UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

UPPER BOUND OF SECOND HANKEL DETERMINANT FOR SUBCLASS OF CLOSE-TO-CONVEX FUNCTIONS

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ABSTRACT

Geometric function theory is an extraordinary area of complex analysis. This area of study is more often associated with geometric properties of analytic function such as extremal properties, radius properties, representation theorem and coefficient bound. Many researchers raised the interest in studying properties in different classes that have been introduced. In this research, we focus on defining new subclasses of analytic functions, $L(\alpha,\delta,t,s)$ afterwards determining the upper bound of second Hankel determinant for the selected class of function.

We introduce a new subclasses of close-to-convex function class, $L(\alpha,\delta,t,s)$ defined in the open unit disc, $U=\{z\in U:|z|<1\}$, for which satisfies $\operatorname{Re}\left[e^{i\alpha}\frac{zf'(z)}{g'(z)}\right]>\delta$ where, $|\alpha|\le\pi$, $\cos\alpha>\delta$, $0\le\delta\le1$, $g'(z)=\frac{1}{(1+tz)(1-sz)}$, $s>t$, $-1\le t<1$ and $-1<s\le1$. From the define subclasses of function, we then focusing on finding the upper bounds of one of the coefficient inequalities in geometric functions theory which is second Hankel determinant, $|a_2a_4-a_2{}^3|$. Consequently, from the objective, we need to use some lemmas to obtain the result. Result will then be verified by reducing it to Kaharudin et al. (2011). If this study is successful, it will lead to development of research in this area of study.

LIST OF SYMBOLS

SYMBOL	DESCRIPTION FOR THE SYMBOLS
$\mathbb{C}$	Set of complex number
$\mathcal{D}$	Domain
S	Univalent functions
U	Open unit disc $U, \{z \in U : \lvert z \rvert < 1\}$
$\mathcal{A}$	Class of normalized analytic functions in the open unit disc, U in the form of : $f(z) = z + a_2 z^2 + \ldots + a_n z^n = z + \sum_{n=2}^{\infty} a_n z^n$
$k(z)$	Koebe functions
P	Class of all function of form $p(z) = 1 + p_1 z + p_2 z^2 + \ldots + p_n z^n + \ldots = 1 + \sum_{n=1}^{\infty} p_n z^n$
S^*	Class for the starlike functions
K	Class for the convex functions
Ct	Class for the close-to-convex