Experimental Analysis to Evaluate the Effect of Dynamic Absorber

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ABSTRACT

Passive Dynamic Absorber (PDA) is a mechanical device used to tackle unwanted vibration resonance within the operational frequency of the system. Theoretically, resonance can be avoided if the excitation (operating) frequency is far away from the natural frequencies of the structures. Hence, it is important to identify the structural natural frequencies, to ensure this condition did not occur. In this study, an experimental main system is designed and fabricated and then, tested using Operational Modal Analysis (OMA) to obtain the natural frequency. PDAs are developed based on Dunkley Method (DM) and Randy Fox Method (RFM) and applied to the main vibrating system. Frequency Response Function (FRF) for both models is obtained using OMA and its effectiveness is evaluated. It is shown that the PDA reduces the magnitude of FRF of the main system at the selected resonance frequency and splitting it into two new resonance frequencies. Hence, it can be predicted that the vibration can be effectively suppressed at a specific frequency when the PDA is attached to the main system. This study enables verification of the effectiveness of dynamic absorber to tackle resonance problem. In future, these results will be helpful to evaluate the performance of PDA based on DM and RFM methods.

Keywords: *Dynamic absorber; Dunkerley method; Randy Fox method; Frequency Response Function; Operational Modal Analysis.*

Introduction

Normally, a vibrating machine or system with wide range of operating speed has high possibility to experience vibration problems. One practicable

method has been developed to suppress resonance on any vibrating system, which is by installing the dynamic absorber. It is an auxiliary system that needs to be installed on the structure (main system) externally. Dynamic absorber can be designed and tested before installation. It can be adjusted in the lab environment with predictable field results [1].

Over the years, different types of dynamics absorber have been established and applied in many application such as in rail structure [2], machines [3], [4] and also civil structures such as building [5], and bridges [6], [7]. It designs and methodologies depend on the applicability and suitability of the required absorber. It is also useful for energy harvesting where the energy is collected from the vibrating absorber and at the same time, controlling the main structures [8].

Passive Dynamic Absorber (PDA) and Active Dynamic Absorber (ADA) are two types of absorber normally used in practice to reduce or eliminate the vibration amplitude arise from harmonically excited system. The difference between both types is PDA uses fixed stiffness and damping while ADA added with force generator. The active and passive terms used interchangeably in suspension for automotive industry and its application is different [9].

Operational Modal Analysis (OMA) technique or often called ambient analysis is used to estimate the experimental modal parameters of many areas of structural engineering. OMA technique provides a valuable tool for determination of the dynamic characteristics of a structure (modal parameters) especially when the input force cannot be controlled or measured. The basis of this technique is measurement of the output signals only from a structure which is naturally excited by ambient and its operating forces as unmeasured input [10]–[12].

In this study, an experimental main mass system is developed and fabricated to assess the effectiveness of PDA consist of tuned stiffness-mass system which is derived based on Randy Fox Method (RFM) and classical Dunkley Method (DM). Graphical User Interface (GUI) templates are established from the derivation of both methods to determine the parameters needed to build the PDA. OMA technique is performed on the main system with and without PDA to determine its resonance frequencies. FRF plots are obtained and observed to evaluate the effect of these PDA on the magnitude of FRF and selection of the resonance frequency is made for further performance evaluation. The text should be right and left justified. The recommended font is Times New Roman, 10 points. In 152 mm x 227 mm paper size, the margins are: left and upper: 22 mm each; right: 20 mm, lower: 25 mm.

Theoretical Background

A dynamic absorber is a device consisting of additional mass and a stiffness element. The additional mass is commonly known as auxiliary mass, M_2. However, when auxiliary mass, M_2 is attached to a main mass, M_1 through a spring of stiffness, k_2 it will resulting a two degree of freedom system as Figure 1 and Figure 2.
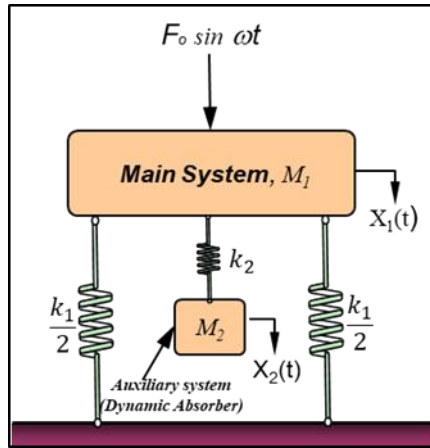

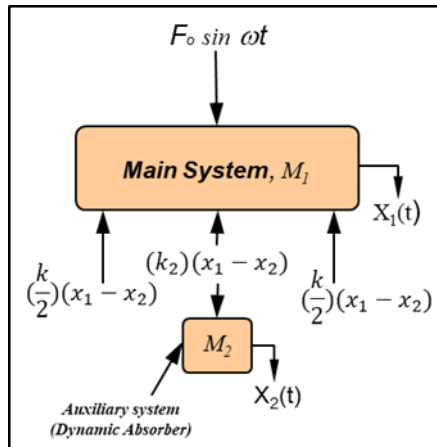Figure 1: Auxiliary system as a dynamic absorber system [13].

Figure 2: Free body diagram of undamped dynamic absorber system [13].

The equations of motion of masses based on free body diagram (FBD) can be grouped into:

For main system, M_1,

$$+ \downarrow \sum F_x = ma$$
$$F_o \sin \omega t - 2[(k_1/2)(x_1)] - k_2(x_1 - x_2) = M_1 \ddot{x}_1$$
$$M_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = F_o \sin \omega \qquad (1)$$

For auxiliary system, M_2

$$+ \downarrow \sum F_x = ma$$
$$k_2(x_1 - x_2) = M_2 \ddot{x}_2$$
$$M_2 \ddot{x}_2 + k_2(x_2 - x_1) = 0 \qquad (2)$$

By assuming harmonic solution,

$$x_1 = x_1 \sin \omega t \qquad (3)$$
$$x_2 = x_2 \sin \omega t \qquad (4)$$

Steady state amplitudes of the masses m_1 and m_2 can be obtained as,

$$X_1 = \frac{(k_2 - M_2 \omega_2)F_o}{(k_1 + k_2 - M_1 \omega^2)(k_2 - M_2 \omega^2) - k_2^2} \qquad (5)$$

$$X_2 = \frac{k_2 F_o}{(k_1 + k_2 - M_1 \omega^2)(k_2 - M_2 \omega^2) - k_2^2} \qquad (6)$$

The main interest is in reducing the amplitude of machine (X_1). In order to make the amplitude of m_1 zero, the numerator of Equation (5) should be set equal to zero. This gives

$$\omega_2 = \frac{k_2}{M_2} \tag{7}$$

Before the addition of dynamic absorber system, if the machine operates near its resonance, $\omega^2 \approx \omega_1{}^2 = \frac{k_1}{M_1}$. Thus if the dynamic absorber is designed such that

$$\omega^2 = \frac{k_2}{M_2} = \frac{k_1}{M_1} \tag{8}$$

The amplitude of vibration of the machine, while operating as its original resonant frequency, will be zero. By defining

$$\delta_{st} = \frac{F_o}{k_1}$$

$$\omega_1 = \sqrt{\frac{k_1}{M_1}}$$

as the natural frequency of the machine or main system, and

$$\omega_2 = \sqrt{\frac{k_2}{M_2}} \tag{9}$$

as the natural frequency of the dynamic absorber or auxiliary system can be written as

$$\frac{x_1}{\delta_{st}} = \frac{1 - \left(\dfrac{\omega}{\omega_2}\right)^2}{\left[1 + \dfrac{k_2}{k_1} - \left(\dfrac{\omega}{\omega_1}\right)^2\right]\left[1 - \left(\dfrac{\omega}{\omega_1}\right)^2\right] - \dfrac{k_2}{k_1}} \tag{10}$$

$$\frac{x_2}{\delta_{st}} = \frac{1}{\left[1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_1}\right)^2\right]-\frac{k_2}{k_1}} \tag{11}$$

Two peaks correspond to the two natural frequencies of the composite system. As seen before, $X_1 = 0$ at $\omega = \omega_1$. At this frequency gives

$$x_2 = -\frac{k_1}{k_2}\delta_{st} = -\frac{F_o}{k_2} \tag{12}$$

This shows that the force exerted by the auxiliary spring is opposite to the impressed force $(k_2 X_2 = -F_0)$ and neutralizes it, thus reducing X_1 to zero. The size of the dynamic absorber can be found from

$$k_2 x_2 = M_2 \omega^2 X_2 = -F_o \tag{13}$$

Thus the values of k_2 and M_2 depends on the allowable value of X_2.
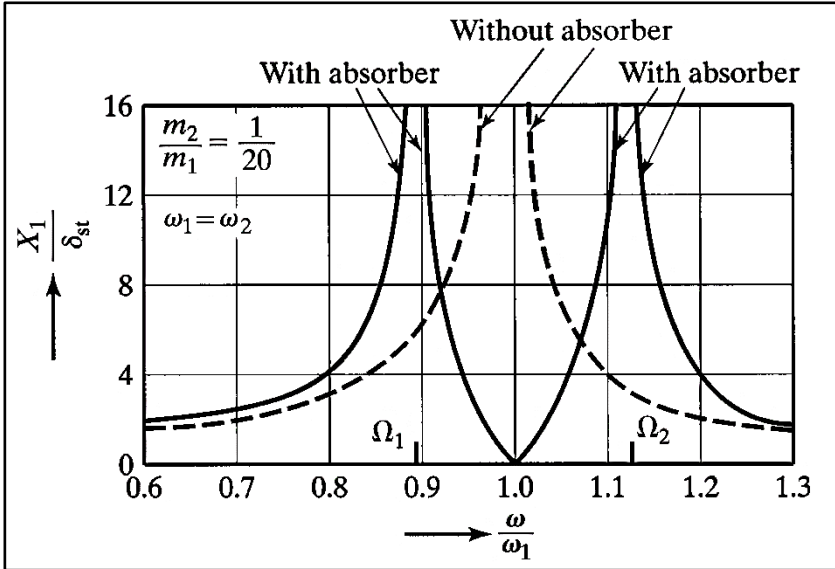
Figure 3: Effect of undamped dynamic absorber on the response [13].

As can be seen in Figure 3, at the frequency ratio of 1.0, dynamic absorber eliminates vibration and introduces two resonant frequencies Ω_1 and Ω_2, at which the amplitude of the machine infinite. The values of Ω_1 and Ω_2 can be found by equating the denominator of Equation (10) and Equation (11) to zero. Where,

$$\frac{k_2}{k_1} = \frac{k_2}{M_2} \cdot \frac{M_2}{M_1} \cdot \frac{M_1}{k_1} = \frac{M_2}{M_1} \cdot \left(\frac{\omega_2}{\omega_1} \right)^2 \tag{14}$$

The setting denominator of Equation (10) to zero leads to

$$\left(\frac{\omega}{\omega_2} \right)^4 \cdot \left(\frac{\omega_2}{\omega_1} \right)^2 - \left(\frac{\omega}{\omega_2} \right)^2 \left[1 + \left(1 + \frac{M_2}{M_1} \right) \left(\frac{\omega_2}{\omega_1} \right)^2 \right] + 1 = 0 \tag{15}$$

The two roots of this equation are given by

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$$\left.\begin{array}{l}\left(\dfrac{\Omega_1}{\omega_2}\right)^2 \\[4mm] \left(\dfrac{\Omega_2}{\omega_2}\right)^2\end{array}\right\} = \dfrac{\left\{\left[1+\left(1+\dfrac{M_2}{M_1}\right)\left(\dfrac{\omega_2}{\omega_1}\right)^2\right]\pm\left\{\left[1+\left(1+\dfrac{M_2}{M_1}\right)\left(\dfrac{\omega_2}{\omega_1}\right)^2\right]^2-4\left(\dfrac{\omega_2}{\omega_1}\right)^2\right\}^{\frac{1}{2}}\right\}}{2\left(\dfrac{\omega_2}{\omega_1}\right)^2}$$

(16)

Based on Equation (16), that Ω_1 is less than and Ω_2 is greater than the operating speed (which is equal to the natural frequency, ω_1) of the machine. Thus the machine must pass through Ω_1 during start-up and stopping. This results in large amplitudes. The variations of $\frac{\Omega_1}{\omega_2}$ and $\frac{\Omega_2}{\omega_2}$ as functions of the mass ratio $\frac{M_2}{M_1}$ are shown in Figure 4 for three different values of the frequency ratio $\frac{\omega_2}{\omega_1}$. The difference between Ω_1 and Ω_2 increases with increasing values of $\frac{M_2}{M_1}$ [13].
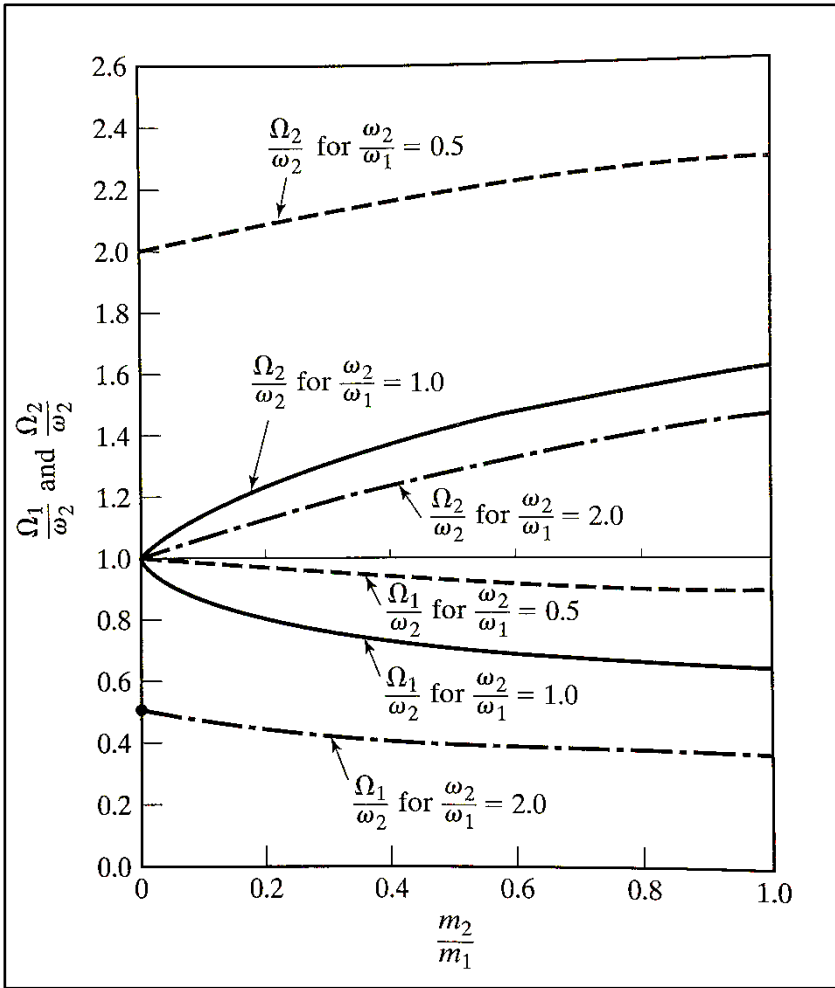
Figure 4: Variations of Ω_1 and Ω_2 given by Equation (16) [13].

Fabrication of Experimental Main System and Dynamic Absorber

Fabrication of experimental main system and dynamic absorber is required in this study as shown by the drawing in Figure 5 and Figure 6. Main system consists of aluminium plate with dimensions of 350 mm height, 100 mm

length and 1.2 mm thickness where for mild steel blocks, the dimensions are 25 mm height, 100 mm length and 9 mm thickness. Two units of mass block are attached at the top of aluminium plate using bolt and nut. A hydraulic swing beam shearing machine and a metal horizontal band saw machine are used to cut of both materials into dimensions.

The resonance frequencies of the main system are obtained from Frequency Response Function (FRF) based on Operational Modal Analysis (OMA) setup. For this study, first resonance frequency of the main system is selected to be excited and its value is applied in GUI templates for both RFM and DM to design and select the parameters of the Passive Dynamic Absorber (PDA).

The PDA is consists of stainless steel bar and mild steel blocks where stainless steel bar dimensions are 28 mm width, 460 mm length and 1 mm thickness. Meanwhile, the dimensions of mild steel block are 25 mm height, 100 mm length and 9 mm thickness. Mass of mild steel block is approximately 0.17 kg each. Two units of mild steel blocks are mounted on the stainless steel bar. Mild steel blocks are also known as tune mass (m_2). Thus, total amount of tune mass is 0.34 kg. Three holes drilled on the stainless steel bar indicate the effective distance of tune mass (a). Mild steel blocks are adjustable.
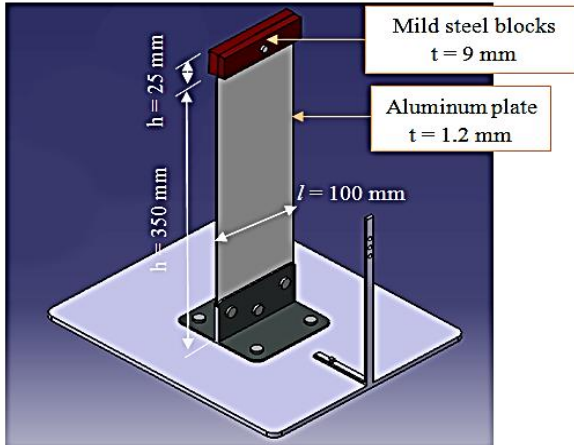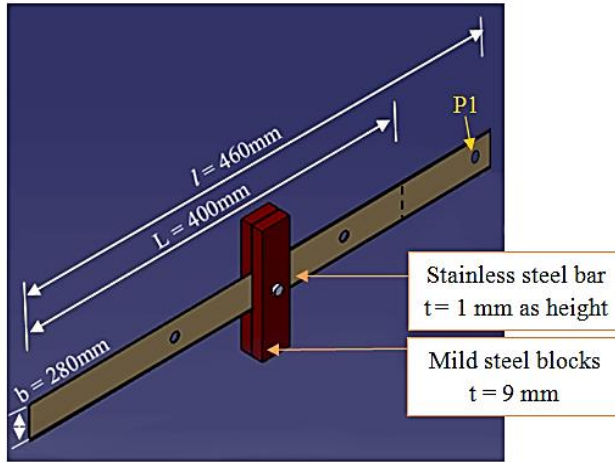

Figure 5: Experimental main system.

Figure 6: Dynamic absorber system.

After both experimental main system and dynamic absorber have been fabricated, dynamic absorber is mounted to the main system at point P1. Figure 7 shows the real test structure consists of experimental main system, roller track and dynamic absorber after undergone fabrication process.
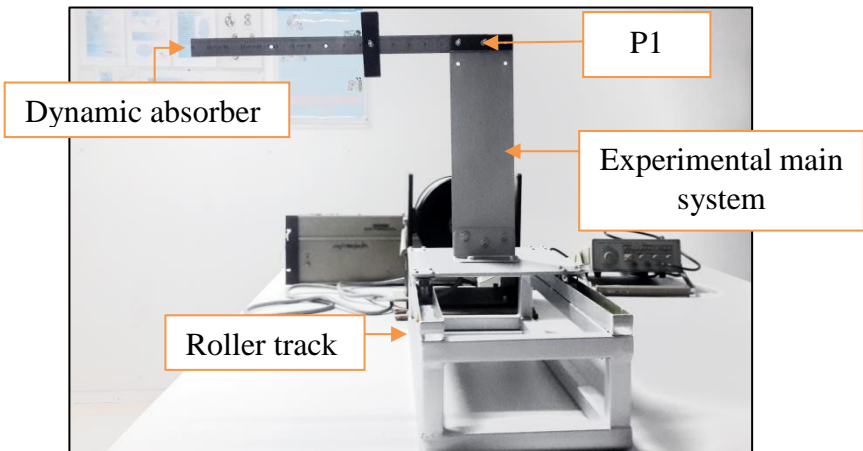

Figure 7: Real test structure.

Operational Modal Analysis (OMA)

OMA was carried out in order to obtain the Frequency Response Function (FRF) plot for natural frequencies on the system without and with influences of dynamic absorber. Measurements were made using a Bruel & Kjaer PULSE^{TM} Multi-Analyzer System. Meanwhile, PULSE LabShop software is used to create structure geometry, allocate measurement degree of freedom and sequence and capture the FRF data. In this OMA experimental setup, frequency span is set to 25 Hz and overlap is about 66.67 %. Averaging is set using linear averaging with 100 averages.

Figure 8: The Bruel & Kjaer PULSE^{TM} Multi-Analyzer System connects to a computer with PULSE LabShop software.

Accelerometers type 4508 with sensitivity of 1.025 mV/ms^{-2} are used in this experimental setup. Since the OMA measurement for both condition (main system with and without absorber) requires only one data set, reference accelerometer is not needed. In OMA testing, the test structure was excited randomly by continuous scrubbing using steel rod with relatively enough energy and its setup is shown in Figure 9 and Figure 10. The excitation should be broadband to have a relevant contribution through all frequency of interest.
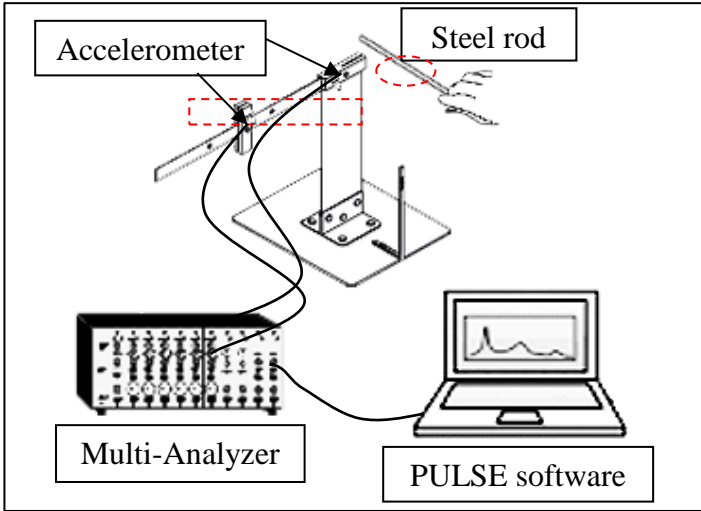
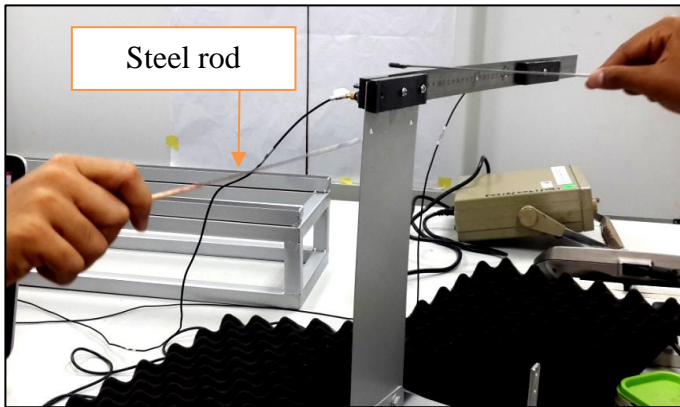Figure 9**:** Diagram for OMA experimental setup.

Figure 10: Random excitation on test structure.

GUI Templates Based on RFM and DM

Since both DM and RFM have different analytical equations, two GUI templates were constructed. Templates for both methods are derived and made available to design the PDA. These templates are developed using MATLAB and used to facilitate efficient selection of the dynamic absorber

parameters. The dynamic absorber for DM and RFM were designed based on the selected natural frequency of the main system obtained from Operational Modal Analysis (OMA) result.
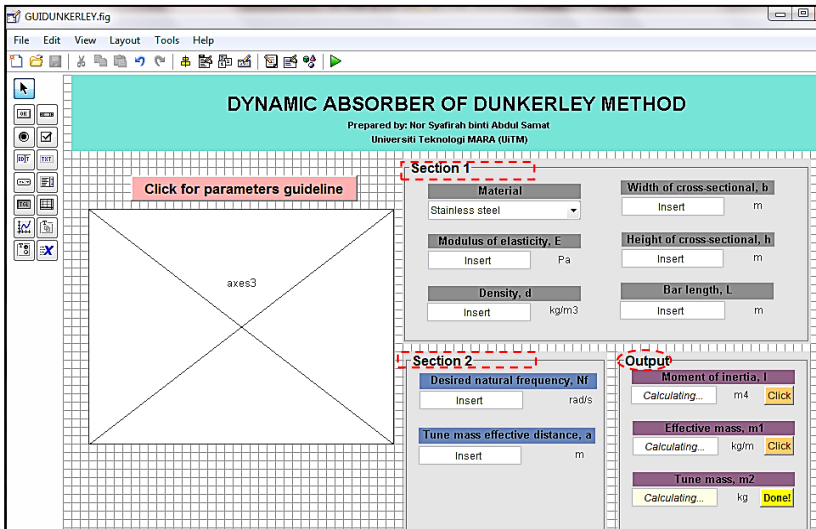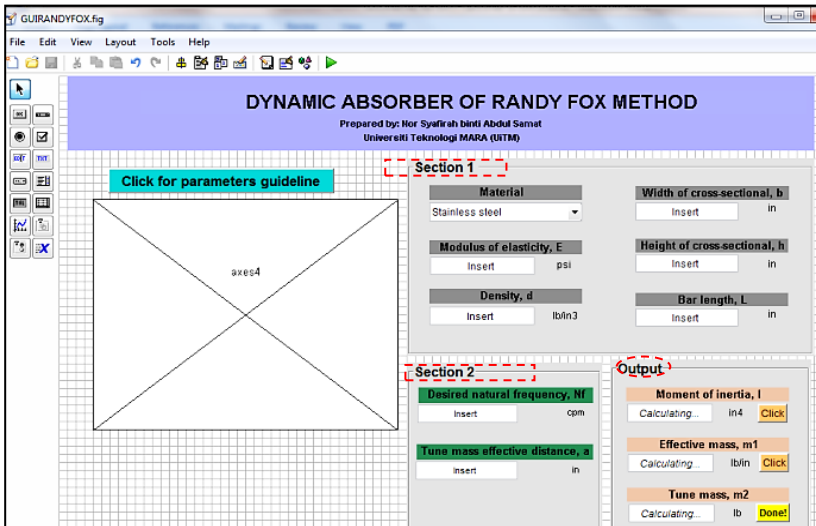

Figure 11: GUI template for DM

Figure 12: GUI template for RFM

Results and Discussions

Analysis of Main System without Dynamic Absorber

Before selecting the right excitation frequency for experimental setup, OMA was carried out in order to check the natural frequencies of the main system. Figure 12 presents the FRF plot of main system without any dynamic absorber attached to it. It is observed that there are two peaks correspond to the first two natural frequencies of the system within 25 Hz frequency span. The magnitude of FRF shows the peaks are at frequency of 2 Hz and 16 Hz which related to the 1st and 2nd mode respectively. The green dash line indicates the peak representing resonance frequencies of the main system without dynamic absorber.
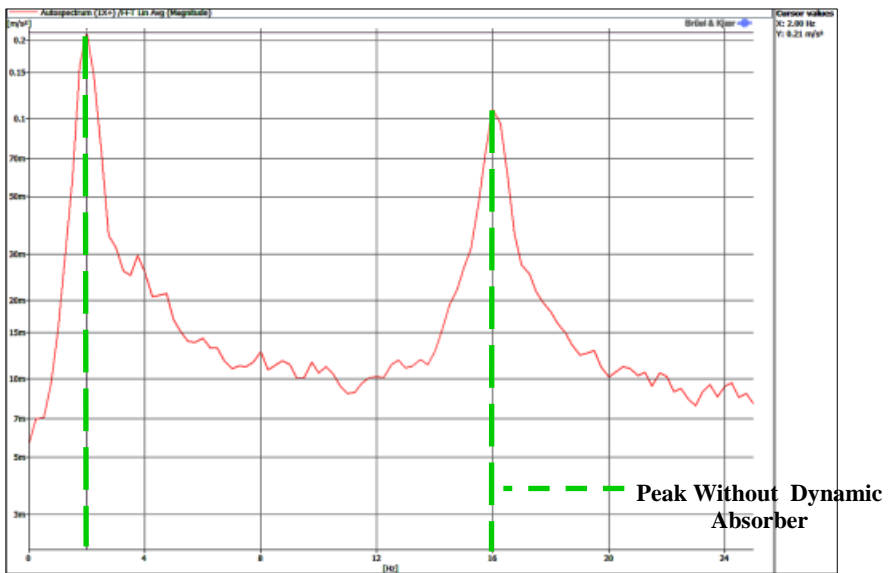

Figure 12: FRF plots of main system without dynamic absorber.

Analysis of Main System with Dynamic Absorber

Since the main system was dominated by natural frequencies of 2 Hz and 16 Hz (as obtained and shown by the FRF plot in Figure 12), thus in this study, the excitation frequency is selected similar to the main system frequency to emulate resonant condition. Using GUI templates, the reasonable parameters of dynamic absorber is determined. The parameters of dynamic absorber are depends on desired resonance frequency (in this study, 2 Hz is applied). For this case, every parameter involves in dynamic absorber design is constant except for the specified distance of tune mass (a). Based on Figure 13 and

Figure 14, the specified distance of tune mass for both methods is dissimilar, even if the same amount of tune mass is used. The overall values of specified distance of tune mass are shown in the Table 1 and its position is illustrated in Figure 15.
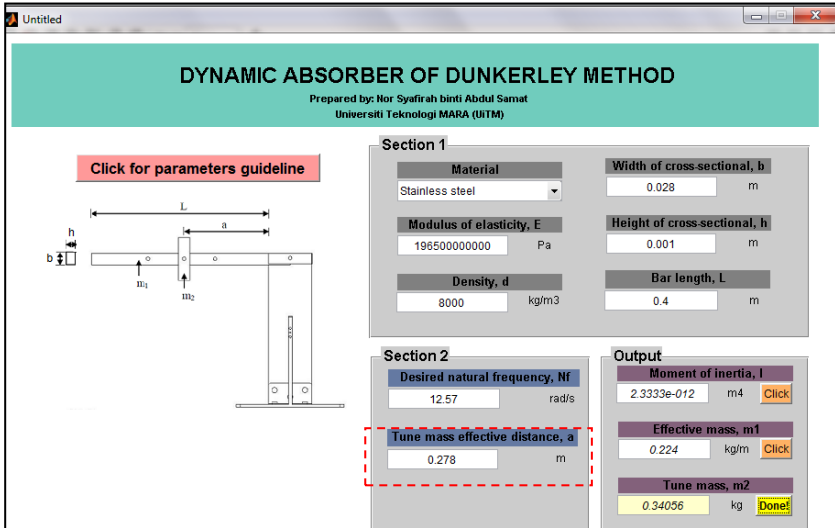

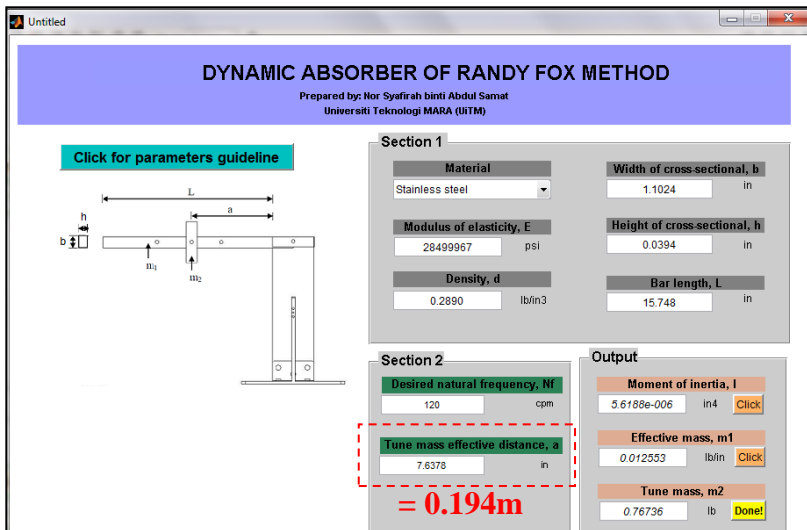Figure 13: Calculation of DM dynamic absorber at 2 Hz

Figure 14: Calculation of RFM dynamic absorber at 2 Hz.

Table 1: Specified distance of tune mass at desired natural frequency.

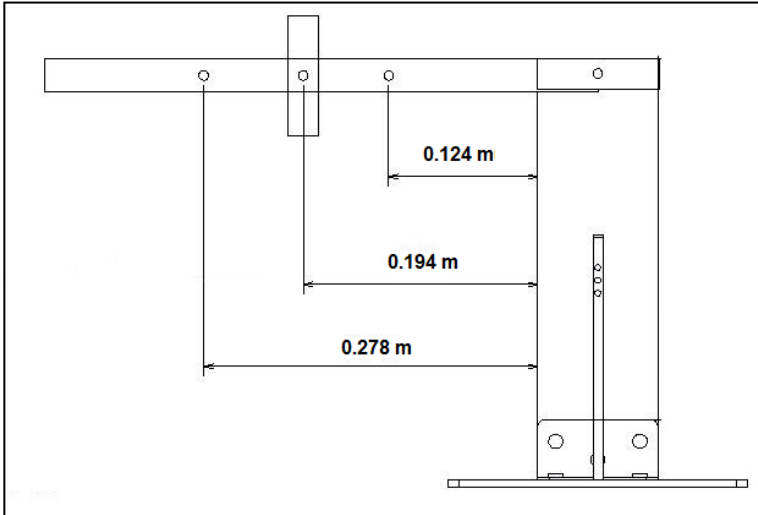| Excitation frequency (Hz)	Specified distance of tune mass, a (m)	
	DM	RFM
2	0.278	0.194

Figure 15: Illustration of tune mass position at specified distance.

OMA experimental setup again was conducted to determine the FRF plots of the combine main system with the final design of RFM and DM dynamic absorber. Theoretically, when dynamic absorber is attached to the system, it reduce the FRF magnitude and introduces two new resonance frequencies (Ω_1 and Ω_2). Referring to Figure 16, it is apparent that dynamic absorbers have significant effect on the main system. The blue line represents new resonance frequencies of the main system with dynamic absorber attached to it.

The experimental results obtained from this study proved that the addition of PDA will change the original natural frequency of the main system and split it into two new frequencies as depicted in Figure 3. The original resonance frequencies at 2 Hz and 16 Hz (Figure 12) are separated into four other frequencies after PDA was attached to the system as shown in Figure 16.

Practically, proper tuning of PDA parameters (M_2 and k_2) should be done to suppress the amplitude of vibration of main system at resonance. As

discuss previously, the tuned parameter in this study is its stiffness and this was achieved by changing the specified distance of tune mass (a).

Figure 16(a)-(b) show the FRF plots when DM dynamic absorber is attached to the main system. In this condition, tune mass was mounted at specified distance of 0.278m. The initial FRF magnitude of the main system frequencies at 2 Hz and 16 Hz is significantly reduced after attaching the DM dynamic absorber and the peaks at both frequencies are disappear.

Similarly for RFM dynamic absorber, the tuned mass was mounted at a difference distance of 0.194m for the same target frequency as DM absorber. Figure 16(c)-(d) show the FRF plots for this design are comparable with DM absorber where the magnitude is reduced and the peaks are disappear.
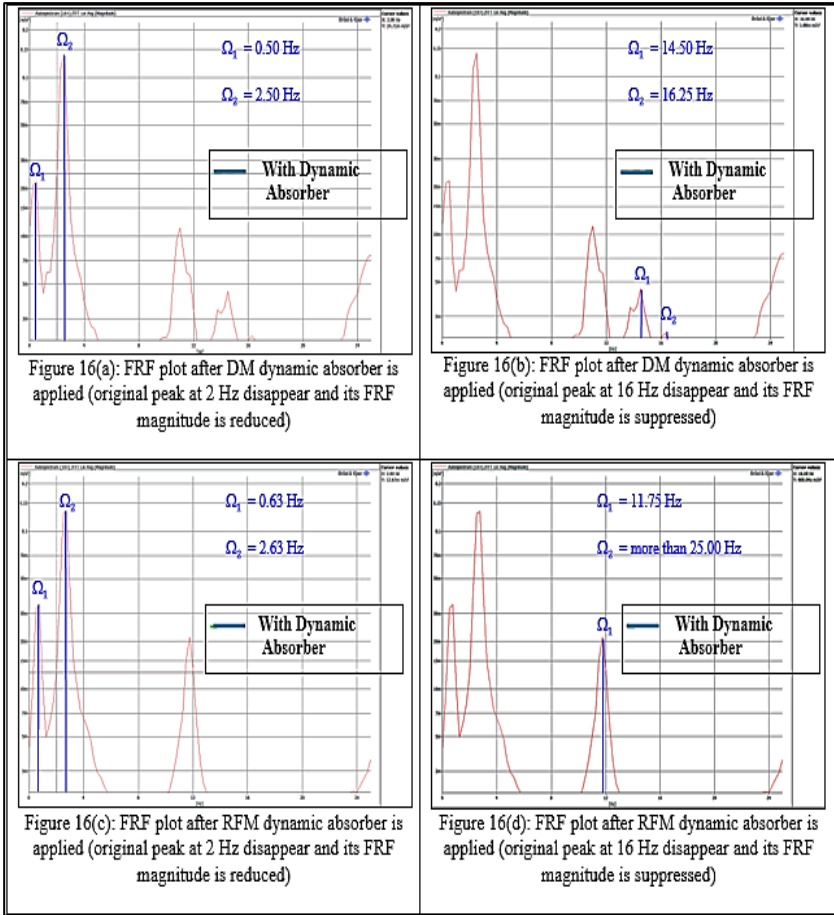
Figure 16(a): FRF plot after DM dynamic absorber is applied (original peak at 2 Hz disappear and its FRF magnitude is reduced)

Figure 16(b): FRF plot after DM dynamic absorber is applied (original peak at 16 Hz disappear and its FRF magnitude is suppressed)

Figure 16(c): FRF plot after RFM dynamic absorber is applied (original peak at 2 Hz disappear and its FRF magnitude is reduced)

Figure 16(d): FRF plot after RFM dynamic absorber is applied (original peak at 16 Hz disappear and its FRF magnitude is suppressed)

Figure 16: FRF plots of main system with dynamic absorber at 2 Hz.

Conclusion

Operational Modal Analysis (OMA) was carried out on a main system with and without PDA attached to it. The structure modal parameter in the form of its frequencies were successfully obtained. The suitable dynamic absorber parameters for main system have been obtained using GUI templates of DM and RFM method. The effectiveness of dynamic absorber of DM and RFM has been observed based on experimental Frequency Response Function (FRF) results. The results show that the addition of dynamic absorber to the target system (main system) will effectively attenuate the motion and amplitude of the main system. Both RFM and DM dynamic show similar

effect on the main system. However, since the efficiency of both methods provide such a valuable insight into the nature of the response, it is recommended that further study be implemented to look at the performance effect of both absorbers. Comparison between the results obtained in this study with that of performance analysis is required to validate the vibration amplitude of the main system will be suppressed and reduced significantly and the energy is absorbed by the tuned dynamic absorber.

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