in a spatially extended version of the model, where *Ms*, *Mi*, and *K* become functions of a space variable *x* (Abramson, 2007). By using a simple diffusion model for the transport mechanism of the population, the extended model is shown by Equation (2):

$$\frac{dMs}{dt} = f(Ms, Mi) + Ds\nabla^2 Ms \qquad (2)$$
$$\frac{dMi}{dt} = g(Ms, Mi) + Di\nabla^2 Mi$$

where *f* and *g* are the right-hand sides of Equation (1) and the included separate diffusion coefficients *Ds* and *Di* for the two classes of mice (Abramson & Kenkre, 2002). The solution of Equation (2) and even its stationary solution may be impossible to find, analytically, for an arbitrary function *K*(*x*) (Abramson, 2007).

Methodology

In this research, the Matlab programming has been used to generate the result when the data is changed. The equation is solved by using Rungge-Kutta 4th Order Method where this method has the advantage of high order local truncation error. Based on the AK result, the value of *K* is divided into few values depending on the interval of years as shown in Table 1. The value at the end of one partition becomes the initial value for next partition. This is to make sure that the figure will be continuous through 20 year

Table 1. Value of K for different intervals

Value of K	Interval of years
40	[0, 2]
30	[2, 4]
20	[4, 6]
5	[6, 8]
40	[8, 10]
30	[10, 12]
20	[12, 14]
60	[14, 16]
40	[16, 18]
30	[18, 20]

Result and Discussion

The numerical experiments on AK equation have been conducted and the result of the simulation has been studied with the initial condition: $Ms = 15$, $Mi = 1$, $a=0.1$, $b = 1$, $c = 0.5$.

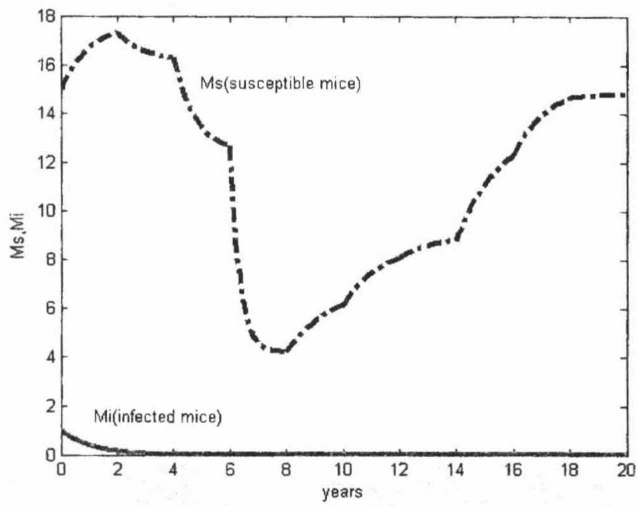

Figure 2: Changes of the number of infected and susceptible mice when the transmission responsible for infection decreased

From Figure 2, the rate of transformation is reduced from 0.1 to 0, which means that there is no transmission from susceptible mice to infected mice. The number of infected mice will drop to zero as time approaches to infinity. As mentioned earlier, mice do not die because of the virus, so it is probably because of competition to obtain the space. The susceptible mice survive due to no transformation of the virus from the infected mice.

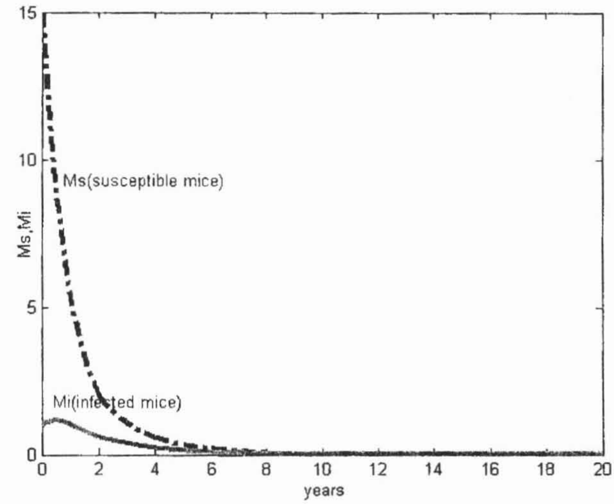

Figure 3: Changes of the number of infected and susceptible mice when the birth rate decreased

Furthermore, graph in Figure 3 shows that when the birth rate decreased from 0.1 to 0, both of the graph *Ms* and *Mi* will be dropped. This situation happened because when there is no increasing of mice so there will be no new infected and susceptible mice.

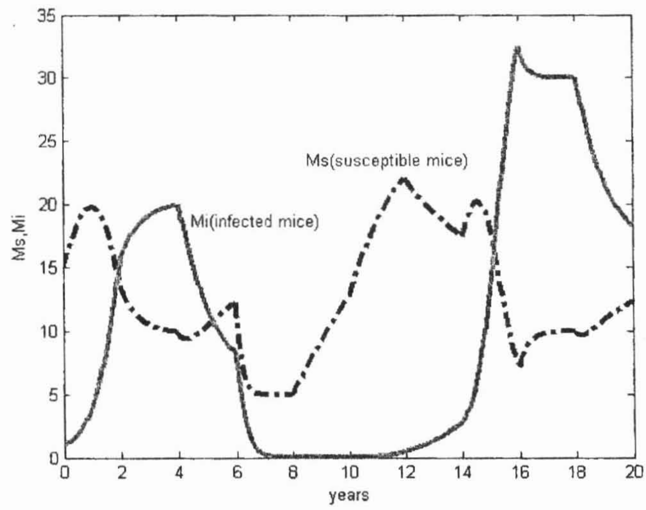

Figure 4: Changes of the number of infected and susceptible mice when the natural death rate decreased

Last but not least, graph in Figure 4 shows the situation that will occur if the natural death rate is decreased. The graph clearly shows that the number of infected mice changed inversely with the number of susceptible mice. Even when the rate of death is decreasing during that time, the rate of birth of the mice is still increasing as usual. This shows that when the number of the infected mice increased, the number of susceptible mice decreased. Conversely, when the number of the infected mice decreased, the number of susceptible mice increased.

Conclusion

This study is conducted to investigate on the properties of AK model and the relationship between K and Ms or Mi due to the changes of data. The result shows that changing in some of the data such as the transmission rate responsible for infection, the birth rate and the natural death rate will affect the values of infected and susceptible mice. As a conclusion, AK model is suitable to demonstrate the Hantavirus infection.

References

Abramson, G. & Kenkre, V. M. (2002). Spatio-temporal patterns in the Hantavirus infection. *Physical Review E, 66, 011912-1-5.*

Abramson, G. & Kenkre, V. M. (2001). Mathematical Modelling of Refugia in the Spread of the Hantavirus. *Physical Review. E, 1.*

Abramson, G. (2007). Of Mice and Viruses. *Actas de la Academia Nacional de Ciencias, 13, 33-40.*

Abramson, G. (2007). Mathematical Modelling Of Hantavirus:From The Mean Field To The Individual Level. *Nova Science Publishers, Inc, ISBN 978-1-60456-171-5*

Yates, T. L., Mills, J. N., Parmenter, C. A., Ksiazek, T. G., Parmenter, R. R., Vande Castle, J. R., Calisher, C. H., Nichol, S. T., Abbott, K. D., Young, J. C., Morrison, M. L., Beaty, B. J., Dunnum, J. L., Baker, R. J., Salazar-Bravo, J. & Peters, C. J., (2002). The ecology and evolutionary history of an emergent disease, *Bioscience 52, no. 11, 989-998.*

Abramson, G., Kenkre, V. M., Yates, T. L. & Parmenter, R. R. (2003). Travelling Waves of infection in the Hantavirus Epidemics. *Bulletin of Mathematical Biology, 65, 519-534.*

Carson-Dewitt, R. S. (2006). Hantavirus Infections. *Encyclopedia of Medicinie.*

Kenkre, V. M. (2005). Statistical Mechanical Considerations in the Theory of the Spread of the Hantavirus. *Physica A, 356, 121-126.*

Kenkre , V. M., Giuggioli, L., Abramson, G. & Camelo-Neto, G. (2005). Theory of Hantavirus Infection Spread Incorporating Localized Adult and Itinerant Juvenile Mice. *Physical Review E.*

Kenkre, V. M., Paramenter, R. R., Peixoto, I. D. & Sadasiv, L. (2004). A Theoretical Framework for the Analysis of the West Nile Virus Epidemic. *Physical review E, 1.*

NORHUDA MOHAMMED, ASYURA ABD NASSIR, NAZIRAH RAMLI, Universiti Teknologi MARA Pahang. hudamohammed@pahang.uitm.edu.my, asyuraan@pahang.uitm.edu.my, nazirahr@pahang.uitm.edu.my