

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**UPPER BOUND OF
FEKETE-SZEGÖ DETERMINANT
FOR α -CLOSE-TO-CONVEX FUNCTIONS**

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ABSTRACT

Every class of functions have their own unique properties, however each class can only be computed by using certain methods hence it has become cumbersome in determining which method is the best suited for which class of functions. The study is conducted to discover new subclass of analytic functions, and find the Fekete-Szegő determinant for the new subclass of analytic functions.

A class of α -close-to convex functions, $\mathcal{Q}(\alpha, \delta, t)$ defined in the unit disk,

$\mathcal{U} = \{z \in \mathbb{C} : |z| = 1\}$ which satisfied the condition $\operatorname{Re} \left\{ e^{i\alpha} \frac{zf'(z)}{g(z)} \right\} > \delta$, for

$z \in \mathcal{U}$, $|\alpha| \leq \pi$, $\cos \alpha > \delta$ and $g(z) = \frac{1}{(1-tz)(1-z)}$ where $-1 \leq t \leq 1$ was investigated.

The technique of comparing two coefficients, c_2 and c_3 from Kaharudin, Akbarally and Mohamad (2010), the application of lemma by Pommerenke (1975), and the triangle inequality, were been utilised with the purpose of acquiring the main results.

The findings had also contributed new results under Fekete-Szegő determinants which could be simplified into Srivastava, Mishra and Das (1999) and Kaharudin et al. (2010). Meanwhile, the class $g(z)$ was similar to the class found by Koebe (1907) and Yahya, Soh and Mohamad (2013) which is also a part of Sakaguchi function (Sakaguchi, 1959), hence the inequalities obtained were also the Fekete-Szegő determinants for Koebe function and Sakaguchi functions respectively. Correspondingly the results obtained would act as a guidance and reference to the others mathematicians and subsequently would strengthen previous studies' result. It is suggested that to always opt for the simplest form of equations specifically for fractions. This is because there are a lot of theorems that need to be proved and one requires a lot of steps, so using the simplest form of fractions really turns out to be useful.