UNIVERSITI TEKNOLOGI MARA

A SUBCLASS OF BOUNDED ANALYTIC FUNCTIONS

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AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

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ABSTRACT

Let S be the class of normalized analytic univalent functions f in the open unit disk $\Delta=\{z:|z|<1\}$ of the form

$$f(z)=z+a_2z^2+a_3z^3+\cdots=z+\sum_{n=2}^{\infty}a_nz^n.$$

Subclasses of S are starlike functions, convex functions and close-to-convex functions. The present work is devoted to the study of a certain subclass of starlike functions, S^* denoted by $G_{\alpha,\beta}(\gamma,\delta)$. The functions in this subclass satisfy the condition

$$\mathrm{Re}\,e^{i\delta}\{\alpha f'(z)+\beta zf''(z)\}>\gamma$$

for some $\alpha>0,\beta>0$ and $0\le\gamma<\alpha\cos\delta$ with $|\delta|\le\frac{\pi}{2}$ and $\alpha\cos\delta-\gamma>0$. By using an extreme function for this class, several extremal properties such as representation theorem, coefficient bound, condition so that the family of $G_{\alpha,\beta}(\gamma,\delta)$ is bounded in Δ, bound for $\mathrm{Re}\,f'(z)$ and $\mathrm{Re}\frac{f(z)}{z}$ in $f\in G_{\alpha,\beta}(\gamma,\delta)$, starlikeness of the class $G_{\alpha,\beta}(\gamma,0)$, some convolution properties and a result on the radius of univalence are obtained. Some coefficient inequalities of the class are also discussed. For the second Hankel determinant problems, we solved for the class $G_{\alpha,\beta}(\gamma,\delta)$ but for the Fekete-Szego problems, we solved for new defined class of function denoted by $J^{\eta}(\alpha,\beta)$ which satisfies

$$\mathrm{Re}\left(\frac{\alpha zf'(z)+\beta z^2f''(z)}{\alpha g(z)}\right)>0,\quad 0\le\beta<1,0<\alpha\le1,$$

where $g(z)\in S^*(\eta),g(z)\neq0\left(\mathrm{Re}\frac{zg'(z)}{g(z)}>\eta\right)$ for $0\le\eta<1$ which can also be reduced to the original defined class, $G_{\alpha,\beta}(\gamma,\delta)$.

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