# UNIVERSITI TEKNOLOGI MARA

# TECHNICAL REPORT

# SECOND HANKEL DETERMINANT OF CERTAIN SUBCLASSES OF ANALYTIC BI-UNIVALENT FUNCTIONS SUBORDINATE TO CHEBYSHEV POLYNOMIALS

**P37S18**

**NADZIRA AMANINA BINTI MOHAMAD**
**NOOR ALEEZA BINTI ABDUL WAHAB**
**AMY AIZAITUL AINA BINTI SHUHAIMEE**

**Report submitted in partial fulfilment of the requirement for the degree of Bachelor of Science (Hons.) Mathematics Faculty of Computer and Mathematical Sciences**

**DECEMBER 2018**

# Acknowledgements

IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL.

First of all, Alhamdulillah and praise to Allah SWT, whom with His willingness, giving us the strength and opportunity to complete this project successfully.

We would like to express our greatest appreciation to our most supportive yet dedicated supervisor, Madam Norlyda binti Mohamed for her valuable technical support and advise upon completing this project. Her willingness to spend her precious time to guide us to finish this group project should be appreciated. Thank you.

In addition, thanks to all group members, Nadzira Amanina binti Mohamad, Noor Aleeza binti Abdul Wahab and Amy Aizaitul Aina binti Shuhaimee for their cooperation, encouragement, constructive suggestion and full of support for the report completion, guidance in the compilation and preparation from the beginning until the end of the report.

Last but not least, we would also like to thank those who have been contributing by supporting our project progress until it is fully complete. Deepest thanks and appreciation to our parents, friends and others for helping us to complete this project. Thank you.

# Abstract


In geometric function theory, there are many types of class of analytic function that had already been introduced by many researchers. For every class of analytic function that have been defined and introduced, it has its own properties to be considered. However, the properties obtained are based on the suitability of the class of analytic functions. To determine the related properties of each class of analytic functions, extreme function must first be determined. Primarily, let $Q_{\Sigma}(\lambda, \mu, t)$ denote the class of analytic bi-univalent function subordinate to Chebyshev polynomials in an open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ and satisfy

$$(1-\lambda)\frac{f(z)}{z} + \lambda f'(z) + \mu z f''(z) \prec H(t, z), z \in U$$

$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) + \mu w g''(w) \prec H(t, w), w \in D_{r_0}$$

where $z, w \in U$ for some $\lambda \in [0, 1]$, $\mu \in [0, 1]$ and $t \in \left(\frac{1}{2}, 1\right)$. Moreover, class of analytic function will significantly provide new contributions to geometric function theory. This project focus on attaining the coefficient bound, $|a_2|$, $|a_3|$, $|a_4|$ and the upper bound of the second Hankel determinant, $H_2(2)$ as the properties of the stated class of analytic function $Q_{\Sigma}(\lambda, \mu, t)$ by using Toeplitz determinant. Furthermore, to obtain the upper bound of the second Hankel determinant, the coefficient bound of the class of function $Q_{\Sigma}(\lambda, \mu, t)$ must be determined first. Apart from that, individual can improve existing proving skills learned from time to time. Meanwhile, by substituting certain values in the variables of the class of function, the class can be reduced to another class of function in which had been developed by other researchers.

# Contents