

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**SECOND HANKEL DETERMINANT OF
CERTAIN SUBCLASSES OF ANALYTIC
BI-UNIVALENT FUNCTIONS SUBORDINATE
TO CHEBYSHEV POLYNOMIALS**

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Abstract

In geometric function theory, there are many types of class of analytic function that had already been introduced by many researchers. For every class of analytic function that have been defined and introduced, it has its own properties to be considered. However, the properties obtained are based on the suitability of the class of analytic functions. To determine the related properties of each class of analytic functions, extreme function must first be determined. Primarily, let $Q_{\Sigma}(\lambda, \mu, t)$ denote the class of analytic bi-univalent function subordinate to Chebyshev polynomials in an open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ and satisfy

$$(1 - \lambda) \frac{f(z)}{z} + \lambda f'(z) + \mu z f''(z) \prec H(t, z), z \in U$$

$$(1 - \lambda) \frac{g(w)}{w} + \lambda g'(w) + \mu w g''(w) \prec H(t, w), w \in D_{r_0}$$

where $z, w \in U$ for some $\lambda \in [0, 1]$, $\mu \in [0, 1]$ and $t \in \left(\frac{1}{2}, 1\right)$. Moreover, class of analytic function will significantly provide new contributions to geometric function theory. This project focus on attaining the coefficient bound, $|a_2|$, $|a_3|$, $|a_4|$ and the upper bound of the second Hankel determinant, $H_2(2)$ as the properties of the stated class of analytic function $Q_{\Sigma}(\lambda, \mu, t)$ by using Toeplitz determinant. Furthermore, to obtain the upper bound of the second Hankel determinant, the coefficient bound of the class of function $Q_{\Sigma}(\lambda, \mu, t)$ must be determined first. Apart from that, individual can improve existing proving skills learned from time to time. Meanwhile, by substituting certain values in the variables of the class of function, the class can be reduced to another class of function in which had been developed by other researchers.

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