

UNIVERSITI TEKNOLOGI MARA

**ON A CLASS OF α -CLOSE-TO-CONVEX
FUNCTIONS**

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Thesis submitted in fulfillment of the requirements
for the degree of
Master of Science

Faculty of Computer and Mathematical Sciences


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ABSTRACT

This thesis is concerned with normalized analytic univalent functions f in the open unit disc $E = \{z : |z| < 1\}$ of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots = z + \sum_{n=2}^{\infty} a_n z^n .$$

The class of normalized analytic univalent functions is denoted by S . In this thesis the concentration is on the class of α -close-to-convex functions, $\mathcal{G}(\alpha, \delta)$ where functions in this class satisfy the condition

$$\operatorname{Re} \left\{ e^{i\alpha} \frac{f'(z)}{g'(z)} \right\} > \delta, \quad (z \in E)$$

with $|\alpha| \leq \pi$, $\cos \alpha > \delta$ and $g'(z) = \frac{1}{1-z}$. The basic properties such as the representation theorem and the extremal properties such as coefficient bounds, distortion theorem, covering theorem and rotation theorem are attained for functions in this class. A result on the radius of convexity, the arc length and the area for functions in this class is also included.

The coefficient inequalities for the class $\mathcal{G}(\alpha, \delta)$ are also discussed which comprise of the upper bounds for the Fekete-Szegő functional, $|a_3 - \mu a_2^2|$ with μ real and the upper bounds for the second Hankel determinant, $|a_2 a_4 - a_3^2|$.

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