Mathematical Modelling of Plastic Injection Molding Process Using Central Composite Design (CCD) Sampling and PRSM

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ABSTRACT

Injection Molding is the most widely used plastic manufacturing process for producing outputs from a mold, particularly in polymer processing. This process is highly recommended to deliver high-quality product with low-cost production in a brief time. A Design of Experiment (DoE) approach is adapted to generate the experimental samples and the sampling method proposed to be studied in this research is the Central Composite Design (CCD). The study presents the application on Central Composite Design (CCD) and Response Surface Methodology (RSM) for the generation of the mathematical model. Four experimental parameters were chosen as the input variables which are pigmentation percentage, barrel temperature, barrel pressure and cooling time, while shrinkage is the output response parameter studied. The quadratic polynomial model is selected to significantly predict the interactions of the parameters and the response factor. The evaluation of the model is done through the analysis of variance (ANOVA) and 3D response surface plots are generated to illustrate the cross factors' relationships. From the ANOVA results obtained, the reduced model offers a better fit with a PRESS RMSE of 6.89 and PRESS R^2 *of 71.2% in comparison to the full model. It can be concluded that CCD sampling is able to soundly predict the response of the parameters studied in the injection moulding process based on the statistical diagnostics obtained. It is also noted in both*

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models that the pigmentation percentage (x_1) is most influential factor in minimizing shrinkage in the plastic injection moulding process.

Keywords: *plastic injection moulding, pigmentation percentage, central composite design, polynomial response surface methodology(PRSM).*

Introduction

Common problems that are usually faced by manufacturers in plastic injection molding (PIM) process are the parameters that affect the process in making the final product. There are so many parameters in this process that must be controlled to ensure that a product is properly manufactured. There are studies showing that there are more than 200 different parameters that has direct or indirect effect on the end product of this process [1]. Injection molding process will produce a different quality of product for every parameter that changes. In this study, the product analyzed is a bookmark with UiTM logo on it. Three most commonly contributing parameters which have been known to contribute in the injection molding process efficacy are the cooling time, barrel temperature and barrel pressure. One newer parameter added which is the percentage of pigmentation used in producing a colored product. Though pigmentation percentage is not a process parameter, nevertheless it interestingly has been highlighted to contribute to the outcome of an injection molded product [13]. Color pigments are of very important use in plastic injection molding as they increase the aesthetic value of the product. Commonly a rule of thumb is used in adding the color pigments to generate colored product [13]. The response factor studied in this research is the shrinkage defect.

To facilitate this study, a Design of Experiment (DoE) method is adapted. In the injection molding process, several established researchers have achieved valuable inputs by adopting this method. Previous researchers have clearly defined the effect of parameters towards various types of response and material.[2]- [3]. A number of past researches which are related to this project also have been performed by the author [4]–[6], whereby the findings was used as the guidance for selecting the factors and level in this study.

The objectives of this study is to generate a set of experimental designs through DoE using the classical Central Composite Design sampling technique and to evaluate the efficacy of the CCD sampling technique through a polynomial response surface mathematical modeling and statistical diagnostics such as the ANOVA. The advantage of the CCD sampling method is the ability to identify the direct and cross effects of each of the process parameters on shrinkage as a defect present.

Optimal process parameters setting are considered as one of the important steps in injection molding for improving the quality of molded product [7]. Previously, production engineers used trial and error method to determine the molding optimal process parameters setting. This is time consuming and costly [5]. The response surface methodology (RSM) was used to optimize the quality characteristics by determining the most appropriate and accurate molding process parameters setting. An additional material setting which is the pigmentation percentage is also studied to identify the extend of effect it has on the shrinkage of a plastic injection molded part.

However, despite the many studies on the impact of different input parameters on to the finished product, there were few similar studies found to investigate the mathematical modeling of the input parameter by using CCD sampling method and the use of polynomial Response Surface Methodology (RSM) as an optimizer to generate the mathematical model. Little has also been done to research the effect of color pigments on the PIM products. Thus, this study will evaluate the relationship of the chosen input parameters to the response defect which is shrinkage by using both the CCD sampling method and RSM modeling. Optimal input parameters that yield an optimal output which is a very crucial factor in producing mass production where by defects should be minimal for marketing advantages [2].

(a)

(b)

Figure 1: (a) Plastic Injection molding machine;
(b) Polypropylenes (PP) white and colors used as raw material

Methodology

The experimental process starts with the identification of methods and recommendation of input and output factors to use throughout the experiment. The chosen sampling technique to be use in this study would be the traditional sampling method, Central Composite Design (CCD) and the mathematical model is obtained through the Polynomial Response Surface Methodology (RSM). The parameters selected were pigmentation (x_1), barrel temperature (x_2), barrel pressure (x_3) and cooling time (x_4) and the response data for (y) is shrinkage.

Design of Experiment (DoE)

In this study, the plastic injection molding process uses the Jinhwa Glotech VDCII -140 machine as the primary method for producing the bookmark. Polypropylene (PP) materials white and colored were used as the raw material. Figure below shows the PIM machine and the PP materials used to produce the bookmark.

A set of 36 experimental runs was generated through the CCD sampling technique, where 16 of them are factorial runs (coded to the usual $\pm$ notation), 8 axial runs and 12 center runs in MATLAB 2016a. Central Composite Designs are commonly preferred as it is very flexible and can be run sequentially, thus it facilitates the experimentation process [14].

In RSM, the second-order models which take the form of quadratic polynomial function used to develop an approximate model which offers and explicit relationship between the response of interest and design variables [11]. Let f(x) be the response function and f'(x) its approximation function obtained by second order polynomial form

$$f(x) = \beta_0 + \sum_{i=1}^{m} \beta_i x_i + \sum_{i=1}^{m} \beta_{ii} x^2{}_i + \sum_{i=1}^{m-1} \sum_{j>1}^{m} \beta_{ij} x_i x_j + \varepsilon \quad (2)$$

where m is the total number of design variables, x is the ith design variable, ε is the error and the βs are unknown coefficient.

Coordinate Measuring Machine (CMM)

Shrinkage was chosen as the response output from the experiment by controlling the 4 selected injection molding parameters. This quality characteristic was measured on specimens after the injection molding process was performed. The specimens that have been ejected from the mold were brought to the Coordinate Measuring Machine (CMM) to measure specimens' thickness. Seven measuring points were identified on the products and the average value was calculated.

Shrinkage is the difference between the size of a mold cavity and the size of the finished part divided by the size of a mold. Usually it is expressed in percentage[5]. Shrinkage was measured using the equations stated in equation (3), where, S = shrinkage value, V_a = actual value, and V_e = experimental value[8].

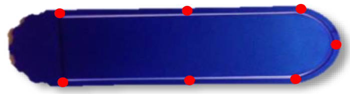
(a)  (b) 

Figure 2: (a) Specimen being measured using CMM;
(b) Red dots represent the seven measuring points
to calculate the thickness of the bookmark

$$S = \frac{V_a - V_e}{V_a} \times 100\% \quad (3)$$

MATLAB R2016a Setup

The mathematical model was generated using the Model-Based Calibration (MBC) Toolbox by MATLAB R2016a. The model class used was linear models and subclass linear model is polynomial. The order specified for the four input parameters and the interaction order is 2, as only the second order was possible with the given data range. In this study, the mathematical modelling is generated for both the full and reduced model.

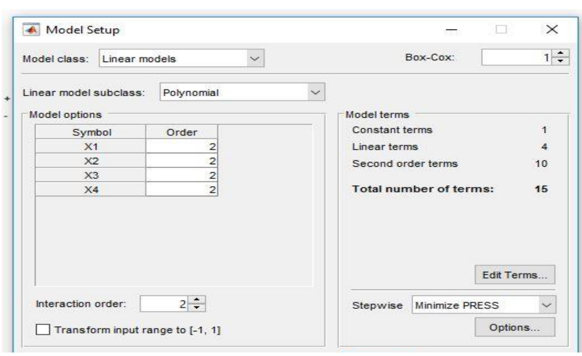


Figure 3: Model setup window in MATLAB 2016a

Result and Discussion

Experimental Result

The experiment was conducted according to the selected input parameters which are pigmentation (x_1), barrel temperature (x_2), barrel pressure (x_3) and cooling time (x_4). While, shrinkage (y) acts as the defect achieved when running the process. From the Design of Experiment (DoE), bookmarks were produced as the product of the experiment. Below are sample of figures product produced based on the different pigmentation values of 1%, 3%, and 5% as well as the other parameters involved.

Pigmentation value of 1%;

Figure 5a: Bookmark produced of experiment no. 5

Figure 5b: Bookmark produced of experiment no. 6

Pigmentation value of 3%;

Figure 6a: Bookmark produced of experiment no. 10

Figure 6b: Bookmark produced of experiment no. 16

Pigmentation value of 5%;

Figure 7a: Bookmark produced of experiment no. 17

Figure 7b: Bookmark produced of experiment no. 25

Table 1: Table of the parameters values and shrinkage values after experiment

Exp. No.	Input, Parameters				Output, Defects
	Pigmentation, % (x_1)	**Barrel Temperature, °C (x_2)**	**Barrel Pressure, bar (x_3)**	**Cooling time, sec (x_4)**	**Shrinkage, % (y)**
1	1	200	80	5	6.20
2	1	200	80	15	27.90
3	1	200	100	5	7.67
4	1	200	100	15	6.24
5	1	300	80	5	6.01
6	1	300	80	15	9.52
7	1	300	100	5	6.05
8	1	300	100	15	5.33
9	5	200	80	5	6.01
10	5	200	80	15	3.19
11	5	200	100	5	7.56
12	5	200	100	15	6.68
13	5	300	80	5	7.70
14	5	300	80	15	10.02
15	5	300	100	5	7.98
16	5	300	100	15	8.50
17	1	250	90	10	48.39
18	5	250	90	10	47.92
19	3	200	90	10	5.49
20	3	300	90	10	3.61
21	3	250	80	10	-7.40
22	3	250	100	10	-15.37
23	3	250	90	5	-7.62
24	3	250	90	15	-5.77
25	3	250	90	10	-2.90

Shrinkage Result

Shrinkage was the output parameter obtained by controlling the injection molding input parameters. The coordinates of the measuring points were all recorded and the average value was calculated. The values obtained were subjected to be subtracted with the radius of the stylus of the CMM machine which was 0.75 mm. As shown in Table 1, the shrinkage values are based on the experimental procedure conducted. The first 25 observations are displayed here to show the variation in the response output as the other 12 experimental runs are a repetition of experiment no 25 and the shrinkage values recorded were approximately similar.

Bookmark produced of experiment 2, 17 and 18, the shrinkage value exceeds up to 27.9%, 48.39%, and 47.92%, respectively, as the specimens were not formed completely. When taking the measurement of the thickness of the specimens, there were not enough data to be calculated. Thus, it gives the result of high percentage of shrinkage occurred. Experiment number 21 until 25 gives the shrinkage percentage of negative value. This is due to the overflow of the material even before the mold is closed. This affected the shrinkage value of the product produced.

Table 2: Sample of coded and true values of the experiment together with shrinkage percentage

Exp. No.	**Coded Value Parameters**				**True Value Parameters**				**Defect**
	Pigmentation (%)	Barrel Temperature (˚C)	Barrel Pressure (bar)	Cooling time (s)	Pigmentation (%)	Barrel Temperature (˚C)	Barrel Pressure (bar)	Cooling time (s)	**Shrinkage (%)**
1	-1	-1	-1	-1	1	200	80	5	**6.20**
2	-1	-1	-1	1	1	200	80	15	**27.90**
3	-1	-1	1	-1	1	200	100	5	**7.67**
4	-1	-1	1	1	1	200	100	15	**6.24**
5	-1	1	-1	-1	1	300	80	5	**6.01**
6	-1	1	-1	1	1	300	80	15	**9.52**
7	-1	1	1	-1	1	300	100	5	**6.05**
8	-1	1	1	1	1	300	100	15	**5.33**
9	1	-1	-1	-1	5	200	80	5	**6.01**
10	1	-1	-1	1	5	200	80	15	**3.19**

Central Composite Design (CCD)

Central Composite Design (CCD) was applied in this study with four factors (parameters) at three levels. The levels were selected based on the preliminary experiments and were coded as -1 (low), 0 (central point), and 1 (high). CCD is automatically generated by MATLAB, which resulting a total number of 36 observations for four factors. Sample of coded values were achieved from CCD sampling method and has been translated into true values as recorded in Table 2. The true values were used to generate the mathematical models.

Mathematical modeling by Response Surface Methodology(RSM)

A second-order quadratic function of shrinkage with interaction terms was obtained, where X_1, X_2, X_3, and X_4, are the coded variable corresponding to the pigmentation, barrel temperature, barrel pressure and cooling time respectively. Two models were generated, the full and reduced model each having 15 and 7 parameters respectively.

Equation (4) represents the mathematical model for the full model and equation (5) is for the reduced model using the stepwise backward reduction method. The stepwise chosen for this study is the Minimizing Predicted Error Sum of Squares (PRESS) because it is a good method for working toward a regression model that provides good predictive capability over the experimental factor space [9].

$$y = -1409.35 - 74.4571\ X_1 + 0.2230\ X_2 + 31.7052\ X_3 + 14.9729\ X_4 + 10.4276\ X_1^2 + 0.01991\ X_1 X_2 + 0.0879 X_1 X_3 - 0.1497\ X_1 X_4 - 0.00076\ X_2^2 + 0.0012\ X_2 X_3 - 0.0027\ X_2 X_4 - 0.1782\ X_3^2 - 0.0340\ X_2 X_3 - 0.5255\ X_4^2 + \varepsilon \quad (4)$$

$$y = -1439.95 - 62.1949\ X_1 + 32.9714\ X_3 + 11.2396\ X_4 - 10.2836\ X_1^2 - 0.1840\ X_3^2 - 0.5486\ X_4^2 + \varepsilon \quad (5)$$

where, the boundaries for equation generated above are within the range of X_1 = [1, 5]; X_2 = [200, 300];
X_3 = [80, 100]; and X_4 = [5, 15].

The stepwise regression technique which is generated by MATLAB has provided a number of methods of selecting the model terms that should be included. This method will take off the unnecessary factors which are not contributing to the model.[9] Figure 6 shows the parameters interaction study, β is representing the value of regression coefficients. The value of the regression coefficients shows to what extent the control parameters affect the response quantitatively. The coefficients that are less significant are eliminated along with the responses with which they are associated with in the reduced model.

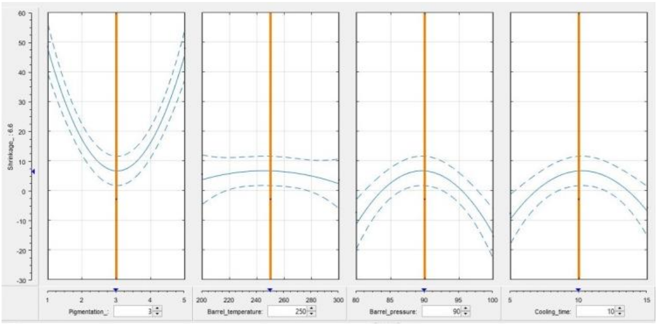


Figure 8: The significance of pigmentation ($\boldsymbol{x_1}$); barrel temperature ($\boldsymbol{x_2}$); barrel pressure ($\boldsymbol{x_3}$); cooling time ($\boldsymbol{x_4}$) against shrinkage (*y*)

Figure 8 displays the relationship of each individual parameter studied to the response which the shrinkage. The parameters which have significant contribution towards the defect shrinkage is identified. The parameter which seems to have the main effect is pigmentation (x_1) with a t value of -12.96, followed by barrel pressure (x_3) with t value of 5.775, cooling time (x_4) with t value of 4.4089, and barrel temperature (x_2) with t value of 0.592. Through quadratic modeling, the interaction of parameters (x_1x_4) against shrinkage (*y*) is also identified with t value of 0.1338.

One of the purpose of this research was to study the effect of pigmentation on the plastic injection molding process. From the results obtained, pigmentation does show an influence in the formation of the plastic molded part.

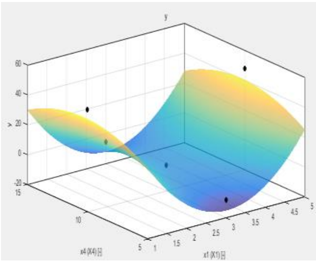


Figure 9: Interactions of pigmentation ($\boldsymbol{x_1}$) and cooling time ($\boldsymbol{x_4}$) towards shrinkage (*y*) for reduced model

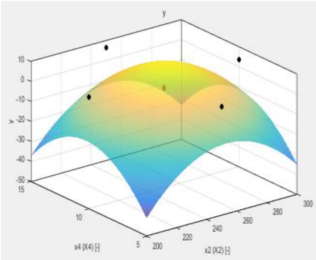


Figure 10: Interactions of barrel temperature ($\boldsymbol{x_2}$) and cooling time ($\boldsymbol{x_4}$) towards shrinkage (*y*) for full model

The surface shown in Figure 9 presents a minimum point of the interaction of pigmentation (x_1) and cooling time (x_4) directly towards shrinkage (y). The graph shows that shrinkage values can be optimum at approximately 3% of pigmentation and 10secs of cooling time. Figure 10 shows the surface interaction plot for barrel temperature (x_2) and cooling time (x_4) towards shrinkage (y) for full the model. Shrinkage is minimal at the optimum points of approximately 250C temperature and 10secs of cooling time. The nature of the polynomial curve fit varies for different interacting parameters but nevertheless the interest of this experiment is to achieve an optimum shrinkage value which in reality should translate to an ideal fill of the mold , leading minimal or no shrinkage occurring.

ANOVA of CCD sampling for full and stepwise reduced model

Table 3: Statistical diagnostic (ANOVA) of RSM using CCD

Statistical Diagnostics	Full Model	Stepwise Reduced Model
Observation Runs	36	36
Parameters	15	7
PRESS RMSE	8.732	6.89
RMSE	5.399	5.376
R^2	0.897	0.859
R^2 adj	0.828	0.83
PRESS R^2	0.538	0.712

Table 3 is the Analysis of Variance (ANOVA) which presents the statistical diagnostics of the Response Surface Methodology (RSM) for this experiment using CCD for both the full and reduced models. A total 36 observation runs were performed in the study. From the statistical diagnostic values achieved, the full model has a fit of 89.7% and the reduced model has 85.9%. Both models generally indicate good fit, however statistically R^2 does not significantly predict the goodness of the model for future data addition.

Thus, in this case, the PRESS statistics is used to better represent the predictability of the models generated. The full model has a PRESS R^2 value of 53.8% and the reduced model has a value of 71.2%. It is shown here that the reduced model offers better predictive capability compared to the full model. This is further backed up by the PRESS RMSE statistics where the full model has a value of 8.732 and the reduced model 6.89. PRESS RMSE indicates the measure of difference between the values predicted by a model and the actual values observed. Therefore, the lower PRESS value in the reduced model signifies that the error of prediction is lesser thus increasing the prediction capability of the model.

Conclusion and Recommendation

In the present work, experimental analysis and mathematical modelling of the selected process parameters of plastic injection molding has been reported. Response Surface Methodology (RSM) has been utilized to investigate the influence of four input parameters – percentage of pigmentation, barrel temperature, barrel pressure and cooling time on bookmark as the product made by using materials, Polypropylene (PP). Shrinkage as a defect was studied as the response output. Central Composite Design (CCD) was employed to conduct the experiments and to develop a correlation between the process parameters and output response.

The important conclusions that can be drawn from the research work are that the CCD sampling offers itself as a suitable method for predicting the response when coupled with RSM. The model's fitness shown are 89% above for both models analyzed. The reduced model however offers a better fit in comparison to the full model. It also noted that pigmentation percentage as a notable influence in the outcome of the studied samples of the plastic injection molded part for the shrinkage defect, though this is not a standard process parameter. Nevertheless, its effect on the PIM process must be exhaustively studied. A mild correlation between the x_1 and x_4 parameters is also noticed though it's not of significant impact to the PIM process.

For further studies, other process parameters can be included and different defects commonly occurring in PIM process can be experimented to validate the effect of pigmentation.

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