Mathematical Modelling and Design of Counterweights for Unbalanced Links

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ABSTRACT

This paper proposes a procedure to mathematically model the counterweight of rotating links based on calculations according to design requirements. The main objective of the research is to introduce an effective method to design the counterweight using CAD drawing. A parameterized equation is formulated with the help of counterweight geometry derived from a typical crankshaft counterweight design. Using the law of moments and Center of gravity of links, the geometry of counterweight can be found using the formula that is parameterized with a single variable. The results were obtained by finding the roots of the final equation. The method was found to be effective in term of time consumption rather than implementing the iterative method.

Keywords: *Mathematical model; Counterweight design; Link balancing; centre of gravity; Centroid location.*

Introduction

Sébastien Briot and Vigen Arakelian [1] stated that whenever an unbalanced mechanism runs at high speeds, both shaking force along with shaking moment is transferred to the surroundings. These instabilities are the sources of vibrations, wear, noise and fatigue problems, and therefore cause the

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ISSN 1823- 5514, eISSN 2550-164X

Received for review: 2017-10-11
Accepted for publication: 2018-06-08
Published: 2018-11-30

decrease in machine efficiency. If the total mass centre of a mechanism is made static, then the vector sum of the forces acting on the mechanism frame disappears.

Arakelian and Smith [2] stated that some of the well-known balancing methods depend on the relocation of the mass of mechanism by the addition of counterweights to the links. This method is usually restricted to uncomplicated mechanisms which have only revolute joints, for overall shaking force balancing. Arakelian & Smith [3] mentioned that in high speed operating machines, vibration can be reduced by mass balancing the moving links that significantly enhance their performances. Nevertheless, absolute shaking force and shaking moment balancing is a complex problem. A counterweight is added on the opposite side of pivot of the link which permits the relocation of the centre of mass of total link to the joint.

Arakelian and Smith [2] mentioned in their review paper about the inertia forces in four-bar linkages. In Figure 1(a) Shaking force balancing is accomplished by two counterweights connected to crank 2 and to rocker 4. This achieves partial force balance but takes no account of coupler 3. An alternative method that is less frequently applied is shown in Figure 1(b) which balances coupler first and based on the effect of increased crank mass, the entire mechanism is balanced.

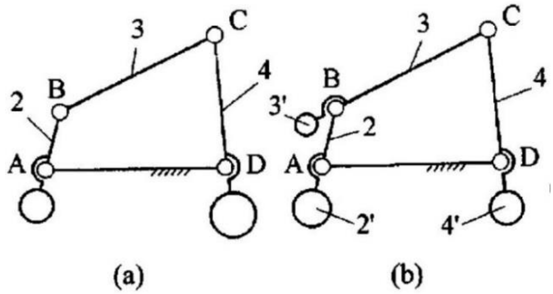

Figure 1: Partial shaking force balanced configuration [2]

Berkof and Lowen [4] proposed a new method called "Method of Linearly Independent Vectors" to completely force balance planar linkages. This way, the overall mass centre can be made static, thus the shaking force disappears. They studied that the minimum number of counterweights required to balance a four-bar mechanism is two, one at the crank and another at rocker. In other words, the counterweights are coupled to the fixed pivots of the input link and the output link. This means that the coupler link remains unchanged. The shaking force and shaking moment are generated within the mechanism due to the inertia loads. By the formulation of mass rearrangement and introduction of secondary masses, the complete shaking force and shaking moment of a four-bar mechanism vanish.

P. Nehemiah et al. [5] presented their research techniques to completely balance the shaking forces of planar linkages of high complexity. These linkages are applied in many automatic machines and engines of various configurations. This force balancing includes the mass redistribution. The planar linkages are replaced by concentrated point masses which were balanced by the addition of counterweights on the other sides of linkages.

S.R. Deepak and G.K. Ananthasuresh [6] discussed in their paper three techniques to statically balance four-bar mechanism using free length springs and auxiliary links. The linkages are made balanced by introducing balanced parallelogram which balances the spring connected between its opposite vertices. The number of additional links added is less than or equal to that of other methods present in literature. S.R. Deepak and G.K. Ananthasuresh [7] later developed a method to statically balance revolute-joint linkages loaded by constant forces or free length springs without any aid of auxiliary bodies. This method iteratively removes the dependence of potential energy on variables of linkages.

Kailash and Himanshu [8] presented an optimization technique to minimize shaking forces using a genetic algorithm (GA) which develops an equivalent system of point masses to represent inertial properties of linkages. Equations of motion are developed related to the point masses and optimization problem is generated to minimize shaking forces.

Researchers tried to find new ways of balancing the linkages and mechanisms and most of them state that the addition of counterweight to links of mechanisms helps in reducing shaking forces. Hence counterweight mass and position calculation can be found using the formulations proposed by the researchers. CAD drawings can be generated for the counterweight design based on those calculations. However, imprecisions in the fabrication of counterweights yields an error in the mass balancing of linkages. A mechanism shows unbalance even if it is theoretically fully balanced, because of the inaccuracies of its manufacturing. Kamenskii [9] stated that the actual accuracy can be increased by experimental balancing on a special stand. The literature on the methods of designing the counterweights can be scarcely found. Engine manufacturers do not make their classified blueprints of engine design available for the public because it can be copied by rival industries. This paper shows a simple method to formulate a parameterized design equation for counterweight which does not require iterations and gives accurate results as needed.

Although the easiest way of designing is by trial-and-error, however, this technique is time-consuming. This paper shows how the counterweight designing is achieved using a mathematical model for an individual link based on an existing counterweight design for crankshafts in automotive. The procedure is achieved by using Law of moments and Centroid positioning using geometric constraints and ratios to restrict the centre of gravity of link on the joint to minimize shaking forces.

Methodology

A typical crankshaft counterweight (CW) design is shown in Figure 2 which is a solid rectangle at the bottom and a semi-circle at the top. This geometry is taken as a benchmark for the design of counterweight of a link for any type of motion in this research.

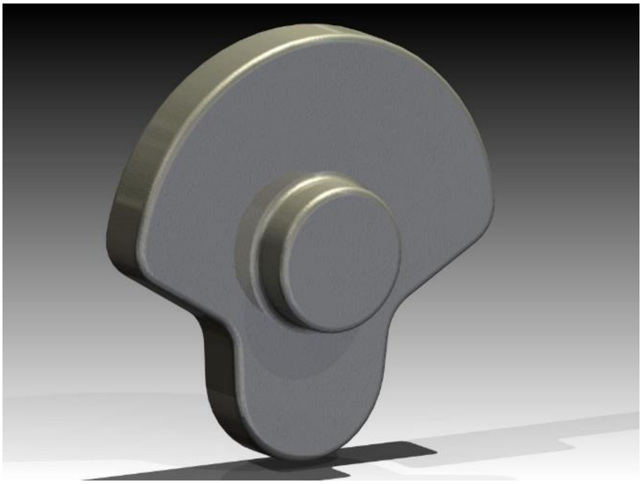

Figure 2: A typical counterweight design for crankshafts

In order to generate a simplified design for the typical counterweight and to formulate a parametric design equation, a rigid crank link is taken. In this case, it is assumed that the link dimensions and mass properties are known. Based on this information, Centroid or Center of gravity (CG) of the link with respect to the pivot point can be obtained using CAD software. The mass-centroid product of the link should be the same as that of its counterweight for complete static balancing according to Law of Moments.

In this study, the counterweight of shape as shown in Figure 3 is used, which includes a semi-circle at the top and a rectangle at the bottom which is directly attached to the link to be balanced. The link is pivoted on one end and the other end of the link may be connected to another link. The counterweight is to be placed on the other side of the pivot. The radius of the semi-circle is designated as r. The width d and thickness t of the rectangular body is set according to the dimensions of the actual link. The distance between the axis of the revolute joint or pivot O of the link and the end of the link is h which is dependent on the design of link and cannot be changed. On the other hand, the length b, which is the length of the rectangular body, is variable according to requirements.

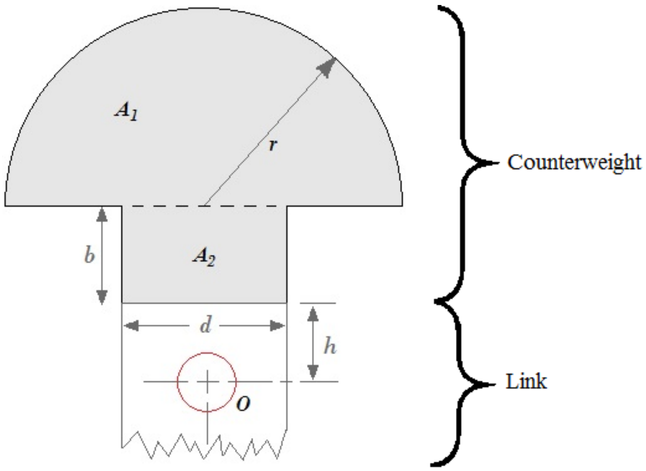

Figure 3: The geometric features of the proposed counterweight

Based on Figure 3, the area of semi-circle and rectangle are given by:

$$A_1 = \frac{\pi r^2}{2} \tag{1}$$
$$A_2 = bd \tag{2}$$

To simplify the counterweight design, the parameters are formulated to have a single variable, which in this case is the radius of a semi-circle. For variation in CG location within the design, b is defined as $b = cr$
where *c* is a user-defined ratio according to requirements. Hence from eq (2).

$$A_2 = crd \tag{3}$$

The total CW area will be
$$A_{TOTAL} = \left(\frac{\pi r^2}{2} + crd\right) \tag{4}$$

The CG position of total geometry can be calculated using the formula of the centroid for composite bodies. Figure 4 shows the CG positions of Semi-circle and Rectangle.

From Figure 4, the centroid composite formula can be represented as follows:
$$A_1 x_1 + A_2 x_2 = A_{TOTAL} x_{CG} \tag{5}$$

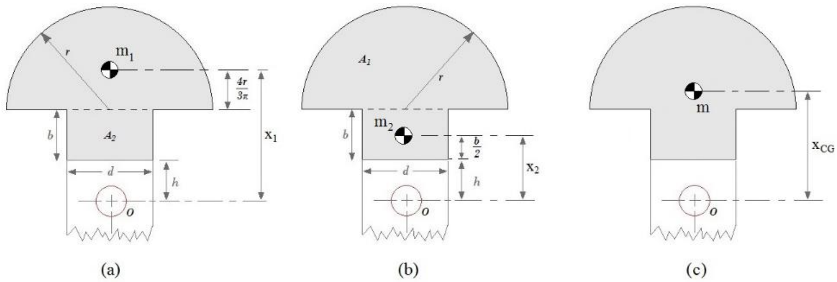

Figure 4: The centroid positions of the semi-circle (a), Rectangle (b) and Combined CW (c)

Substituting the values of A_1 and A_2 in eq. (5) yields

$$\frac{\pi r^2}{2}x_1 + crdx_2 = A_{TOTAL}x_{CG} \tag{6}$$

Where x_1, x_2 and x_{CG} are the CG position of the area of semi-circle A_1, the rectangular area A_2 and the combined counterweight geometry from pivot O respectively. x_{CG} is the overall CG position of the combined counterweight geometry.

Since the centroid of the semi-circle is given by $y = \frac{4r}{3\pi}$ and that of the rectangle by $y = \frac{b}{2}$ therefore,

$$x_1 = \left(h + b + \frac{4r}{3\pi}\right) \tag{7}$$

$$x_2 = \left(h + \frac{b}{2}\right) \tag{8}$$

For mass properties of CW, the following formula holds

$$V = A_{TOTAL}\, t = \frac{m}{\rho} \tag{9}$$

$$\Rightarrow V = A_{TOTAL} = \frac{m}{\rho t} \tag{10}$$

Where ρ is the density, m is the mass and v is the volume of counterweight. Putting A_{TOTAL} in equation (6).

$$\frac{\pi r^2}{2}\left(h + c.r + \frac{4r}{3\pi}\right) + (c.r.d)\left(h + \frac{c.r}{2}\right) = \frac{mx_{CG}}{\rho t} \tag{11}$$

To achieve a complete static balance of the link, by Law of moments, the counterweight must have the same mass moment as that of the actual link. In this case, $m.x_{CG}$ is the mass moment of CW, but the value of the mass moment of the actual link will be used in the equation to achieve static balance.

Simplifying eq (11), the following formula is obtained

$$\left(\frac{\pi}{2}c+\frac{2}{3}\right)r^3+\left(\frac{\pi}{2}h+\frac{c^2}{2}d\right)r^2+(c.h.d)r=\frac{mx_{CG}}{\rho t} \qquad (12)$$

This is a 3rd Order Equation with unknown variable r which can be solved for the roots. It gives two complex roots and a real root. The real value of r will be used to design the counterweight. If smaller or bigger value of r is desired, then the constant c can be varied to give different geometrical results.

Example

Assume a crank link as shown in Figure 5 with dimensions and properties given in Table 1.

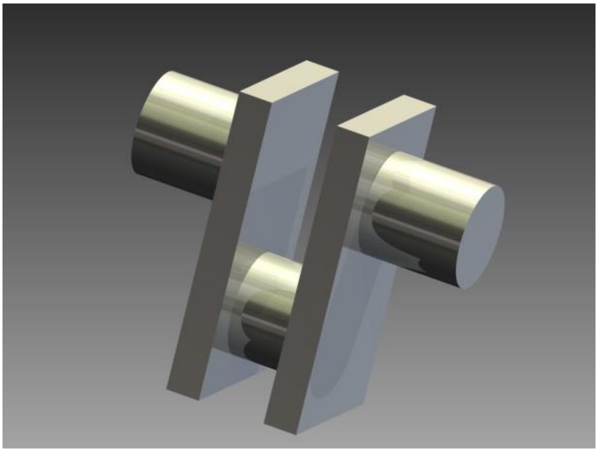

Figure 5: Unbalanced crank link

Assuming the value of $c = 0.2$
Therefore, substituting the values in Eq. (12)

$$\left(\frac{\pi}{2(0.2)}+\frac{2}{3}\right)r^3+\left(\frac{\pi}{2(12)}+\frac{(0.2)^2}{2(24)}\right)r^2+(0.2 \text{ x } 12 \text{ x } 24)r=\frac{0.139 \text{ x } 7.995}{7.86\text{x}10^{-6} \text{ x } 10} \qquad (13)$$

Simplifying yields

$$0.981r^3+19.33r^2+57.6r-14138.74=0 \qquad (14)$$

Table 1 Crank link dimensions and mass properties

Dimension/Property	Denotation	Value
Mass	m	0.139 kg
Density (Mild Steel)	ρ	$7.86\text{x}10^{-6}$ kg/mm^3
CG Position (from pivot)	x_{CG}	7.995 mm
Width	d	24 mm
Thickness	t	10 mm
Pivot to End-point distance	h	12 mm

The roots of Eq. (12) are:

$$r = 18.63$$
$$r = -19.17 + 20.14i$$
$$r = -19.17 - 20.14i \qquad (15)$$

There are two complex roots (16,17) and one real root (15) of the equation. Therefore, the real value of $r = 18.63$ mm will be used to design the counterweight of the crank link.

Since

$$b = c.r$$
$$b = 0.2 \text{ x } 18.63 = 3.72\text{mm} \qquad (16)$$

After designing the counterweight based on the calculated radius r, the final crank link transformed as shown in Figure 6.

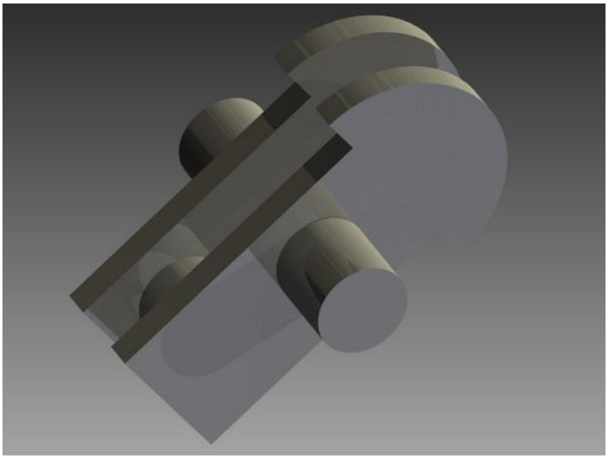

Figure 6: Balanced crank link

The centroid location of the balanced link from the pivot of the crank link is 0.019mm. This small round-off error of 1.9% rises because of using 2 significant figures of values used in calculations and CAD drawing. As this balanced crank link has its CG precisely at its axis of rotation, the link will generate almost negligible shaking forces on the frame at high-speed rotation.

Conclusion

A method of designing an actual counterweight using CAD drawing has been proposed. In brief, an equation is formulated with a single variable to represent the geometry of counterweight. One of the roots of equation denotes the required dimension of the counterweight. This method was found to be very effective in terms of time consumption as an iterative methodology requires solving some higher order equations with multiple variables for each timestep. Iterations are run until the solution converges to a root value which is not guaranteed in a specific time duration. However, this method gives a parametrized equation in terms of a single variable for any desired shape which can be calculated in a single step in comparatively no time.

This method gives the initial shape of the design of counterweight. A finished product will include chamfers and fillets on the link which will affect the mass of link and thus give rise to a small error in Center of Gravity location with some offset from the actual position. Iterative methods work by narrowing the solution to a convergence point from a starting guess. If the timesteps or number of iterations are not chosen appropriately, the final answer will have inaccuracies. However, this procedure is more accurate than the lengthy trial-and-error methods as this does not involve iterations. This method solves the single parametrized equation and gives result in the form of a real value.

Acknowledgements

This research is a part of a Crank-Rocker Engine development Project funded by UTP. The authors hereby acknowledge financial support from a Government Research Grant under the Yayasan Universiti Teknologi PETRONAS (YUTP), Malaysia.

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