Vibration Suppression Control of Three Inertial Systems

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ABSTRACT

This paper concerns with vibration suppression control for three inertia resonance system which can be used as the model for many moving mechanisms widely used in industry such as steel rolling mills, flexible arms, large-scale space structures, etc. Since the mechanical resonance which causes vibration is unavoidable, especially at high speed operation, vibration suppression plays a key role in improving the accuracy of the system. In this research, a linear quadratic regulator-based speed controller with an integrator that can effectively suppress the torsional vibration is applied to control the system. Instead of using conventional state observer that is sensitive to noise and model uncertainties, the extended state observer is used to overcome the disturbance and uncertainty problems. Moreover, this observer also gives us the load torque information that is used to improve the load response. Simulation results show the effectiveness of the proposed controller. In addition, the comparison to conventional PID controller also be done to verify the advantages of the proposed approach.

Keywords: *Three inertial system; extended state observer; linear quadratic regulator; vibration suppression*

Introduction

Nowadays, high precision and fast response motor driver systems are widely used in industry such as steel rolling mill, robot manipulator, electrical vehicle, etc. Generally, these moving mechanisms can be regarded as multi-inertia systems with several inertia moments, gears and springs. Theoretically, vibration is unavoidable in above mentioned systems where motion is involved due to the mechanical resonance. For most motions, conventional proportional-integral-differential (PID) controller is sufficient to meet the desired requirements due to the fact that the nature frequency of the mechanical

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system is quite high in comparison with the motion maneuver. However, for applications where both fast dynamic and high precision positioning are required, vibration suppression plays a key role in improving the system performance.

To deal with the vibration problem, most researches treat the motion mechanism as a 2-inertia system which comprised of mass, spring and damping. Then, various control methods are employed to suppress the vibration such as resonant ratio control [1], state feedback control [2,3], linear quadratic Gaussian (LQG) control [4,5], linear quadratic control with extended state observer (ESO) [6], active disturbance rejection control [7], fractional order PID-*k* controller [8], model predictive control [9], back-stepping position control [10], adaptive speed control [11]. Since the simplified model and the real system may be different in the number of resonant frequencies, i.e., the number of links of a manipulator is three or more, the effectiveness of the control system based on the 2-inertia model may be degraded in practice.

To further improve the system performance in term of vibration suppression, multi inertia model, i.e., three and more inertia system, is considered [12]. In [13], a three-inertia system is controlled by using PI/PID control. In addition, a modified integral plus proportional plus derivative (m-IPD) controller is used to suppress vibration in 3-inertia system. Fuzzy controller with differential evolution is also employed to control this 3-inertia system [12]. Despite of improved performance, the tuning procedure of the aforementioned methods are quite complicated due to the presence of the modeling error and uncertainties.

In this research, a linear quadratic regulator (LQR) control with an extended state observer (ESO) is proposed to handle the vibration problem of the 3-inertia system. The proposed method is not only effective in torsional vibration suppression but also robust against parameters variation since the ESO is employed to estimate the immeasurable state variables and load disturbance. In advanced, the tuning procedure of the method is simplified which plays a very importance role in practical applications. Particularly, the method can also be extended to higher order inertia systems.

To this end, the paper is organized as follows. The mathematical model of the 3-inertia system is introduced in section II. Section III shows the design procedure of the ESO and the LQR. Numerical simulations which verify the validity and effectiveness of the proposed strategy are shown in section IV. The conclusions are shown in the last section of the paper.

Three-Inertia System Model

A typical 3-inertia system, which consists of three rigid inertias and two torsional shafts, is shown in Figure 1.

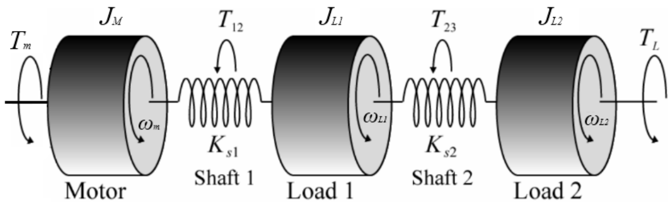


Figure 1: Three inertial system.

In this figure, ω_m is the motor angular speed, ω_{L1} is the angular speed of load 1, ω_{L2} is the angular speed of load 2, J_m is the motor inertia, J_{L1} is the inertia of load 1, J_{L2} is the inertia of load 2, T_m is the motor torque, T_L is the load torque, T_{12} is the torsional torque of shaft 1, T_{23} is the torsional torque of shaft 2, K_{s1} is the stiffness of shaft 1 and K_{s2} is the stiffness of shaft 2. The 3-inertia system can be modeled by the following equations:

$$\begin{cases} J_m \frac{d\omega_m}{dt} = T_m - T_{12} \\ \frac{dT_{12}}{dt} = K_{s1}(\omega_m - \omega_{L1}) \\ J_{L1} \frac{d\omega_{L1}}{dt} = T_{12} - T_{23} \\ \frac{dT_{23}}{dt} = K_{s2}(\omega_{L1} - \omega_{L2}) \\ J_{L2} \frac{d\omega_{L1}}{dt} = T_{23} - T_L \end{cases} \tag{1}$$

Equation (1) can be rewritten in state-space form as:

$$\begin{aligned} \dot{x} &= Ax + B_1 T_L + B_2 T_m \\ y &= Cx \end{aligned} \tag{2}$$

where

$$x = [\omega_m \quad T_{12} \quad \omega_{L1} \quad T_{23} \quad \omega_{L2}]^T$$

$$A = \begin{bmatrix} 0 & -1/J_m & 0 & 0 & 0 \\ K_{s1} & 0 & -K_{s1} & 0 & 0 \\ 0 & 1/J_{L1} & 0 & -1/J_{L1} & 0 \\ 0 & 0 & K_{s2} & 0 & -K_{s2} \\ 0 & 0 & 0 & 1/J_{L2} & 0 \end{bmatrix} \tag{3}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1/J_{L2} \end{bmatrix} \qquad B_2 = \begin{bmatrix} 1/J_m \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$C = [1 \quad 0 \quad 0 \quad 0 \quad 0];$$

From Equation (2), the transfer function from T_m to ω_m is:

$$\frac{\omega_m(s)}{T_m(s)} = \frac{(s^2+\omega_{a1}^2)(s^2+\omega_{a2}^2)}{J_m(s^2+\omega_{r1}^2)(s^2+\omega_{r2}^2)} \tag{4}$$

In which, ω_{a1}, ω_{a2} are anti-resonant frequencies and ω_{r1}, ω_{r2} are resonant frequencies of the system and are given by.

$$\omega_{a1} = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right) - \sqrt{\left(\frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right)^2 - 4\frac{K_{s1}K_{s2}}{J_m}}} \tag{5}$$

$$\omega_{a2} = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right) + \sqrt{\left(\frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right)^2 - 4\frac{K_{s1}K_{s2}}{J_m}}} \tag{6}$$

$$\omega_{r1} = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{K_{s1}}{J_m} + \frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right) - \sqrt{\left(\frac{K_{s1}}{J_m} + \frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right)^2 - 4\frac{J_m+J_{L1}+J_{L2}}{J_{L1}J_{L2}}K_{s1}K_{s2}}} \tag{7}$$

$$\omega_{r2} = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{K_{s1}}{J_m} + \frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right) + \sqrt{\left(\frac{K_{s1}}{J_m} + \frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s2}}{j_{L2}}\right)^2 - 4\frac{J_m+J_{L1}+J_{L2}}{J_{L1}J_{L2}}K_{s1}K_{s2}}} \tag{8}$$

These resonant frequencies cause vibrations and therefore degrade the system performance. The controller must be designed not only for speed/position tracking, but also for vibration suppression.

Control of 3-Inertia System

State and Disturbance Observer

To implement the state feedback controller, all the state variables should be known. However, not all state variables are available for measure because of the high cost, mounting constraints or noise. In this case, a state observer is a good solution to estimate the immeasurable state variables. In this paper, the ESO [14] is chosen because of its better performance over other observers such as high-gain and sliding-mode observers in term of robustness against disturbance.

Consider the fifth order system (2), in order to estimate the states and loads disturbance of this system by the ESO, the following transformation matrix T is employed:

$$T=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & -J_m & 0 & 0 & 0\\ 1 & 0 & \frac{J_m}{K_{s1}} & 0 & 0\\ 0 & -(J_m+J_{L1}) & 0 & -\frac{J_{L1}J_m}{K_{s1}} & 0\\ 1 & 0 & \frac{J_m}{K_{s1}}+\frac{J_m+J_{L1}}{K_{s2}} & 0 & \frac{J_{L1}J_m}{K_{s1}K_{s2}}\end{bmatrix} \tag{9}$$

Let $x^* = T^{-1}x$. Then, Equation (2) can be written as:

$$\begin{cases}\dot{x}_1^* = x_2^* + \frac{1}{J_m}u\\ \dot{x}_2^* = x_3^*\\ \dot{x}_3^* = x_4^* - \frac{K_{s1}}{J_m^2}u\\ \dot{x}_4^* = x_5^*\\ \dot{x}_5^* = a(t) + \frac{K_{s1}^2}{J_m^2}\left(\frac{1}{J_m}+\frac{1}{J_L}\right)u\\ y = x_1^*\end{cases} \tag{10}$$

with

$$a(t) = (-K_{s1}K_{s2}\frac{J_m+J_{L1}+J_{L2}}{J_mJ_{L1}J_{L2}})x_2^* + (\frac{-1}{J_{L2}}+\frac{K_{s1}+K_{s2}}{J_{L1}}+\frac{K_{s1}}{J_m})x_4^* - \frac{K_{s1}K_{s2}}{J_mJ_{L1}J_{L2}}T_L \tag{11}$$

By treating $a(t) = x_6^*(t)$ as an augmented state variable, $h(t)$ as the derivative of $a(t)$ which is unknown, the system state-space model in Equation (10) becomes:

$$\begin{cases} \dot{x}_1^* = x_2^* + \frac{1}{J_m}u \\ \dot{x}_2^* = x_3^* \\ \dot{x}_3^* = x_4^* - \frac{K_{s1}}{J_m^2}u \\ \dot{x}_4^* = x_5^* \\ \dot{x}_5^* = x_6^* + \frac{K_{s1}^2}{J_m^2}\left(\frac{1}{J_m} + \frac{1}{J_L}\right)u \\ \dot{x}_6^* = h(t) \\ y = x_1^* \end{cases} \quad (12)$$

To estimate the new states variable x^*, the following ESO is used:

$$\begin{cases} e = z_1 - y \\ \dot{z}_1 = z_2 - \beta_1 g_1(e) + \frac{1}{J_m}u \\ \dot{z}_2 = z_3 - \beta_2 g_2(e) \\ \dot{z}_3 = z_4 - \beta_3 g_3(e) - \frac{K_{s1}}{J_m^2}u \\ \dot{z}_4 = z_5 - \beta_4 g_4(e) \\ \dot{z}_5 = z_6 - \beta_5 g_5(e) + \frac{K_{s1}^2}{J_m^2}\left(\frac{1}{J_m} + \frac{1}{J_L}\right)u \\ \dot{z}_6 = \beta_6 g_6(e) \end{cases} \quad (13)$$

where β_i (*i=1,..,6*) are observer gains, z_i (*i=1,..,6*) are estimated values of x_i^*, and $g_i(e)$ (*i=1,..,6*) can be either linear or nonlinear functions. In this paper, $g_i(e) = e$ is chosen. With appropriate values of β_i, the estimation of x_i^* and the system states $\hat{x} = T\hat{x}^*$ can be obtained. Then the estimation of load torque $\hat{T}_L$ can be determined by

$$\hat{T}_L = \left(-a(t) + \left(-K_{s1}K_{s2}\frac{J_m+J_{L1}+J_{L2}}{J_mJ_{L1}J_{L2}}\right)x_2^* + \left(\frac{-1}{J_{L2}} + \frac{K_{s1}+K_{s2}}{J_{L1}} + \frac{K_{s1}}{J_m}\right)x_4^*\right)\frac{J_mJ_{L1}J_{L2}}{K_{s1}K_{s2}} \quad (14)$$

Controller Design

To extend the bandwidth of the motion mechanism, vibration caused by mechanical resonant must be suppressed. Besides, other requirements of a standard controlled system such as stability, minimal tracking error, etc., must also be fulfilled. To achieve the aforementioned goals simultaneously, a state feedback control with an extended integral of error state variable is utilized to eliminate the steady-state error. The design procedure is as followings.

The block diagram of the state feedback control system is shown in Figure 2 in which K_i is the integral gain, K_d is feed forward gain to compensate the load torque, and $F = [f_1\, f_2\, f_3\, f_4\, f_5]$ is the state feedback gain.

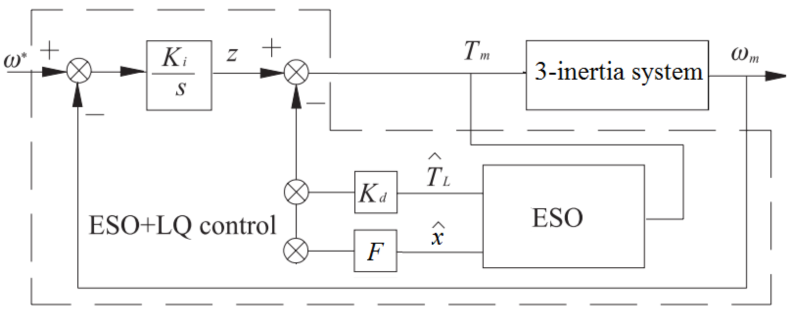


Figure 2: Proposed controller schema

In order to determine the integral gain and the state feedback gain, we first define a new state variable S as:

$$S = \int_0^t (\omega_m - \omega^*) d\tau \tag{15}$$

Then, by differentiating both sides of Equation (15), it yields:

$$\dot{S} = \omega_m - \omega^* = Cx - \omega^* \tag{16}$$

Combining Equation (2) and Equation (16), a new state-space model in which the torque T_m is substituted by the control signal u is derived as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ S \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} u + \begin{bmatrix} B_1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} T_L \\ \omega^* \end{bmatrix} \tag{2}$$

In steady-state, $\dot{x} = 0$ and $\dot{S} = 0$, then the steady-state solution x_s, S_s and u_s must satisfy the following equation:

$$\begin{bmatrix} B_1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} T_L \\ \omega^* \end{bmatrix} = -\begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ S_s \end{bmatrix} - \begin{bmatrix} B_2 \\ 0 \end{bmatrix} u_s \tag{18}$$

Substituting Equation (18) into Equation (17), it gives:

$$\begin{bmatrix} \dot{x} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x - x_s \\ S - S_s \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} (u - u_s) \tag{19}$$

Define a new state variable Z as:

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} x - x_s \\ S - S_s \end{bmatrix}, \dot{Z} = \begin{bmatrix} \dot{x} \\ \dot{S} \end{bmatrix}, q = u - u_s \quad (20)$$

Then Equation (19) becomes:

$$\dot{Z} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} q = \bar{A}Z + \bar{B}q \quad (21)$$

Choose the performance index J as follows:

$$J = \int_0^\infty [\alpha(\omega_m - \omega_m^*)^2 + \beta(\omega_{L1} - \omega_m^*)^2 + \varepsilon(\omega_{L2} - \omega_m^*)^2 + \delta(S - S_s)^2 + \gamma(u - u_s)^2] dt \quad (22)$$

or

$$J = \int_0^\infty (Z^T Q Z + R q^2) dt \quad (23)$$

where

$$Q = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta \end{bmatrix} \text{ and } R = \gamma$$

Then, the state feedback gain $K = [K_1 K_2]$ of the LQR control for system in Equation (21) subject to performance index in Equation (22) can easily be computed. Consequently, the control input q is:

$$q = -KZ = -K_1 Z_1 - K_2 Z_2 \text{ or } u - u_s = -K_1(x - x_s) - K_2(S - S_s) \quad (24)$$

Since $u_s = -K_1 x_s - K_2 S_s$ in steady state, Equation (24) becomes:

$$u = -K_1 x - K_2 S = -K_1 x - K_2 \int_0^t (\omega_m - \omega^*) d\tau \quad (25)$$

By considering Figure 2, it can be realized that $F = [f_1\, f_2\, f_3\, f_4\, f_5] = K_1$ and $K_i = K_2$. Since the load torque of which the sudden change may cause speed drop and initiate hazardous torsional vibration is observed by the ESO, the feed forward compensation gain $K_d = 1$ is sufficient.

With the ESO, the state estimation errors can approach zero with arbitrarily dynamic [15]. Then, the stability of the control system can be guaranteed by the LQR controller [16].

Numerical Simulation

In this section, numerical simulations are carried out to verify the effectiveness of the proposed control strategy. The systems parameters are provided in Table 1.

Table 1: Systems parameters

Name	Symbol	Value
Motor inertia	J_m	1552 Kgm^2
Inertia of load 1	J_{L1}	1000 Kgm^2
Inertia of load 2	J_{L2}	542 Kgm^2
Stiffness of shaft 1	K_{S1}	$5.93 x 10^6$ $Nmrad^{-1}$
Stiffness of shaft 2	K_{S2}	$5.93 x 10^6$ $Nmrad^{-1}$
Load Disturbance	T_L	$0.2 x 10^6$ Kgm^2

After being well tuned, weighting matrices Q and R in Equation (23) are chosen with $\alpha = 1000$, $\beta = 10^7$, $\varepsilon = 7.10^6$, $\delta = 10^{13}$ and $\gamma = 1$. Then, the state-feedback gain is K = [$3{,}0560.10^5$ 2,2702 $2{,}0347.10^4$ -1.0770 $2.7274.10^3$ $3{,}1623.10^6$]. The gains of ESO are $\beta_1 = 6\omega_0$, $\beta_2 = 15\omega_0^2$, $\beta_2 = 20\omega_0^3$, $\beta_4 = 15\omega_0^4$, $\beta_5 = 6\omega_0^5$ and $\beta_6 = \omega_0^6$ where $\omega_0 = 1000$.

The performance of the proposed control strategy is first investigated by step-response analysis. A constant reference speed, i.e., 30 rad/s, is used and the corresponding systems response is shown in Figure 3. It can be observed that the speed of the motor and loads quickly track the reference speed without vibration. In addition, the robustness of the controller against the load disturbance is also verified in this simulation. In detail, an external disturbance is introduced at time instance *1s*. It can be seen that the influence of the disturbance is quickly compensated by the controller in about 0.1 s.

In order to show the validity of the ESO, the error between the real and the observed speeds of motor, load 1 and load 2 are calculated and shown in Figure 4. It can be realized that these errors are extremely small even in transient-state, and quickly converge to zero in steady-state.

To show the advantage of the proposed control strategy over the conventional PID controller, comparative simulation is also carried out. A PID controller of which parameters are well tuned by particle swarm optimization (PSO) technique is used for system (2). The comparative speed responses of the PID controller and the proposed controller are provided in Figure 5. It can be seen that although being optimized, the PID controller still shows poor performance, i.e., large overshoot with torsional vibration. In contrast, the proposed controller shows much better performance with smooth transient-state whilst the vibration is completely removed.

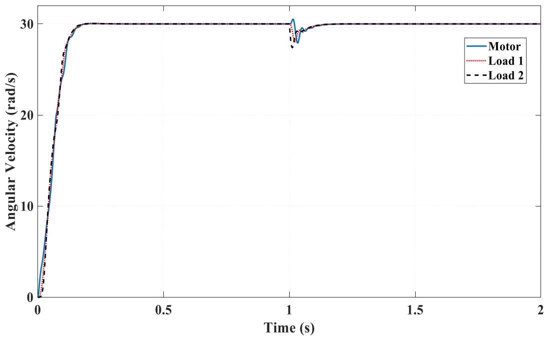


Figure 3: Velocity responses of the system with the proposed controller

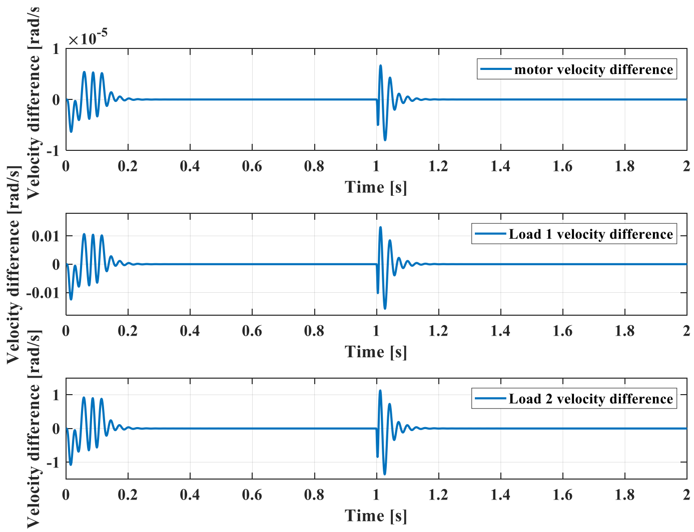
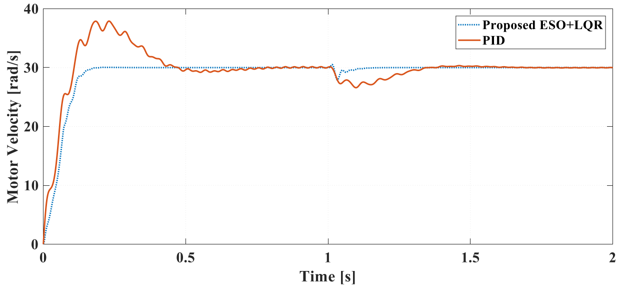
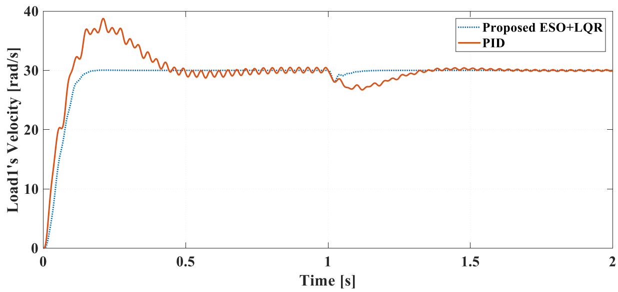


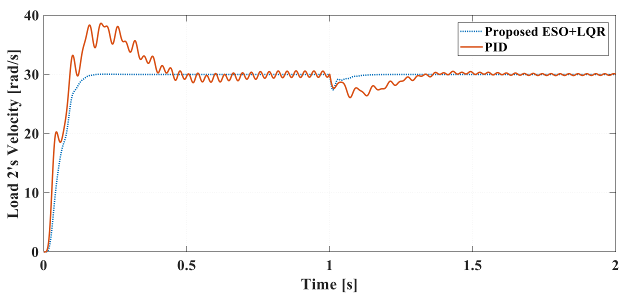
Figure 4: Speed estimation errors



(a) Motor velocity



(b) Load 1 velocity



(c) Load 2 velocity

Figure 5: Comparison between PID and the proposed controller

Finally, the robustness of the proposed control strategy against the parameter variation is test. The moment of inertial of the loads used in simulation is two times larger than the one used in control design. The simulation result in this case is shown in Figure 6. Although the system model is inaccurate, the system speed still tracks the reference one without vibration. However, the overshoot is unavoidable in this case. The reason may come from the transient response of the ESO.

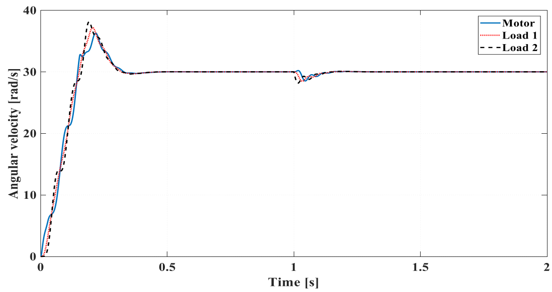


Figure 6: Velocity responses of the system with the proposed controller in case of parameter uncertainty.

Conclusion

In this paper, we proposed the LQR controller based ESO for a three-inertia system in order to achieve vibration suppression and disturbance rejection. The unknown states and load torque are estimated using ESO. Then, the state feedback and feedforward compensation for load torque are employed in control design. Theoretical analysis and numerical simulations show that the proposed control strategy guarantees the system stability, good performance and robustness with parameters variation as well as load disturbance. Particularly, the control strategy can easily be extended to be applied to any order inertia systems.

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